MIXED NUCLEON-MESON CASCADES IN FINITE ABSORBERS

by S. Ganguly, *Department of Mathematics and Geophysics, Bengal Engineering College, Howrah*

(Communicated by S. K. Chakrabarty, F.N.I.)

(*Received November 5, 1955 ; read January 1, 1956*)

I. Introduction

Different processes for the production of π-mesons in nucleon-nucleus collisions have been worked out theoretically by various authors. They have proceeded on two different assumptions, viz. the 'plural production' and the 'multiple production'. Heisenberg (1939), Fermi (1950) and Lewis, Oppenheimer and Wouthuysen (1948) have put forward different models for the energy loss of an incident nucleon through the process of 'multiple production' of mesons. Heitler and Janossy (1949, 1950), Peng (1944), and Messel (1951, 1952) have advocated processes involving the 'plural theory' of meson production. Results of observation, however, do not conclusively exclude or corroborate either of these processes. On the contrary, these tend to point out that the actual process may be some form of a mixed multiple and plural production. Further improvements in the theoretical analysis is therefore desirable in order to properly correlate the observed data and thus get a clear picture of the generation and multiplication of mesons and nucleons.

Peng (1944) and Chakrabarty (1947) obtained theoretical results for the spectra of mesons and nucleons on the assumption that no showers are initiated by the mesons produced. They derived cross-sections for the energy loss of a nucleon from the meson theories and Chakrabarty also included in his scheme the loss in the number of mesons through the decay process. Heitler and Janossy (1949) proceeded in a phenomenological way and obtained the cross-sections for the energy loss through meson production. Heitler and Janossy (1950) further showed that when the generated mesons have energies greater than 10^9 e.v., they are capable of starting new showers. Messel, Potts and McCusker (1952) worked out detailed theoretical results on the number of mesons and nucleons in a mixed meson-nucleon cascade, when the primary suffers a collision with a finite nucleus. In their scheme, the production of recoil nucleons was also taken into account. They, however, did not take into consideration the effect of the decay of mesons, as its effect was not appreciable for passage through a finite nucleus. It makes, however, a significant contribution in the case of cascades in light elements and more so in air and other gases. The size of the showers will depend to a great extent on the value of the decay constant. In the present paper, we have worked out the effect of the decay of mesons on the sizes of the nucleon and meson showers. The results show clearly the effect of the decay on the intensities of the low and high energy particles. In a later section, we have given the average number of mesons and nucleons produced by a primary of given energy at different depths. Expressions for the energy spectra of mesons and nucleons have also been given and the results have been put in a form which indicate the effect of meson decay in the production of showers.

II. The diffusion equations

As in the analysis of Messel, Potts and McCusker (1952) we have also assumed that, in the high energy regions, the generated mesons will interact with nucleons and give rise to secondary mesons and recoil nucleons. We have, therefore, considered two processes, viz.

(A) meson+nucleon→meson+meson+nucleon,
(B) nucleon+nucleon→nucleon+nucleon+meson.

The mesons and nucleons created may cascade in the nucleus and also in dispersed matter. Let $F\left(\frac{E_1}{E}, \frac{E_2}{E}\right)\frac{dE_1 dE_2}{E^2}$ be the cross-section for the production by a nucleon of energy E colliding with another nucleon, of two nucleons in the energy ranges E_1, E_1+dE_1, and E_2, E_2+dE_2, and a meson of energy $E-E_1-E_2$. Then following *MPM*, we have

$$F\left(\frac{E_1}{E}, \frac{E_2}{E}\right) = \frac{120 E_1 E_2}{E^2}\left(1-\frac{E_1}{E}-\frac{E_2}{E}\right). \quad \ldots \quad \ldots \quad (1)$$

Let

$$k\left(\frac{E''}{E'}\right)\frac{dE''}{E'} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

be the cross-section for the production by a meson of energy E' of a nucleon of energy between E'' and $E''+dE''$ and

$$l\left(\frac{E''}{E'}\right)\frac{E''}{E'} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

be the cross-section for the production of a meson of energy between E'' and $E''+dE''$ by a meson of energy E' in a meson-nucleon collision.

Consider a layer of the substance on which the primary of energy E_0 impinges normally. Let $n^{(i)}(E, x)dE$ be the average number of nucleons with energies in the range E, $E+dE$ at a depth x collision units in homogeneous nuclear matter due to a primary particle (i) (when $i=1$, it is a nucleon, and when $i=2$, it is a meson). Let $\pi^{(i)}(E, x)dE$ be the corresponding quantity for the π-mesons, which also includes uncharged mesons and the number of charged mesons in a shower will be taken as two-thirds of the total number. For the protons again, we can roughly take a value equal to half of the theoretically calculated value for the nucleons.

We have thus for the diffusion equations

$$\frac{\partial}{\partial x}n^{(i)}(E,x)+n^{(i)}(E,x) = \int_E^\infty n^{(i)}(E',x)\bar{F}\left(\frac{E}{E'}\right)\frac{dE'}{E'} + \int_E^\infty \pi^{(i)}(E',x)\, k\left(\frac{E}{E'}\right)\frac{dE'}{E'} \quad \ldots \quad (4)$$

$$\frac{\partial}{\partial x}\pi^{(i)}(E,x)+\pi^{(i)}(E,x) = \tfrac{1}{2}\int_E^\infty n^{(i)}(E',x)\bar{F}\left(\frac{E}{E'}\right)\frac{dE'}{E'} + \int_E^\infty \pi^{(i)}(E',x)\, l\left(\frac{E}{E'}\right)\frac{dE'}{E'} - \frac{b}{E}\pi^{(i)}(E,x) \quad \ldots \quad (5)$$

where

$$\bar{F}(\eta_1) = \int_0^{1-\eta_1} \{F(\eta_1, \eta_2)+F(\eta_2, \eta_1)\}\, d\eta_2$$

$$b = \frac{l}{c\tau_\pi}$$

l = length in cm. of the matter corresponding to one collision unit.

c = velocity of light.

τ_π = average lifetime of the π-mesons.

$= 2{\cdot}56\times 10^{-8}$ secs.

The energies and cross-sections are all expressed in natural meson units. The values of l and b are given below for some elements.

TABLE I

	Pb	Al	O_2	N_2
l in cm.	7·4	31·2	58800	67300
b	0·01	0·04	75·36	86·21

We define two functions $p^{(i)}(s, x)$, $q^{(i)}(s, x)$ such that

$$\left.\begin{aligned} p^{(i)}(s, x) &= \int_0^\infty \left(\frac{E}{E_0}\right)^{s-1} n^{(i)}(E, x)dE \\ q^{(i)}(s, x) &= \int_0^\infty \left(\frac{E}{E_0}\right)^{s-1} \pi^{(i)}(E, x)dE \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

By Mellins' transform, we have

$$\left.\begin{aligned} n^{(i)}(E, x) &= \frac{1}{2\pi i E_0}\int_{\sigma_1-i\infty}^{\sigma_1+i\infty} \left(\frac{E_0}{E}\right)^s p^{(i)}(s, x)\, ds \\ \pi^{(i)}(E, x) &= \frac{1}{2\pi i E_0}\int_{\sigma_2-i\infty}^{\sigma_2+i\infty} \left(\frac{E_0}{E}\right)^s q^{(i)}(s, x)\, ds \end{aligned}\right\} \quad \ldots \quad \ldots \quad (7)$$

where σ_1 and σ_2 are such that when $R(s)>\sigma_1$, σ_2, $p^{(i)}(s, x)$, $q^{(i)}(s, x)$ are analytic. We thus have

$$\frac{\partial}{\partial x}p^{(i)}(s, x)+[1-A(s)]\,p^{(i)}(s, x) = q^{(i)}(s, x)B(s) \quad \ldots \quad \ldots \quad (8)$$

$$\frac{\partial}{\partial x}q^{(i)}(s, x)+[1-C(s)]\,q^{(i)}(s, x) = \tfrac{1}{2}A(s)p^{(i)}(s, x) - \frac{b}{E_0}q^{(i)}(s-1, x) \quad \ldots \quad \ldots \quad (9)$$

where

$$\left.\begin{aligned} A(s) &= \int_0^1 z^{s-1}\overline{F}(z)\,dz \\ B(s) &= \int_0^1 z^{s-1}k\,(z)\,dz \\ C(s) &= \int_0^1 z^{s-1}l(z)\,dz \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

Eliminating $p^{(i)}(s, x)$ between equations (8) and (9), we obtain

$$\frac{\partial^2}{\partial x^2} q^{(i)}(s, x)+[2-A(s)-C(s)]\frac{\partial}{\partial x} q^{(i)}(s, x)$$

$$+\Big\{[1-A(s)][1-C(s)]-\tfrac{1}{2}A(s)B(s)\Big\}q^{(i)}(s, x)$$

$$=-\frac{b}{E_0}\Big\{\frac{\partial}{\partial x}q^{(i)}(s-1, x)+[1-A(s)]\,q^{(i)}(s-1, x)\Big\} \quad \ldots \quad (11)$$

The solution $q^{(i)}(s, x)$ of (11) when the decay of mesons is neglected is given by

$$q_0^{(i)}(s, x) = G(s)e^{\lambda_1(s)x}+H(s)e^{\lambda_2(s)x} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

where

$$\begin{matrix}\lambda_1(s)\\ \lambda_2(s)\end{matrix} = \frac{-[2-A(s)-C(s)]\pm\sqrt{\{A(s)-C(s)\}^2+2A(s)B(s)}}{2} \quad \ldots \quad (13)$$

The complete solution of (11) is given by

$$q^{(i)}(s, x) = q_0^{(i)}(s, x)+q_p^{(i)}(s, x) \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

where

$$q_p^{(i)}\,(s, x) = -\frac{b}{E_0}\frac{1}{\lambda_1(s)-\lambda_2(s)}\Big[q^{(i)}\,(s-1, 0)\Big\{e^{\lambda_2(s)x}-e^{\lambda_1(s)x}\Big\}$$

$$+\Big\{\lambda_1(s)+1-A(s)\Big\}e^{\lambda_1(s)x}\int_0^x q^{(i)}(s-1, x')e^{-\lambda_1(s)x'}\,dx'$$

$$-\Big\{\lambda_2(s)+1-A(s)\Big\}e^{\lambda_2(s)x}\int_0^x q^{(i)}(s-1, x')e^{-\lambda_2(s)x'}\,dx'\Big] \quad \ldots \quad (15)$$

Assume further that

$$q^{(i)}(s, x) = \sum_{n=0}^{\infty} q_n^{(i)}\,(s, x)\left(-\frac{b}{E_0}\right)^n \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

It can then be shown that

$$q_1^{(i)}(s, x) = \frac{1}{\lambda_1(s)-\lambda_2(s)} \Big[q^{(i)}(s-1, 0)\Big\{e^{\lambda_2(s)x} - e^{\lambda_1(s)x}\Big\} + \Big\{\lambda_1(s)+1-A(s)\Big\} e^{\lambda_1(s)x} \int_0^x e^{-\lambda_1(s)x'} q_0^{(i)}(s-1, x')\, dx' - \Big\{\lambda_2(s)+1-A(s)\Big\} e^{\lambda_2(s)x} \int_0^x e^{-\lambda_2(s)x'} q_0^{(i)}(s-1, x')\, dx'\Big] \quad (17)$$

and

$$q_n^{(i)}(s, x) = \frac{1}{\lambda_1(s)-\lambda_2(s)} \Big[\Big\{\lambda_1(s)+1-A(s)\Big\} e^{\lambda_1(s)x} \int_0^x e^{-\lambda_1(s)x'} q_{n-1}^{(i)}(s-1, x')\, dx' - \Big\{\lambda_2(s)+1-A(s)\Big\} e^{\lambda_2(s)x} \int_0^x e^{-\lambda_2(s)x'} q_{n-1}^{(i)}(s-1, x')\, dx'\Big] \quad \ldots \quad \ldots \quad (18)$$

when $n > 1$.

We then have

$$\pi^{(i)}(E, x) = \frac{1}{2\pi i E_0} \int_{\sigma_1 - i\infty}^{\sigma_1 + i\infty} \left(\frac{E_0}{E}\right)^s \sum_{n=0}^{\infty} q_n^{(i)}(s, x) \left(-\frac{b}{E_0}\right)^n ds \quad \ldots \quad (19)$$

Proceeding in a similar way it can be shown that

$$p^{(i)}(s, x) = \sum_{n=0}^{\infty} p_n^{(i)}(s, x) \left(-\frac{b}{E_0}\right)^n \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where

$$p_0^{(i)}(s, x) = K(s)e^{\lambda_1(s)x} + M(s)e^{\lambda_2(s)x} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

$$p_1^{(i)}(s, x) = \frac{B(s)}{B(s-1)[\lambda_1(s)-\lambda_2(s)]} \cdot \Big[p^{(i)}(s-1, 0)\Big\{e^{\lambda_2(s)x} - e^{\lambda_1(s)x}\Big\} + \Big\{\lambda_1(s)+1-A(s-1)\Big\} e^{\lambda_1(s)x} \int_0^x e^{-\lambda_1(s)x'} p_0^{(i)}(s-1, x')\, dx' - \Big\{\lambda_2(s)+1-A(s-1)\Big\} e^{\lambda_2(s)x} \int_0^x e^{-\lambda_2(s)x'} p_0^{(i)}(s-1, x')\, dx'\Big] \quad (22)$$

$$p_n^{(i)}(s, x) = \frac{B(s)}{B(s-1)[\lambda_1(s)-\lambda_2(s)]} \Big[\{\lambda_1(s)+1-A(s-1)\}e^{\lambda_1(s)x} \times \int_0^x e^{-\lambda_1(s)x'} p_{n-1}^{(i)}(s-1, x')\, dx' - \{\lambda_2(s)+1-A(s-1)\}e^{\lambda_2(s)x} \times \int_0^x e^{-\lambda_2(s)x'} p_{n-1}^{(i)}(s-1, x')\, dx'\Big] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

when $$n>1$$

where $K(s)$ and $M(s)$ are functions of s and depend on the boundary conditions. We have then

$$n^{(i)}(E, x) = \frac{1}{2\pi i E_0} \int_{\sigma_2 - i\infty}^{\sigma_2 + i\infty} \left(\frac{E_0}{E}\right)^s \sum_{n=0}^{\infty} p_n^{(i)}(s, x)\left(-\frac{b}{E_0}\right)^n ds \quad \ldots \quad (24)$$

For a primary nucleon, we have

$$\left.\begin{aligned} n^{(1)}(E, O) &= \delta(E - E_0) \\ \pi^{(1)}(E, O) &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

and for a primary meson, we have

$$\left.\begin{aligned} n^{(2)}(E, O) &= 0 \\ \pi^{(2)}(E, O) &= \delta(E - E_0) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

From (12) and (21), we thus get for a primary nucleon

$$\left.\begin{aligned} G(s) &= \frac{A(s)}{2[\lambda_1(s) - \lambda_2(s)]}, \qquad K(s) = -\frac{\lambda_2(s) + 1 - A(s)}{\lambda_1(s) - \lambda_2(s)} \\ H(s) &= \frac{A(s)}{2[\lambda_2(s) - \lambda_1(s)]}, \qquad M(s) = -\frac{\lambda_1(s) + 1 - A(s)}{\lambda_2(s) - \lambda_1(s)} \end{aligned}\right\} \quad \ldots \quad (27)$$

and when the primary is a meson

$$\left.\begin{aligned} G(s) &= -\frac{\lambda_2(s) + 1 - C(s)}{\lambda_1(s) - \lambda_2(s)}, \quad K(s) = \frac{B(s)}{\lambda_1(s) - \lambda_2(s)} \\ H(s) &= -\frac{\lambda_1(s) + 1 - C(s)}{\lambda_2(s) - \lambda_1(s)}, \quad M(s) = \frac{B(s)}{\lambda_2(s) - \lambda_1(s)} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (28)$$

The solutions (19) and (24) have been obtained for path in homogeneous nuclear matter. These are absolutely and uniformly convergent. For the calculation of the average number of mesons and nucleons in a finite absorber at any depth θ gm./cm.^{2}, we find from (17) and (18) that

$$q_n^{(i)}(s, x) = \sum_r \phi_{nr}^{(i)}(s) e^{-a_{nr}^{(i)}(s)x} \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

where the functions $\phi_{nr}^{(i)}(s)$ do not depend on x, and can be worked out from the recurrence formulae (17) and (18). We then have

$$\pi^{(i)}(E, x) = \frac{1}{2\pi i E_0} \int_{\sigma_1 - i\infty}^{\sigma_1 + i\infty} \left(\frac{E_0}{E}\right)^s \sum_{n=0}^{\infty} \sum_r \phi_{nr}^{(i)}(s) e^{-a_{nr}^{(i)}(s)x} \left(-\frac{b}{E_0}\right)^n ds \quad \ldots \quad \ldots \quad (30)$$

If we use the length of the nuclear diameter as a unit, we have

$$\pi^{(i)}(E, X) = \frac{1}{2\pi i E_0} \int_{s_0 - i\infty}^{s_0 + i\infty} \left(\frac{E_0}{E}\right)^s \sum_{n=0}^{\infty} \sum_r \phi_{nr}^{(i)}(s) e^{-a_{nr}^{(i)}(s) D_A X} \left(-\frac{b}{E_0}\right)^n ds$$

where D_A is the length of the nuclear diameter and may be taken as 3·7 and 8·8 for N_2 and Pb respectively.

In a finite absorber we have

$$\pi^{(i)}(E, \bar{p}) = \frac{1}{2\pi i E_0} \int_{\sigma - i\infty}^{\sigma + i\infty} \left(\frac{E_0}{E}\right)^s \sum_{n=0}^{\infty} \left(-\frac{b}{E_0}\right)^n \sum_r \phi_{nr}^{(i)}(s) e^{-\bar{p}f\left[a_{nr}^{(i)}(s)D_A\right]} ds \quad .. \quad (31)$$

where

$$f(\lambda) = 1 - 2\frac{1-(1+\lambda)e^{-\lambda}}{\lambda^2} \quad .. \quad .. \quad .. \quad (32)$$

$\bar{p} = \theta n \phi_A$, giving the average number of collisions when passing through a thickness θ gm./cm.^{2}; n is the number of nuclei per gram and ϕ_A is the geometrical cross-section of the nucleus.

In the table below, values of θ for different elements have been given for $\bar{p} = 1$.

TABLE II

	Pb	Al	O_2	N_2
θ in $\frac{\text{gm.}}{\text{cm.}^2}$	158·6	80·4	67·5	64·6

If $\Pi^{(i)}(E, \bar{p})$ gives the total number of mesons of energies greater than E at a depth $\bar{p}$, we have

$$\Pi^{(i)}(E, \bar{p}) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \left(\frac{E_0}{E}\right)^{s-1} \frac{ds}{s-1} \sum_{n=0}^{\infty} \left(-\frac{b}{E_0}\right)^n \sum_r \phi_{nr}^{(i)}(s)\, e^{-\bar{p}f\left[a_{nr}^{(i)}(s)D_A\right]} \quad (33)$$

Similarly $N^{(i)}(E, \bar{p})$, the total number of nucleons of energies greater than E at a depth $\bar{p}$, is given by

$$N^{(i)}(E, \bar{p}) = \frac{1}{2\pi i} \int_{t - i\infty}^{t + i\infty} \left(\frac{E_0}{E}\right)^{s-1} \frac{ds}{s-1} \sum_{n=0}^{\infty} \left(-\frac{b}{E_0}\right)^n \sum_r \psi_{nr}^{(i)}(s)\, e^{-\bar{p}f\left[b_{nr}^{(i)}(s)D_A\right]} \quad (34)$$

where

$$p_n^{(i)}(s, x) = \sum_r \psi_{nr}^{(i)}(s)\, e^{-b_{nr}^{(i)}(s)\, x} \quad .. \quad .. \quad .. \quad (35)$$

and $\psi_{nr}^{(i)}(s)$ are functions of s which can be determined from the recurrence relations (22) and (23).

Equation (33) may be written in the form

$$\Pi^{(i)}(E, \bar{p}) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \left(\frac{E_0}{E}\right)^{s-1} \frac{ds}{s-1} \sum_{n=0}^{\infty} \left(-\frac{b}{E_0}\right)^n Q_n^{(i)}(s, \bar{p}) \quad .. \quad (36)$$

where

$$Q_n^{(i)}(s, \bar{p}) = \sum_r \phi_{nr}^{(i)}(s)\, e^{-\bar{p}f\left[a_{nr}^{(i)}(s)\, D_A\right]} \quad .. \quad .. \quad (37)$$

Writing $(s+n)$ for s and shifting the contour we have

$$\Pi^{(i)}(E,\bar{p}) = \frac{1}{2\pi i}\int_c \left(\frac{E_0}{E}\right)^{s-1} \sum_{n=0}^{\infty} \frac{1}{s+n-1}\left(-\frac{b}{E}\right)^n Q_n^{(i)}(s+n,\bar{p})\,ds.$$

We now let

$$\left.\begin{aligned} a_n^{(i)}(s,\bar{p}) &= \frac{\Gamma(s)}{\Gamma(s+n)} Q_n^{(i)}(s+n,\bar{p}) \\ g^{(i)}(s,\bar{p}) &= \frac{a_1^{(i)}(s,\bar{p})}{a_0^{(i)}(s,\bar{p})} \\ f_n^{(i)}(s,\bar{p}) &= \left\{a_0^{(i)}(s,\bar{p})\frac{\left[g^{(i)}(s,\bar{p})\right]^n}{n!} - a_1^{(i)}(s,\bar{p})\frac{\left[g^{(i)}(s,\bar{p})\right]^{n-1}}{(n-1)!} + \cdots\right\} \end{aligned}\right\} \qquad (38)$$

We thus arrive at the result

$$\Pi^{(i)}(E,\bar{p}) = \frac{1}{2\pi i}\int_C \left(\frac{E_0}{E+bg^{(i)}}\right)^{s-1} \sum_{n=0}^{\infty} \frac{1}{s+n-1}\left(\frac{b}{E+bg^{(i)}}\right)^n \frac{\Gamma(s+n)}{\Gamma(s)} f_n^{(i)}(s,\bar{p})\,ds \qquad (39)$$

From equation (34), we get in a similar manner

$$N^{(i)}(E,\bar{p}) = \frac{1}{2\pi i}\int_{C'} \left(\frac{E_0}{E+b\,h^{(i)}}\right)^{s-1} \sum_{n=0}^{\infty} \frac{1}{s+n-1}\left(\frac{b}{E+b\,h^{(i)}}\right)^n \frac{\Gamma(s+n)}{\Gamma(s)}\, l_n^{(i)}(s,\bar{p})\,ds \quad (40)$$

where

$$\left.\begin{aligned} h^{(i)}(s,\bar{p}) &= \frac{d_1^{(i)}(s,\bar{p})}{d_0^{(i)}(s,\bar{p})} \\ l_n^{(i)}(s,\bar{p}) &= \left\{d_0^{(i)}(s,\bar{p})\frac{[h^{(i)}(s,\bar{p})]^n}{n!} - d_1^{(i)}(s,\bar{p})\frac{[h^{(i)}(s,\bar{p})]^{n-1}}{(n-1)!} + \cdots\right\} \\ d_n^{(i)}(s,\bar{p}) &= \frac{\Gamma(s)}{\Gamma(s+n)} \cdot P_n^{(i)}(s+n,\bar{p}) \end{aligned}\right\} \quad \ldots \quad (41)$$

$$P_n^{(i)}(s,\bar{p}) = \sum_r \psi_{nr}^{(i)}(s) e^{-\bar{p}f\left[b_{nr}^{(i)}(s)D_A\right]} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

III. Numerical evaluation

Following *MPM* (1952) we also take

$$\left.\begin{aligned} k(z) &= \tfrac{1}{2}\bar{F}(z) \\ l(z) &= \bar{F}(z) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

and hence we get

$$\left.\begin{aligned} A(s) &= 40\times\left[\frac{1}{s+1}-\frac{3}{s+2}+\frac{3}{s+3}-\frac{1}{s+4}\right] \\ B(s) &= \tfrac{1}{2}A(s) \\ C(s) &= A(s) \\ \lambda_1(s),\lambda_2(s) &= -1+\frac{3A(s)}{2},\ -1+\frac{A(s)}{2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (44)$$

By the help of (44), equations (27) and (28) reduce to

$$\left.\begin{aligned} G(s) &= \tfrac{1}{2}, \quad K(s) = \tfrac{1}{2} \\ H(s) &= -\tfrac{1}{2}, \quad M(s) = \tfrac{1}{2} \end{aligned}\right\} \quad \ldots \quad (45)$$

for a primary nucleon, and

$$\left.\begin{aligned} G(s) &= \tfrac{1}{2}, \quad K(s) = \tfrac{1}{2} \\ H(s) &= \tfrac{1}{2}, \quad M(s) = -\tfrac{1}{2} \end{aligned}\right\} \quad \ldots \quad (46)$$

for a primary meson.

For the actual numerical work, we take only two terms in (33) and (34), where the second term in each, as we have checked, contributes less than 4% of the first term, for the case of Pb. Collecting only the first two terms we arrive after some simplifications at the following results:—

$$\begin{aligned} \Pi^{(i)}(E,\bar{p}) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\left(\frac{E_0}{E}\right)^{s-1}\frac{ds}{s-1}\Bigg[&\left\{G^{(i)}(s)+\frac{b}{E_0}\left(\frac{\delta_{2,i}}{\lambda_1(s)-\lambda_2(s)}+F_3(s)G^{(i)}(s-1)\right)\right. \\ &\left.+F_4(s)H^{(i)}(s-1)\right\}e^{-\bar{p}f[-\lambda_1(s)D_A]}-\frac{b}{E_0}G^{(i)}(s-1)F_1(s)e^{-\bar{p}f[-\lambda_1(s-1)D_A]} \\ &+\left\{H^{(i)}(s)-\frac{b}{E_0}\left(\frac{\delta_{2,i}}{\lambda_1(s)-\lambda_2(s)}+F_5(s)G^{(i)}(s-1)+F_6(s)H^{(i)}(s-1)\right)\right\}e^{-\bar{p}f[-\lambda_2(s)D_A]} \\ &-\frac{b}{E_0}H^{(i)}(s-1)F_2(s)e^{-\bar{p}f[-\lambda_2(s-1)D_A]}\Bigg] \quad \ldots \quad (47) \end{aligned}$$

$$\begin{aligned} N^{(i)}(E,\bar{p}) = \frac{1}{2\pi i}\int_{t-i\infty}^{t+i\infty}\left(\frac{E_0}{E}\right)^{s-1}\frac{ds}{s-1}\Bigg[&\left(K^{(i)}(s)+\frac{b}{E_0}\left\{\frac{A(s)}{A(s-1)}\cdot\frac{\delta_{1,i}}{\lambda_1(s)-\lambda_2(s)}\right.\right. \\ &\left.\left.+D_3(s)K^{(i)}(s-1)+D_4(s)M^{(i)}(s-1)\right\}\right)e^{-\bar{p}f[-\lambda_1(s)D_A]} \\ &+\left(M^{(i)}(s)-\frac{b}{E_0}\left\{\frac{A(s)}{A(s-1)}\cdot\frac{\delta_{1,i}}{\lambda_1(s)-\lambda_2(s)}+D_5(s)K^{(i)}(s-1)\right.\right. \\ &\left.\left.+D_6(s)M^{(i)}(s-1)\right\}\right)e^{-\bar{p}f[-\lambda_2(s)D_A]} \\ &-\frac{b}{E_0}K^{(i)}(s-1)D_1(s)e^{-\bar{p}f[-\lambda_1(s-1)D_A]} \\ &-\frac{b}{E_0}M^{(i)}(s-1)D_2(s)e^{-\bar{p}f[-\lambda_2(s-1)D_A]}\Bigg] \quad \ldots \quad (48) \end{aligned}$$

where $G^{(i)}(s)$, $H^{(i)}(s)$ are different for different primaries so that $i = 1$, and $i = 2$, correspond to a primary nucleon and a primary meson and their values are given by (45) and (46). We have also

$$\left.\begin{aligned}
F_1(s) &= \tfrac{2}{3}\frac{3A(s-1)-2A(s)}{[A(s-1)-A(s)][3A(s-1)-A(s)]}; & F_4(s) &= \frac{1}{A(s-1)-3A(s)} \\
F_2(s) &= \frac{2[A(s-1)-2A(s)]}{[A(s-1)-3A(s)][A(s-1)-A(s)]}; & F_5(s) &= -\frac{1}{3A(s-1)-A(s)} \\
F_3(s) &= \frac{1}{3[A(s-1)-A(s)]}; & F_6(s) &= -\frac{1}{A(s-1)-A(s)}
\end{aligned}\right\} \quad (49a)$$

$$\left.\begin{aligned}
D_1(s) &= \frac{2A(s)}{3[A(s-1)-A(s)][3A(s-1)-A(s)]}; & D_4(s) &= \frac{3A(s)-2A(s-1)}{A(s-1)[A(s-1)-3A(s)]} \\
D_2(s) &= \frac{-2A(s)}{[A(s-1)-3A(s)][A(s-1)-A(s)]}; & D_5(s) &= \frac{A(s)-2A(s-1)}{A(s-1)[3A(s-1)-A(s)]} \\
D_3(s) &= \frac{3A(s)-2A(s-1)}{3A(s-1)[A(s-1)-A(s)]}; & D_6(s) &= \frac{A(s)-2A(s-1)}{A(s-1)[A(s-1)-A(s)]}
\end{aligned}\right\} \quad (49b)$$

The integrals in (47) and (48) have been evaluated by the saddle point method and the results are given in Tables III and IV. We have also plotted in Fig. 1 the values of $\log_e N^{(1)}(E, \bar{p})$ against the thickness of the material traversed for some different values of $\log_e \frac{E_0}{E}$ and E.

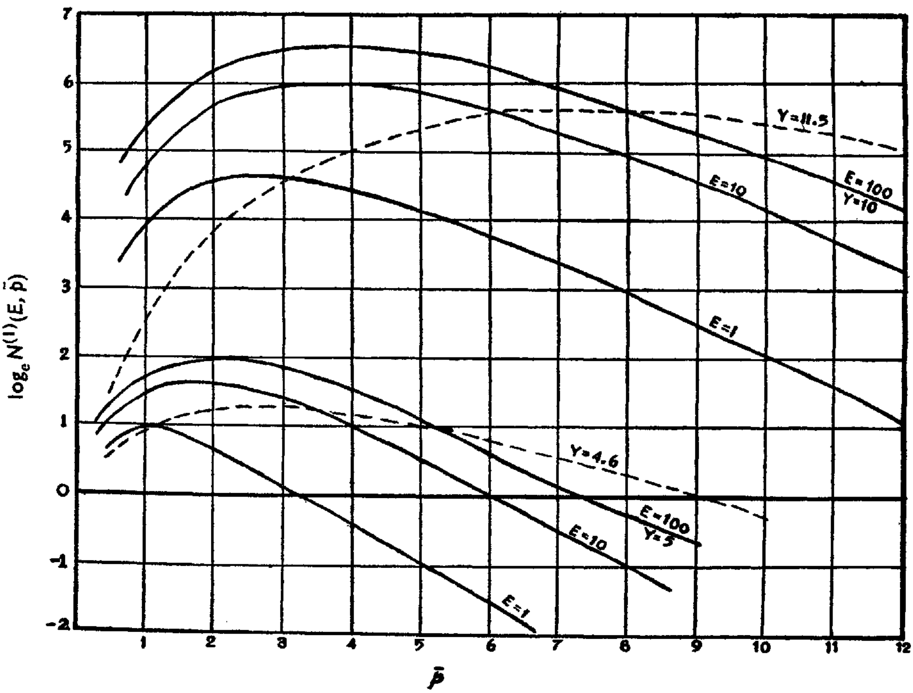

FIG. 1. Logarithm of the average number of nucleons above a given energy E, at different depths for some values of $\log_e E_0/E = Y$.
(——— present paper; - - - - derived from the analysis of Messel)

TABLE III-A

Values of the average number of mesons $\Pi^{(1)}(E, \bar{p})$, $\Pi^{(2)}(E, \bar{p})$, above energy E produced by a primary nucleon or meson of energy E_0 in Nitrogen in terms of the thicknesses travelled. The value in the first row corresponds to $\Pi^{(1)}(E, \bar{p})$, that in the second row to $\Pi^{(2)}(E, \bar{p})$

$\log_e \frac{E_0}{E}$	E \ $\bar{p}$	1	2	3	4	6	8	10
	1	0·37 0·87	0·23 0·47	0·13 0·24				
5	10	1·57 2·72	1·25 2·10	0·85 1·50	0·58 0·96	0·20 0·23		
	100	4·50 5·23	4·65 5·42	3·65 4·45	2·45 3·20	0·95 1·30	0·65 0·85	
	1	3·30 5·20	3·0 4·8	1·8 3·2	1·2 2·2			
8	10	14·8 17·5	20·0 25·6	17·2 21·6	13·7 16·8	6·2 8·1	2·1 2·8	
	100	39·5 43·0	67·0 70·8	66·8 70·3	58·0 61·0	33·0 35·8	15·0 16·6	7·8 8·4
	1	12·0 17·0	17·5 24·0	16·5 20·0	12·0 14·5	5·0 6·0		
10	10	63·0 69·0	112 125	121 137	104 116	62 68	27 30	7·0 8·0
	100	135 147	402 425	475 488	452 463	315 326	180 187	89 92

TABLE III-B

Values of the average number of mesons $\Pi^{(1)}(E, \bar{p})$, $\Pi^{(2)}(E, \bar{p})$ in Lead, produced by a primary E_0

$\log_e \frac{E_0}{E}$	E \ $\bar{p}$	1	2	3	4	6	8	10
5	1	5·06 5·52	3·75 4·15	2·40 2·68	1·35 1·60	0·42 0·52	0·21 0·23	
8		77·4 79·6	73·0 74·6	50·4 52·0	30·0 30·8	11·0 11·2	3·5 3·5	
10		450 "	508 "	410 "	290 "	105 "	44 "	15 "

TABLE IV-A

Values of the average number of nucleons $N^{(1)}(E, \bar{p})$, $N^{(2)}(E, \bar{p})$ above energy E produced by a primary nucleon or meson of energy E_0 in Nitrogen in terms of the thicknesses travelled. The value in the first row gives $N^{(1)}(E, \bar{p})$, that in the second row $N^{(2)}(E, \bar{p})$

$\log_e \frac{E_0}{E}$	E \ $\bar{p}$	1	2	3	4	6	8	10
	1	2·52 0·78	1·70 0·56	1·06 0·31	0·62 0·16			
5	10	4·90 3·37	4·95 2·95	3·64 2·25	2·50 1·53	0·98 0·50		
	100	6·16 5·75	6·70 6·12	5·50 5·05	4·02 3·70	1·78 1·50	1·00 0·82	
	1	17·5 9·0	18·6 10·0	15·0 7·5	11·0 5·6	5·2 2·0		
8	10	39·0 32·0	66·0 52·0	65·8 49·6	55·0 41·0	30·8 22·2	14·2 9·2	6·0 2·4
	100	54·0 50·2	92·0 86·0	103·6 98·8	91·2 87·4	58·0 57·0	30·0 29·0	12·8 12·0
	1	55·0 37·0	93·0 65·0	88 52	75 48	40 25	18 12	
10	10	135 115	320 260	395 337	382 320	265 210	140 117	65 55
	100	230 180	460 405	615 555	645 585	472 422	292 273	145 130

TABLE IV-B

Values of the average number of nucleons $N^{(1)}(E, \bar{p})$, $N^{(2)}(E, \bar{p})$ in Lead produced by a primary E_0

$\log_e \frac{E_0}{E}$	E \ $\bar{p}$	1	2	3	4	6	8	10
5	1	5·75 5·10	4·13 3·71	2·40 2·26	1·30 1·15	0·42 0·30	0·18 0·15	
8		80·0 77·0	74 73	53 ,,	38 ,,	12 ,,	4 ,,	1·8 ,,
10		475 ,,	510 ,,	380 ,,	275 ,,	145 ,,	74 ,,	25 ,,

IV. Discussion of the results

Values of Π and N have been obtained for $\log_e\left(\frac{E_0}{E}\right) = 5$, 8 and 10. In the case of Pb, it is found that the values depend only on the ratio of E_0/E and not on E alone which shows that effect of meson decay is not significant in Pb. In the case of Nitrogen, however, the dependence on E is quite pronounced. This is due to the decay of mesons which is more predominant for smaller values of E. We get much bigger values for the average numbers in Nitrogen in the case of $E = 10$, or $E = 100$ than in the case of $E = 1$, assuming the same value of $\log_e\left(\frac{E_0}{E}\right)$. For the estimation of the showers in Nitrogen, air or any other light elements, the decay of mesons produces a significant effect and its neglect in the analysis altogether changes the form of the showers.

In Fig. 1, we have compared the results of the present paper with those of Messel (1951). Messel neglected the nucleons generated by mesons and his analysis allowed for only $\frac{5}{24}$ths of the total energy for the creation of mesons. In the present paper, we have assumed that the total energy lost is shared equally by meson and the recoil nucleon. Accordingly, we get a more rapid absorption of the shower and a greater intensity at small depths. Messel has calculated his results for air, and we have worked out the results for Nitrogen, but the difference on that account will not be appreciable. The different curves for $E = 1$, 10 and 100 indicate the effect of the decay of mesons and also that produced by the contribution of mesons in the production of nucleon.

The average number of mesons generated by a primary nucleon was calculated by Peng (1944) but his numerical results are few. He also neglected the decay of mesons in his work and he found the average numbers of mesons associated with the passage of a primary through water. His results indicate large values for the number of mesons even at large depths. In the table below, some of his values are compared with the results of Pb, worked out in the present paper.

Table V

Average number of mesons above a given energy E, created by a primary of energy E_0. (Peng's values are those for water, values of the present paper are given for Pb)

$\bar{p}$ \ $\log_e \frac{E_0}{E}$	6·2 (Peng)	5 (present paper)	$\bar{p}$ \ $\log_e \frac{E_0}{E}$	7·5 (Peng)	8 (present paper)
1	3·0	5·0	1	15·0	77·4
2	5·5	3·8	2	26·0	73·0
4	10·5	1·4	4	40·2	30·0
8	17·4	0·2	8	48·5	3·5

The results of the present paper clearly indicate that low energy mesons undergo earlier decay as has been observed by different authors and the effect of meson decay must be taken into consideration in any quantitative analysis of showers of mesons and nucleons particularly in light elements.

The process envisaged will be utilized in a subsequent work for the estimation of the average number of mesons and nucleons in the atmosphere at different depths.

ACKNOWLEDGEMENT

In conclusion, the author thanks Dr. S. K. Chakrabarty, F.N.I., for his kind help and guidance during the progress of this work.

ABSTRACT

Nucleon-nucleon collisions produce mesons and recoil nucleons. The recoil nucleons as also the mesons are capable of starting new showers. The cross-sections used by Messel, Potts and McCusker have been used in the estimation of the showers and the decay of mesons has been taken into account. The diffusion equations have been solved and their solutions have been obtained in a form suitable for numerical computations. Numerical results have been obtained for the energy spectra of nucleons and mesons at different depths. The large effect produced by the decay of mesons on the formation of the shower is clear from a comparison of the sizes of showers in heavy and light elements.

REFERENCES

Chakrabarty, S. K. (1947). Generation of mesons and its dependence on meson spin. *Bull. Cal. Math. Soc.*, **39**, 166.

Fermi, E. (1950). High energy nuclear events. *Prog. Theo. Phys.*, **5**, 570.

Heisenberg, W. (1939). Zur Theorie der explosionsarligen Schaner in der Kosmischen Strahlung II. *Zeits. f. Physik*, **113**, 61, 61–86.

Heitler, W., and Janossy, L. (1949). On the absorption of meson producing nucleons. *Proc. Phy. Soc.*, A **62**, 374.

——— (1950). Further investigation on the plural production of meson showers. *Helv. Phys. Acta*, **23**, 417.

Lewis, H. W., Oppenheimer, J. R., and Wouthuysen, S. A. (1948). The multiple production of mesons. *Phys. Rev.*, **73**, 127, Pt. 127–140.

Messel, H. (1951). On the theory of a nucleon cascade. *Com. Dub. Ins. for Advanced Studies*, A, 7.

——— (1951). On the nucleon cascade with ionisation loss. *Phys. Rev.*, **83**, 21.

Messel, H., Potts, R. B., and McCusker, C. B. A. (1952). Theoretical results on the high energy nuclear collisions in light and heavy elements. *Phil. Mag.*, **43**, 889.

Peng, H. W. (1944). On the cascade production of mesons. *Proc. R. Irish Ac.*, **49** A, **14**, 245.

Issued August 28, 1956.