

LATENT HEAT OF EVAPORATION OF LIQUID He⁴ AND LIQUID He³.

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1. INTRODUCTION

The latent heat of evaporation of liquid He³ has been calculated by Abraham, Osborne and Weinstock (1950), using the Clausius-Clapeyron equation. The results, however, do not have the certainty of direct experimental observation, as the latent heat of evaporation of liquid He³ has not been determined experimentally. In order to form an idea about the accuracy of the theoretical results, we have calculated the latent heat of evaporation of liquid He⁴ by following the same procedure. The results thus obtained are compared with experimental measurements of Keesom (1942). We have then recalculated the latent heat of evaporation of liquid He³ by using recent values of virial coefficients as computed by Kilpatrick, Keller, Hammel and Metropolis (1954), and liquid densities of He³ as determined recently by Kerr (1954). The calculations have also been extended to low temperatures (1.0 to 0.0° K.) by making use of Chen and London (1953) vapour pressure equation. Using the latent heat values of He⁴ and He³ at absolute zero, the effective mass of He³ atom has been estimated.

2. LATENT HEAT OF LIQUID He⁴

According to Bleaney and Simon (1939), the pressure temperature relation obeyed by He⁴ in the region 0.0° K. to 1.6° K. is

$$\log_{10} p_{mm} = -\frac{3.117}{T_A} + 2.5 \log_{10} T_A + 2.196 - 2.27 \times 10^{-3} \cdot T_A^3 + \Delta, \quad \dots (1)$$

where Δ is a small correction term whose dependence on T_A is given graphically by Keesom (1942) and p_{mm} is in mm. of Hg at 0° C. Here T_A is the 'Agreed Temperature' calculated from the He⁴ pressure and the table given by van Dijk and Shoenberg (1949).

For the region from 1.6 to 4.3° K., Keesom and Lignac have given the following interpolation formula

$$\log_{10} p_{mm} = -\frac{4.7921}{T_A} + 0.00783 T_A + 0.017601 T_A^2 + 3.6730. \quad \dots (2)$$

For the region 4.3° K. to 5.2° K. (the critical point) experimentally determined data of p and T due to Kamerlingh Onnes and Weber can be represented, as pointed out by Squire (1953), by the interpolation formula

$$T = 2.967 + 1.905 \times 10^{-3} p_{mm} - 3.52 \times 10^{-7} p_{mm}^2. \quad \dots (3)$$

We have used for calculating (dp/dT_A) the vapour-pressure formulas given in equations (1) to (3) and the vapour-pressure data of liquid He⁴.

We have calculated V_g , the molar volume of He^4 in the gaseous state, by making use of the equation

$$pV_g = RT \left[1 + \frac{B(T)}{V_g} \right], \quad \dots \quad \dots \quad \dots \quad (4)$$

the values of second virial coefficient $B(T)$ being taken from the computations of Kilpatrick *et al.* (1954).

V_l , the molar volume of liquid He^4 , is obtained from Keesom's (1942) value for density under saturated vapour pressure. Then the latent heat of evaporation can be calculated by using the Clausius-Clapeyron equation:

$$\left(\frac{dP}{dT} \right) = \frac{L}{T(V_g - V_l)}. \quad \dots \quad \dots \quad \dots \quad (5)$$

The calculations of L could not be carried out for temperatures greater than 4.5°K . for lack of data of liquid densities for He^4 . The results are tabulated in Table I and are also shown in Figure 1.

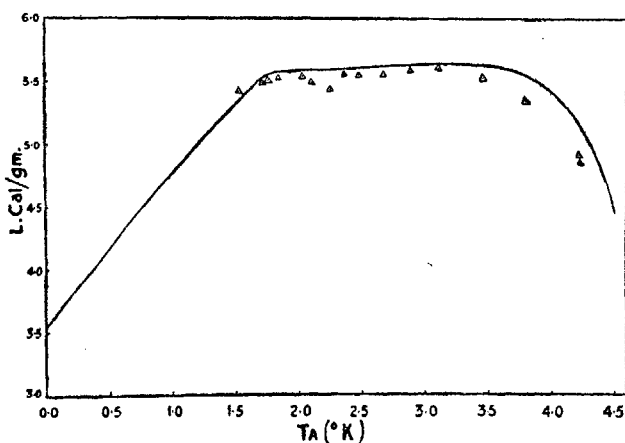


FIG. 1. Latent Heat of Evaporation of Liquid He^4 versus T_A °K. The curve indicates the calculated values, while the points represent experimental results of Keesom (1942).

TABLE I
Latent Heat of Evaporation of Liquid He^4 .

T_A °K.	T_k °K.	L cal./gm. (calculated).	L cal./gm. (experimental).
0.000	0.000	3.56
0.500	0.500	4.18
1.000	1.000	4.78
1.500	1.500	5.32	5.40
2.000	1.989	5.59	5.55
2.500	2.490	5.62	5.54
3.000	2.995	5.64	5.60
3.500	3.500	5.63	5.52
4.000	4.001	5.42	5.17
4.500	4.501	4.46

We notice from Fig. 1 that the calculated values are in good agreement with those determined experimentally. Table I also includes the theoretically extrapolated value of L at 0.0° K. which is 3.56 cal./gm.

3. LATENT HEAT OF LIQUID He³

The saturated vapour pressure of liquid He³ was observed by Abraham, Osborne and Weinstock (1950) in the region 1.0 to 3.3° K. The experimental data given by them are represented by the equation

$$\log_{10} p_{mm} = -\frac{0.97796}{T_K} + 2.5 \log_{10} T_K + 0.000302 T_K^3 + 1.91594, \quad \dots (6)$$

which is valid in the region 1.0 to 3.3° K. Here T_k is the temperature calculated by applying to T_A the corrections of Kistemaker (1946) smoothed as shown by Abraham *et al.* (1950).

The value of (dP/dT) is obtained from equation (6), while V_g , as before, is obtained from (4) making use of virial coefficient computations of Kilpatrick *et al.* (1954). The molar volume V_1 of the liquid is obtained from liquid densities measured by Kerr (1954). The heat of evaporation of He³ is then obtained from the thermodynamical relation (5).

No calculations have been made above 2.5° K. because the equation of state, as given by (4), is inadequate for the saturated vapour above this temperature.

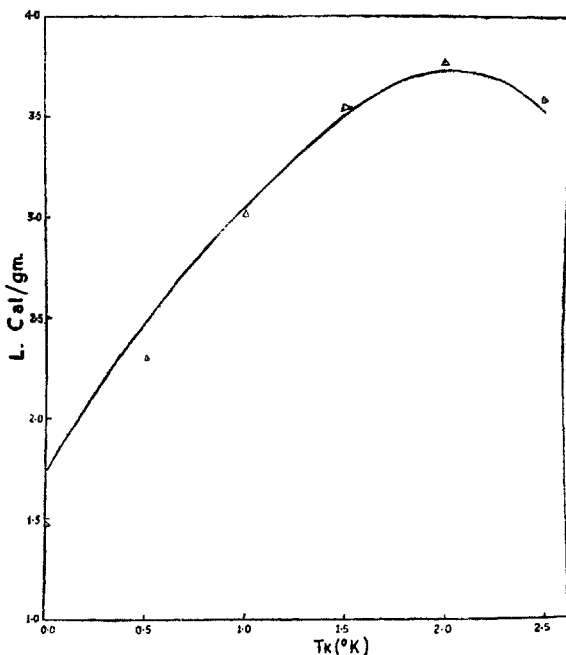


FIG. 2. Latent Heat of Evaporation of Liquid He³ versus T_k ° K. The curve indicates calculated values, while the points represent the computations by Abraham *et al.* (1950).

In order to extend the calculations of latent heat below 1.0° K. we make use of the Chen and London (1953) vapour-pressure equation

$$\log_{10} \left(\frac{p_{mm}}{T_K^{5/2}} \right) = 2.3126 - \frac{1.1561}{T_K} - 0.25254T_K - 0.00667T_K^2 + 0.05266T_K^3 - 0.01210T_K^4. \quad \dots (7)$$

This equation represents the vapour pressure measurements of Abraham *et al.* satisfactorily up to 2.5° K. The entropy curve of liquid He³ derived from it, in agreement with the third law of thermodynamics, goes smoothly to zero at 0° K. and does not lead to any residual entropy like equation (6) given earlier by Abraham *et al.*

The heat of vaporisation of He³ thus calculated is tabulated in Table 2 and is shown in Fig. 2. It has a maximum value of 3.72 cal./gm. at 2.0° K. Further, we conclude that at 0.0° K. the vaporisation heat is 1.75 cal./gm.

TABLE 2
Latent Heat of Evaporation of Liquid He³.

T_k ° K.	T_A ° K.	L cal./gm.	T_k ° K.	T_A ° K.	L cal./gm.
0.000	0.000	1.75	1.300	1.300	3.34
0.100	0.100	1.91	1.400	1.400	3.38
0.200	0.200	2.07	1.500	1.500	3.51
0.300	0.300	2.21	1.600	1.603	3.57
0.400	0.400	2.35	1.700	1.705	3.63
0.500	0.500	2.49	1.800	1.807	3.67
0.600	0.600	2.61	1.900	1.909	3.70
0.700	0.700	2.73	2.000	2.011	3.72
0.800	0.800	2.84	2.100	2.112	3.71
0.900	0.900	2.94	2.200	2.213	3.69
1.000	1.000	3.02	2.300	2.312	3.66
1.100	1.100	3.13	2.400	2.411	3.60
1.200	1.200	3.24	2.500	2.510	3.51

As has already been mentioned, the vapour-pressure equation proposed by Abraham *et al.* (1950) leads to a non-vanishing entropy for liquid He³ at the absolute zero. Thus it seems that the extrapolation of their curve to absolute zero is not quite justified and, in fact, various authors have tried to look for a transition in He³ below 1.0° K. in order to account for this non-vanishing entropy at zero degree Kelvin. On the other hand, no such difficulty exists with the Chen and London (1953) vapour-pressure equation. Now Fig. 2 clearly shows that our results (derived on the basis of the Chen and London equation) differ considerably from those of Abraham *et al.* (1950) below 1.0° K. Consequently, it appears that observation on the heat of vaporisation of He³ may provide conclusive evidence regarding the existence of a transition.

4. THE EFFECTIVE MASS OF HE³ ATOM

At the absolute zero the latent heat of He⁴ differs considerably from that of He³. This difference may, in part, be attributed to the 'Statistical Zero Point Energy' of He³ which in case of He⁴ is zero. Equating it to

$$\frac{3N}{5} \left\{ \frac{h^2}{2m_3^*} \cdot \left(\frac{3N}{8\pi V_3} \right)^{\frac{2}{3}} \right\}, \quad \dots \dots \dots (8)$$

(where N is Loschmidt's number and V_3 the molar volume of liquid He³ at 0° K.), we obtain the ratio of the effective to the actual mass of He³ atom as

$$m_3^*/m_3 = 0.66.$$

This is, however, a crude method of estimating the effective mass of He³ atom, because we have ignored the effect of the interactions. The evaluation of the effective mass, taking interactions into account, will be published in a subsequent paper.

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SUMMARY

Latent heat of evaporation of He⁴ and He³ is calculated by using the Clausius-Clapeyron equation. Since in case of He⁴, satisfactory agreement between the theoretical and the experimental latent heat values is observed, therefore the calculated He³ values can be used with confidence in further thermodynamical computations. The latent heat values for He³, as calculated by using Chen and London (1953) vapour-pressure equation, differ considerably from those calculated by Abraham *et al.* (1950) in the temperature range 1.0° K. to 0.0° K. Thus experimental results of latent heat of He³ below 1.0° K. will enable us to decide whether or not a transition in He³ exists below 1.0° K.

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