THE EQUATIONS OF MOTION OF PARTICLES IN THE UNIFIED FIELD THEORY OF EINSTEIN (1953)

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1. INTRODUCTION

As is well known the equations of motion of particles can be deduced from the field equations of general relativity by solving the latter so that the field is regular everywhere except at the particles. The usual method of approximation in general relativity utilizes the existence of the singularities of the field at the particles. The method followed by Einstein and co-workers (1938, 1949) is based on expansions in the powers of a parameter λ for the field potentials, the field equations having been split into terms according to the powers of λ. The split field equations demand integrability conditions. These integrability conditions are the equations of motion of the singularities of the field. Thus the equations of motion of the particles in general relativity turn out to be the integrability conditions of the field equations.

Einstein (1953) has postulated a set of field equations to represent the combined field of electromagnetism and gravitation. We find, on following the same technique for deriving the equations of motion, that these field equations also demand integrability conditions which are the required equations of motion. Our problem is to find out whether the equations of motion of the singularities contain terms representing Coulomb's law of force between charge particles and Newton's law of attraction between inertial particles.

Infeld (1951) and Callaway (1953) have applied the method of approximation to the sets of field equations postulated by Einstein in 1950 and 1953 respectively and shown that the equations of motion of charge particles, up to the first order of approximation, do not contain the term which accounts for Coulomb's law of force. We shall show here that a solution of the field equations (Einstein, 1953) can be found out so that the equations of motion of the singularities of the field contain a term accounting for Coulomb's law of force. The singularities considered by us here carry with them a charge cloud with density diminishing with distance and, in this respect, are different from those considered by Callaway.

2. PRELIMINARIES

The field equations postulated by Einstein (1953) are

$$\frac{\partial g_{\mu\nu}}{\partial x^\sigma} - g_{\rho\nu}\Gamma^\rho_{\mu\sigma} - g_{\mu\rho}\Gamma^\rho_{\sigma\nu} = 0, \quad \ldots \quad \ldots \quad \ldots \quad (2.1)$$

$$\partial(\sqrt{-g}\, g^{\underset{\vee}{\mu\nu}})/\partial x^\nu = 0, \quad \ldots \quad \ldots \quad \ldots \quad (2.2a)$$

$$\underset{\vee}{\Gamma^\sigma_{\mu\sigma}} = 0, \quad \ldots \quad \ldots \quad \ldots \quad (2.2b)$$

$$R_{\underline{\mu\nu}} = 0, \quad \ldots \quad \ldots \quad \ldots \quad (2.3)$$

$$\frac{\partial R_{\underset{\vee}{\mu\nu}}}{\partial x^{\sigma}} + \frac{\partial R_{\underset{\vee}{\sigma\mu}}}{\partial x^{\nu}} + \frac{\partial R_{\underset{\vee}{\nu\sigma}}}{\partial x^{\mu}} = 0, \qquad \ldots \quad \ldots \quad \ldots \quad (2.4)$$

where

$$g = \text{det.}\, |g_{\mu\nu}|,$$

$$g^{\mu\sigma} g_{\nu\sigma} = g^{\sigma\mu} g_{\sigma\nu} = \delta^{\mu}_{\nu},$$

$$R_{\mu\nu} \equiv R_{\underset{-}{\mu\nu}} + R_{\underset{\vee}{\mu\nu}} \equiv \frac{\partial \Gamma^{\sigma}_{\mu\nu}}{\partial x^{\sigma}} - \frac{1}{2}\left[\frac{\partial \Gamma^{\sigma}_{\underset{-}{\mu\sigma}}}{\partial x^{\nu}} + \frac{\partial \Gamma^{\sigma}_{\underset{-}{\nu\sigma}}}{\partial x^{\mu}}\right] - \Gamma^{\sigma}_{\mu\rho}\Gamma^{\rho}_{\sigma\nu} + \Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho}.$$

Here and in what follows the dummy suffix summation convention will be used. Greek indices refer to both space and time, running over the values 0, 1, 2, 3 and Latin indices refer to space co-ordinates only, running over the values 1, 2, 3. The suffix 0 refers to the time co-ordinate. A bar below two indices indicates symmetry and a hook in the same position indicates antisymmetry. $\Gamma^{\sigma}_{\mu\nu}$ are the affine connections to be determined by (2.1) in terms of the fundamental tensor $g_{\mu\nu}$ which consists of sixteen independent components. Equations (2.2*a*), (2.3) and (2.4) which are sixteen independent equations are to be solved for $g_{\mu\nu}$. The condition (2.2*b*) will be ensured by the equations (2.1) and (2.2*a*).

3. The method of approximation

The field equations will be solved for the case of quasi-stationary fields. For any field quantity ϕ, its derivatives with respect to the space co-ordinates x^s is of the order of ϕ while the derivative with respect to the time co-ordinate x^0 is of the order of $\lambda\phi$ where λ is of the order of smallness of v/c, v being a typical particle velocity and c the velocity of light. We define a new time $\tau (= \lambda x^0)$ so that the derivative of ϕ with respect to τ is of the order of ϕ. A derivative of ϕ with respect to (τ, x^s) is denoted by a comma:

$$\left.\begin{aligned} \frac{\partial \phi}{\partial \tau} &= \phi_{,0}\,; \\ \frac{\partial \phi}{\partial x^s} &= \phi_{,s}. \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3.1)$$

The expansion of the field variables $g_{\mu\nu}$ in powers of λ may be taken as given in (3.2*b*) (Einstein, 1949):

$$\left.\begin{aligned} g_{\mu\nu} &= g_{\underset{-}{\mu\nu}} + g_{\underset{\vee}{\mu\nu}}\,; \\ g_{\underset{-}{\mu\nu}} &= \eta_{\mu\nu} + h_{\mu\nu}\,; \\ \gamma_{\mu\nu} &= h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}\eta^{\rho\sigma}h_{\rho\sigma}\,; \\ \eta_{11} &= \eta_{22} = \eta_{33} = -\eta_{00} = -1\,; \\ \eta_{\mu\nu} &= 0,\ \mu \neq \nu\,; \\ \eta^{\mu\sigma}\eta_{\nu\sigma} &= \delta^{\mu}_{\nu}\,; \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad (3.2a)$$

$$\left.\begin{aligned}
\gamma_{00} &= \lambda^2\underset{2}{\gamma}_{00}+\lambda^4\underset{4}{\gamma}_{00}+\ldots.+\lambda^{2l}\underset{2l}{\gamma}_{00}+\ldots.;\\
\gamma_{0m} &= \lambda^3\underset{3}{\gamma}_{0m}+\lambda^5\underset{5}{\gamma}_{0m}+\ldots.+\lambda^{2l+1}\underset{2l+1}{\gamma}_{0m}+\ldots.;\\
\gamma_{mn} &= \lambda^4\underset{4}{\gamma}_{mn}+\lambda^6\underset{6}{\gamma}_{mn}+\ldots.+\lambda^{2l+2}\underset{2l+2}{\gamma}_{mn}+\ldots.;\\
\underset{\vee}{g}_{0m} &= \lambda^3\underset{3\,\vee}{f}_{0m}+\lambda^5\underset{5\,\vee}{f}_{0m}+\ldots.+\lambda^{2l+1}\underset{2l+1\,\vee}{f}_{0m}+\ldots.;\\
\underset{\vee}{g}_{mn} &= \lambda^2\underset{2\,\vee}{f}_{mn}+\lambda^4\underset{4\,\vee}{f}_{mn}+\ldots.+\lambda^{2l}\underset{2l\,\vee}{f}_{mn}+\ldots.;
\end{aligned}\right\} \qquad \ldots \quad (3.2b)$$

In the above formulae an index below an entity indicates its order of smallness. Now the field equations (2.2*a*), (2.3) and (2.4) are expanded in powers of λ by substituting (3.2) for $g_{\mu\nu}$. The coefficients of λ^l ($l = 1, 2, 3 \ldots.$) in the field equations must vanish if the field equations are to be satisfied. The field equations will be solved here up to the order of λ^3 to find out the equations of motion up to the order of λ^4. The field equations (2.2*a*), (2.3) and (2.4) up to the order of λ^3 give

$$\underset{2\,\vee}{f}_{mn,\,n} = 0, \qquad \ldots \quad (3.3a)$$

$$\underset{3\,\vee}{f}_{0m,\,m} = 0, \qquad \ldots \quad (3.3b)$$

$$\underset{2}{R}_{00} \equiv \tfrac{1}{4}\underset{2}{\gamma}_{00,\,ss} = 0, \qquad \ldots \quad (3.4a)$$

$$\underset{2\,-}{R}_{mn} \equiv \tfrac{1}{4}\delta_{mn}\underset{2}{\gamma}_{00,\,ss} = 0, \qquad \ldots \quad (3.4b)$$

$$\underset{3\,-}{R}_{0m} \equiv \tfrac{1}{2}\left(\underset{3}{\gamma}_{0m,\,ss}-\underset{3}{\gamma}_{0s,\,sm}+\underset{2}{\gamma}_{00,\,0m}\right) = 0, \qquad \ldots \quad (3.4c)$$

$$\tfrac{1}{6}\,\epsilon^{mnl0}\left(\underset{2\,\vee}{R}_{mn,\,l}+\underset{2\,\vee}{R}_{lm,\,n}+\underset{2\,\vee}{R}_{nl,\,m}\right)$$

$$= \tfrac{1}{12}\,\epsilon^{mnl0}\left(\underset{2\,\vee}{f}_{nm,\,l}+\underset{2\,\vee}{f}_{ml,\,n}+\underset{2\,\vee}{f}_{ln,\,m}\right)_{,\,ss} = 0, \qquad \ldots \quad (3.5a)$$

$$\tfrac{1}{2}\,\epsilon^{0mnl}\left(\underset{3\,\vee}{R}_{0m,\,n}+\underset{3\,\vee}{R}_{n0,\,m}+\underset{2\,\vee}{R}_{mn,\,0}\right)$$

$$= \tfrac{1}{4}\,\epsilon^{0mnl}\left(\underset{3\,\vee}{f}_{m0,\,n}+\underset{3\,\vee}{f}_{0n,\,m}+\underset{2\,\vee}{f}_{nm,\,0}\right) = 0, \qquad \ldots \quad (3.5b)$$

where δ_{mn} is Kronecker's delta symbol, $\epsilon^{\mu\nu\sigma\rho}$ is antisymmetric in all the four indices and ϵ^{1230} is unity.

The solution of the equations (3.3) and (3.4) is given by

$$\underset{2\,\vee}{f}_{mn} = \epsilon_{mns0}\underset{2}{\phi}_{0,\,s}, \qquad \ldots \quad (3.6a)$$

$$\underset{3\,\vee}{f}_{0m} = \epsilon_{mst0}\underset{3}{\phi}_{s,\,t}, \qquad \ldots \quad (3.6b)$$

$$\underset{2}{\gamma}_{00} = -4\sum_{k=1}^{N}\overset{k}{m}\overset{k}{\psi}, \qquad \ldots \quad (3.7a)$$

$$\underset{3}{\gamma}_{0s} = 4\sum_{k=1}^{N} \overset{k}{m}\, \overset{k}{\dot{\xi}}{}^{s}\, \overset{k}{\psi}, \qquad \ldots \qquad (3.7b)$$

$$\overset{k}{\psi} = \left[\left(x^{s}-\overset{k}{\xi}{}^{s}\right)\left(x^{s}-\overset{k}{\xi}{}^{s}\right)\right]^{-\frac{1}{2}},$$

$$\epsilon_{\mu\nu\sigma\rho} = \epsilon^{\mu\nu\sigma\rho},$$

where $\underset{2}{\phi}_0$, $\underset{3}{\phi}_s$ are arbitrary functions of the co-ordinates x^{μ}, $\overset{k}{m}$ are constants and $\overset{k}{\xi}{}^{s}$ are the space co-ordinates of the k *th* particle at time τ. Here it may be noted that an index above an entity stands for the distinguishing number of the particle and a dot above a function stands for a differentiation with respect to the time τ. By the relation (3.6), equations (3.5a) and (3.5b) reduce to

$$-\tfrac{1}{2}\underset{2}{\phi}_{0,\,ssrr} = 0, \qquad \ldots \qquad (3.8a)$$

$$-\tfrac{1}{2}\left(\underset{3}{\phi}_{r,\,ss} - \underset{3}{\phi}_{s,\,sr} - \underset{2}{\phi}_{0,\,0r}\right)_{,\,tt} = 0\,. \qquad \ldots \qquad (3.8b)$$

A solution of (3.8a) may be taken as

$$\underset{2}{\phi}_0 = \overset{k}{e}\,\overset{k}{\psi} + \tfrac{1}{2}\overset{k}{q}\,\overset{k}{r}, \qquad \ldots \qquad (3.9)$$

where $\overset{k}{e}$ and $\overset{k}{q}$ are functions of time only and $\overset{k}{r}$, usually called as the distance from x^{s} to $\overset{k}{\xi}{}^{s}$, is defined as

$$\overset{k}{r} = \left[\left(x^{s}-\overset{k}{\xi}{}^{s}\right)\left(x^{s}-\overset{k}{\xi}{}^{s}\right)\right]^{\frac{1}{2}}.$$

It may be seen from (3.6) that the addition of the gradient of any function to $\underset{3}{\phi}_s$ does not alter $\underset{3\,v}{f}_{0m}$. Such an addition is superfluous and may be considered as similar to the gauge transformation in the case of the classical electromagnetic theory. The integrability of (3.8b) demands

$$\overset{k}{\dot{q}} = 0\,. \qquad \ldots \qquad (3.10)$$

With the condition (3.10), a solution of (3.8b) may be given as

$$\underset{3}{\phi}_s = \left(\overset{k}{e}\,\overset{k}{\psi} + \tfrac{1}{2}\overset{k}{q}\,\overset{k}{r}\right)\overset{k}{\dot{\xi}}{}^{s}. \qquad \ldots \qquad (3.11)$$

Now we are in a position to find out the equations of motion of the particles in the combined field of gravitation and electromagnetism, the gravitational field being given by (3.7) and the electromagnetic field being given by (3.6), (3.9), (3.10) and (3.11). Up to this approximation, there is no interaction between electromagnetic and gravitational fields.

4. The equations of motion

The field equations that determine the motion of the particles are

$$\left.\begin{aligned} \underset{4}{R}_{mn} &= 0, \\ \underset{4}{R}_{00} &= 0. \end{aligned}\right\} \qquad \ldots \qquad (4.1)$$

We shall show now that the equations (4.1) demand integrability conditions. Equations (4.1) imply

$$\underset{4}{R}_{mn}+\tfrac{1}{2}\,\delta_{mn}\Big(\underset{4}{R}_{00}-\underset{4}{R}_{ss}\Big)=0. \quad \ldots \quad (4.2)$$

The equation (4.2) when written out explicitly gives

$$\begin{aligned}
&\tfrac{1}{2}\Big(\underset{4}{\gamma}_{mn,\,ss}-\underset{4}{\gamma}_{ms,\,sn}-\underset{4}{\gamma}_{ns,\,sm}+\delta_{mn}\,\underset{4}{\gamma}_{rs,\,rs}\Big)\\
&=\tfrac{1}{2}\Big(-\underset{3}{\gamma}_{0m,\,0n}-\underset{3}{\gamma}_{0n,\,0m}+2\,\delta_{mn}\,\underset{3}{\gamma}_{0s,\,0s}-\delta_{mn}\,\underset{2}{\gamma}_{00,\,00}\Big)\\
&-\tfrac{1}{4}\,\underset{2}{\gamma}_{00}\,\underset{2}{\gamma}_{00,\,mn}-\tfrac{1}{8}\,\underset{2}{\gamma}_{00,\,m}\,\underset{2}{\gamma}_{00,\,n}+\tfrac{3}{16}\,\delta_{mn}\,\underset{2}{\gamma}_{00,\,s}\,\underset{2}{\gamma}_{00,\,s}\\
&-\tfrac{1}{2}\,\delta_{mn}\,\underset{2}{\phi}_{0,\,rs}\,\underset{2}{\phi}_{0,\,rs}+\tfrac{1}{2}\,\underset{2\vee}{f}_{mr,\,s}\,\underset{2\vee}{f}_{ns,\,r}\\
&-\tfrac{1}{4}\,\delta_{mn}\,\underset{2}{\phi}_{0,\,ss}\,\underset{2}{\phi}_{0,\,rr}+\underset{2}{\phi}_{0,\,ss}\,\underset{2}{\phi}_{0,\,mn}\\
&+\underset{2}{\phi}_{0,\,s}\,\underset{2}{\phi}_{0,\,smn}-\tfrac{1}{2}\,\underset{2}{\phi}_{0,\,m}\,\underset{2}{\phi}_{0,\,nrr}\\
&-\tfrac{1}{2}\,\underset{2}{\phi}_{0,\,n}\,\underset{2}{\phi}_{0,\,mrr}-\tfrac{1}{2}\,\delta_{mn}\,\underset{2}{\phi}_{0,\,s}\,\underset{2}{\phi}_{0,\,stt} \quad \ldots \quad (4.3)
\end{aligned}$$

The equation (4.3) may be written as

$$\underset{4}{\Phi}_{mn}=\underset{4}{\Lambda}_{mn}, \quad \ldots \quad (4.4)$$

where $\underset{4}{\Phi}_{mn}$ stands for the expression on the left-hand side of the equation (4.3) and $\underset{4}{\Lambda}_{mn}$ stands for the expression on the right side of (4.3). It may be seen (Einstein, 1949) that

$$\left.\begin{aligned}
&\underset{4}{\Phi}_{mn,\,n}=0,\\
&\oint_{\overset{k}{S}}\underset{4}{\Phi}_{mn}\,n_n\,dS=0,
\end{aligned}\right\} \quad \ldots \quad (4.5)$$

where $\overset{k}{S}$ stands for a closed two-dimensional surface enclosing the *k th* singularity and n_n is the unit outward normal to the surface $\overset{k}{S}$. According to (4.4) and (4.5), $\underset{4}{\Lambda}_{mn}$ should satisfy the equations

$$\underset{4}{\Lambda}_{mn,\,n}=0, \quad \ldots \quad (4.6a)$$

$$\oint_{\overset{k}{S}}\underset{4}{\Lambda}_{mn}\,n_n\,dS=0. \quad \ldots \quad (4.6b)$$

$\underset{4}{\Lambda}_{mn}$ contains only known functions of the co-ordinates. It may be seen that (4.6*a*) is satisfied everywhere in the regular region. The equation (4.6*a*) ensures that the left-hand side of (4.6*b*) is independent of x^s. The integral on the left side of (4.6*b*) can be evaluated since all those terms that are contained in $\underset{4}{A}_{mn}$ are known functions of the co-ordinates. On calculation one finds that

$$\oint_{\overset{k}{S}} \underset{4}{A}_{mn}\, n_n\, dS = 8\Pi \left[\overset{k}{m}\, \ddot{\overset{k}{\xi}}{}^{m} - \sum_{p=1}^{N}{}' \overset{k}{m} \overset{p}{m} \overset{p,k}{\psi}{}_{,m} - \frac{1}{4}\sum_{p=1}^{N}{}' \overset{k}{e} \overset{p}{q} \overset{p,k}{\psi}{}_{,m} - \frac{1}{4}\sum_{p=1}^{N}{}' \overset{k}{q} \overset{p}{e} \overset{p,k}{\psi}{}_{,m} - \frac{1}{8}\sum_{p=1}^{N}{}' \overset{k}{q} \overset{p}{q} (r_{p,k})_{,m} \right], \quad .. \quad (4.7)$$

where

$$\overset{p,k}{\psi}{}_{,m} = \left(\overset{p}{\psi}{}_{,m}\right)(x^s = \overset{k}{\xi}{}^s),$$

$$(r_{p,k})_{,m} = \left(\overset{p}{r}{}_{,m}\right)(x^s = \overset{k}{\xi}{}^s),$$

and $\sum_{p=1}^{N}{}'$ stands for the summation for p from 1 to N excepting k.

Equations (4.6b) and (4.7) give

$$\overset{k}{m}\, \ddot{\overset{k}{\xi}}{}^{s} - \sum_{p=1}^{N}{}' \left[\overset{k}{m} \overset{p}{m} \overset{p,k}{\psi}{}_{,s} - \frac{1}{4} \overset{k}{e} \overset{p}{q} \overset{p,k}{\psi}{}_{,s} - \frac{1}{4} \overset{k}{q} \overset{p}{e} \overset{p,k}{\psi}{}_{,s} - \frac{1}{8} \overset{k}{q} \overset{p}{q} (r_{p,k})_{,s} \right] = 0 \quad .. \quad (4.8)$$

These are the equations of motion of the particles up to the order of λ^4.
By considering

$$\overset{k}{q} = q \overset{k}{e}, \quad .. \quad .. \quad .. \quad .. \quad (4.9)$$

one gets the equations of motion as

$$\overset{k}{m}\, \ddot{\overset{k}{\xi}}{}^{s} - \sum_{p=1}^{N}{}' \left[\overset{k}{m} \overset{p}{m} \overset{p,k}{\psi}{}_{,s} - \frac{q}{2} \overset{k}{e} \overset{p}{e} \overset{p,k}{\psi}{}_{,s} - \frac{q^2}{8} \overset{k}{e} \overset{p}{e} (r_{p,k})_{,s} \right] = 0. \quad .. \quad (4.10)$$

5. Discussion

The equation (4.10) may be compared to the equations of motion of a charge particle. The third term in (4.10) corresponds to Coulomb's law of force. When $\overset{k}{m}$, r and the distances between the particles are taken in the relativistic units q will be given by the equation (Bonnor, 1954)

$$q = 2k/c^4, \quad .. \quad .. \quad .. \quad .. \quad (5.1)$$

where k is the constant of gravitation and c the velocity of light. Now the equations of motion (4.10) may be written as

$$\overset{k}{m}\, \ddot{\overset{k}{\xi}}{}^{s} - \sum_{p=1}^{N}{}' \left[\overset{k}{m} \overset{p}{m} \overset{p,k}{\psi}{}_{,s} - \frac{k}{c^4} \overset{k}{e} \overset{p}{e} \overset{p,k}{\psi}{}_{,s} - \frac{k}{2c^8} \overset{k}{e} \overset{p}{e} (r_{p,k})_{,s} \right] = 0. \quad .. \quad (5.2)$$

The fourth term in (5.2) is of the order of k^2/c^8 and hence may be considered as small when compared to Coulomb's law of force. This additional term in the equation (5.2) is a correction to Coulomb's law of force which is to be tested by observation.

It was suggested by Einstein (1953) that the vector

$$\tfrac{1}{6}\,\epsilon^{\mu\nu\sigma\rho}\left(g_{\underset{\vee}{\mu\nu},\,\sigma}+g_{\underset{\vee}{\sigma\mu},\,\nu}+g_{\underset{\vee}{\nu\sigma},\,\mu}\right) \qquad \text{(5.3)}$$

may be identified with the electric current vector. There are several alternatives in this connection. For instance, a vector

$$\tfrac{1}{6}\,\epsilon^{\mu\nu\sigma\rho}\left(\alpha_{\underset{\vee}{\mu\nu},\,\sigma}+\alpha_{\underset{\vee}{\sigma\mu},\,\nu}+\alpha_{\underset{\vee}{\nu\sigma},\,\mu}\right)$$

may be identifiable with the current vector where $\alpha_{\underset{\vee}{\mu\nu}}$ may be any of the tensors constructed from the fundamental tensor $g_{\mu\nu}$.

By taking (5.3) as the current vector we can find out the charge distribution in this special case. The charge density σ_0 is given by the equation:

$$\sigma_0 = \tfrac{1}{6}\,\epsilon^{mnl0}\left(g_{\underset{\vee}{mn},\,l}+g_{\underset{\vee}{lm},\,n}+g_{\underset{\vee}{nl},\,m}\right). \qquad \text{(5.4)}$$

Up to the order of λ^2, σ_0 is given by the expression

$$\tfrac{1}{6}\,\epsilon^{mnl0}\left(\underset{2}{f}_{\underset{\vee}{mn},\,l}+\underset{2}{f}_{\underset{\vee}{lm},\,n}+\underset{2}{f}_{\underset{\vee}{nl},\,m}\right). \qquad \text{(5.5)}$$

According to the solutions (3.6), (3.9) and (4.9) the charge distribution is

$$\overset{k}{q}\,\overset{k}{e}\,\psi \equiv \overset{k}{q}\,\overset{k}{e}\big/\overset{k}{r}. \qquad \text{(5.6)}$$

Hence the equations (5.2) are the equations of motion of the apexes of charge distributions located at $\overset{k}{\xi}{}^{s}$.

With the identification of the vector (5.3) with the current vector it was shown by Schrödinger (1954) that the field equations of Einstein (1953) do not favour the existence of discrete charge particles. The solution given in this paper is a particular case of a charge distribution, the motion of the apexes being given by the equation (5.2).

Abstract

It has been shown that mass particles carrying charges can be represented in the 1953 version of Einstein's unified field theory provided each particle singularity is surrounded by a continuous distribution of charge. The equations of motion worked out according to the technique developed by Einstein and Infeld in 1949 for a pure gravitational field reveal the gravitational interaction as well as the expected Coulomb force.

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