INTERNAL BALLISTICS OF COMPOSITE CHARGE WHEN THEY BURN ACCORDING TO THE GEOMETRIC FORM FUNCTIONS FOR SPHERES

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INTRODUCTION

We know that the general form function for most of the propellants is of the form $Z = (1-f)\ (1+\theta f)$ where θ is a measure of the change in area of the burning surface as burning proceeds. Those shapes for which θ is positive are called 'degressive' and those for which θ is negative are called 'progressive'. Shapes for which $\theta = 0$, e.g. tube, are called 'neutral'. Also the possible range of θ is restricted. It must lie within -1 and $+1$. But this does not mean that cord ($\theta = 1$) is the most degressive shape possible. Shapes such as spheres decrease even more rapidly. The true geometric form function for spheres is $Z = (1 - f^3)$ and in this case the web size D is the diameter of the sphere. The general form function for degressive shapes is $Z = (1-f)\ (1+\theta f+\theta f^2)$. But the need for such a degressive form rarely arises except with composite charges and also with chopped cord which are sometimes used in small arms and infantry mortars.

In this paper the author has solved the interior ballistic equations of an orthodox gun for composite charge, when they burn according to form functions for spheres or cubes, using the 'R. D. 38' method which is based upon the isothermal approximation, that is a mean temperature of the propellant gases is assumed throughout the period of burning of the charges. This is a fair approximation, since the continuing conversion of thermal energy of the gas to kinetic energy of the shot is largely compensated by the generation of energy by the reaction of more propellants.

2. THE BASIC EQUATIONS

The equations of internal ballistics for a single charge have been derived by Corner (1950) and in our case they become

$$D_1 \frac{df_1}{dt} = -\beta_1 P \quad \ldots \quad (1a)$$

$$D_2 \frac{df_2}{dt} = -\beta_2 P \quad \ldots \quad (1b)$$

$$\left(W_1 + \frac{C_1+C_2}{2}\right)\frac{dV}{dt} = AP \quad \ldots \quad (2)$$

where W_1 is the effective shot weight taking into account the resistance to motion, the recoil of the gun and the rotation of the shot.

$$AP = \frac{F_1C_1Z_1+F_2C_2Z_2}{x+l}\,\frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad (3)$$

$$Z_1 = 1-f_1^3 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4a)$$

$$Z_2 = 1-f_2^3 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4b)$$

where C_1, F_1, β_1, D_1, f_1, and Z_1 refer to the first charge and C_2, F_2, β_2, D_2, f_2, and Z_2 refer to the second charge.

3. Simultaneous and Non-simultaneous Burning of the Charges

Dividing (1*a*) and (1*b*) we get

$$\frac{df_1}{df_2} = \frac{\beta_1 D_2}{\beta_2 D_1} = K \text{ (say)}$$

Integrating and applying the conditions $f_1 = f_2 = 1$ (initially) we get

$$(1-f_1) = K(1-f_2) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Now two cases arise, viz.:

(I) Both the propellants burn out simultaneously.

(II) The two propellants burn out at different times.

Case I :—If the two charges burn out simultaneously then since at all-burnt position $f_1 = f_2 = 0$, we have from (5) the condition for simultaneous burning as $K = 1$, i.e. $\beta_1 D_2 = \beta_2 D_1$.

Case II :—If charge C_1 burns out first, then if suffix 'b' denotes this position, we have from (5),

$$f_{2b} = 1 - \frac{1}{K}$$

But since f_{2b} must be a positive fraction we have the condition for burning out of the charge C_1 first as $K > 1$, i.e. $\beta_1 D_2 > \beta_2 D_1$.

If charge C_2 burns out first then similarly as above we have the condition as $K < 1$, i.e. $\beta_1 D_2 < \beta_2 D_1$.

For the sake of definiteness we will assume that charge C_1 burns out first.

4. Solution of the Equation

Eliminating P from (1*a*) and (2) and integrating we get

$$V = \frac{A D_1(1-f_1)}{\beta_1 \left(W_1 + \dfrac{C_1+C_2}{2}\right)} \qquad \ldots \quad \ldots \quad \ldots \quad (6)$$

If we use equation (6) in equation (2) and eliminate P by using equation (3),

$$-M_1(1-f_1)\frac{df_1}{dx} = \frac{Z_1 + \dfrac{F_2 C_2}{F_1 C_1} Z_2}{x+l} \qquad \ldots \quad \ldots \quad \ldots \quad (7)$$

where

$$M_1 = \frac{A^2 D_1^2 \left(1 + \dfrac{C_1+C_2}{3W_1}\right)}{\beta_1^2 W_1 F_1 C_1 \left(1 + \dfrac{C_1+C_2}{2W_1}\right)^2}$$

Putting the values of Z_1 and Z_2 and making use of relation (5) equation (7) reduces to

$$\frac{dx}{x+l} = -\frac{M_1 df_1}{\left\{1+\frac{F_2C_2}{F_1C_1K}\left(3-\frac{3}{K}+\frac{1}{K^2}\right)\right\}+f_1\left\{1+\frac{F_2C_2}{F_1C_1K^2}\left(3-\frac{2}{K}\right)\right\}+f_1^2\left\{1+\frac{F_2C_2}{F_1C_1K^3}\right\}}$$

$$= -\frac{M_1 df_1}{a+bf_1+cf_1^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

where

$$\left.\begin{aligned} a &= 1+\frac{F_2C_2}{F_1C_1K}\left(3-\frac{3}{K}+\frac{1}{K^2}\right) \\ b &= 1+\frac{F_2C_2}{F_1C_1K}\left(3-\frac{2}{K}\right) \\ c &= 1+\frac{F_2C_2}{F_1C_1K^3} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Integrating equation (8) and applying the conditions that $x=0$ when $f_1=1$, we obtain

$$\log\frac{x+l}{l} = \frac{2M_1}{\sqrt{(4ac-b^2)}}\tan^{-1}\left\{\frac{\sqrt{(4ac-b^2)}(1-f_1)}{(2a+b)+f_1(2c+b)}\right\} \quad \ldots \quad \ldots \quad (10)$$

Therefore from equation (3), pressure P is given by

$$P = \frac{F_1C_1(1-f_1^3)+\frac{F_2C_2}{K}(1-f_1)\left[\left(3-\frac{3}{K}+\frac{1}{K^2}\right)+\frac{f_1}{K}\left(3-\frac{2}{K}\right)+\frac{f_1^2}{K^2}\right]}{A(x+l)} \frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad (11)$$

where $(x+l)$ is given by (10).

This completes the determination of x, V and P as functions of f when both the charges are burning.

Case I :—If the two charges burn out simultaneously then all the above equations are true for $K=1$.

Values at all-burnt for this case :—If suffix B denotes the position of all-burnt, then

$$V_B = \frac{AD_1}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$\log\frac{x_B+l}{l} = \frac{\pi M_1F_1C_1}{3\sqrt{3}(F_1C_1+F_2C_2)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

and

$$P_B = \frac{F_1C_1+F_2C_2}{A(x_B+l)} \frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Solution after all-burnt in this case:—

Let

$$r = \frac{x+l}{x_B+l} \quad \ldots \quad (15)$$

Since after all-burnt the expansion of the gases is adiabatic, the pressure at any travel $x > x_B$ is

$$P = P_B r^{-\gamma} \quad \ldots \quad (16)$$

Also the equation of the motion of the shot is

$$\left(W_1 + \frac{C_1+C_2}{2}\right)\frac{dv}{dt} = AP \quad \ldots \quad (17)$$

Therefore the velocity is given by

$$V^2 - V_B^2 = \frac{2A}{\left(W_1+\frac{C_1+C_2}{2}\right)} \int_{x_B}^{x} Pdx$$

$$= \frac{2AP_B(x_B+l)}{\left(W_1+\frac{C_1+C_2}{2}\right)(\gamma-1)}\left(1-r^{1-\gamma}\right)$$

$$= \frac{(F_1C_1+F_2C_2)\Phi}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad \ldots \quad (18)$$

where

$$\Phi = \frac{2}{\gamma-1}\left(1-r^{1-\gamma}\right) \quad \ldots \quad (19)$$

But

$$V_B^2 = \frac{F_1C_1M_1}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad \ldots \quad (20)$$

Thus the velocity at any travel after all-burnt is given by

$$V^2 = \frac{F_1C_1(M_1+\Phi)+F_2C_2\Phi}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad \ldots \quad (21)$$

Hence if suffix E refers to the quantities at the muzzle, the muzzle velocity is

$$V_E^2 = \frac{F_1C_1(M_1+\Phi_E)+F_2C_2\Phi_E}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad \ldots \quad (22)$$

where

$$\Phi_E = \frac{2}{\gamma-1}\left[1-\left(\frac{x_E+l}{x_B+l}\right)^{1-\gamma}\right]$$

Case II :—In this case all the above equations from (1) to (11) will hold and so when the first charge is all-burnt we have

$$V_b = \frac{AD_1}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

$$\log\frac{x_b+l}{l} = \frac{2M_1}{\sqrt{(4ac-b^2)}}\tan^{-1}\left\{\frac{\sqrt{(4ac-b^2)}}{(2a+b)}\right\} \quad \ldots \quad \ldots \quad (24)$$

and

$$P_b = \frac{F_1C_1+\frac{F_2C_2}{K}\left(3-\frac{3}{K}+\frac{1}{K^2}\right)}{A(x_b+l)}\frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots \quad (25)$$

After C_1 has burnt out the equations of motion are:

$$AP = \frac{F_1C_1+F_2C_2Z_2}{x+l}\frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

$$D_2\frac{df_2}{dt} = -\beta_2 P \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

$$Z_2 = (1-f_2^3) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

and

$$\left(W_1+\frac{C_1+C_2}{2}\right)\frac{dV}{dt} = AP \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

Eliminating P from (27) and (29) and integrating we get

$$V = \frac{AD_2(1-f_2)}{\beta_2\left(W_1+\frac{C_1+C_2}{2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

Using equation (30) in equation (29) and eliminating P by making use of equation (26), we have

$$-M_2(1-f_2)\frac{df_2}{dx} = \frac{\frac{F_1C_1}{F_2C_2}+Z_2}{(x+l)} \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

where

$$M_2 = \frac{A^2D_2^2\left(1+\frac{C_1+C_2}{3W_1}\right)}{\beta_2^2W_1F_2C_2\left(1+\frac{C_1+C_2}{2W_1}\right)^2}$$

Putting the value of Z_2 in (31) we obtain

$$\frac{dx}{x+l} = -\frac{M_2(1-f_2)df_2}{(\alpha^3-f_2^3)} \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

where

$$\alpha=\left(1+\frac{F_1C_1}{F_2C_2}\right)^{\frac{1}{3}}$$

Thus

$$\frac{dx}{x+l}=-M_2\left[\frac{(1-\alpha)}{3\alpha^2}\,\frac{1}{(\alpha-f_2)}+\frac{(1-\alpha)f_2+\alpha(2+\alpha)}{3\alpha^2(\alpha^2+\alpha f_2+f_2^2)}\right]df_2$$

$$=-M_2\left[\frac{(1-\alpha)}{3\alpha^2}\,\frac{1}{(\alpha-f_2)}+\frac{(1-\alpha)}{6\alpha^2}\,\frac{(2f_2+\alpha)}{(\alpha^2+\alpha f_2+f_2^2)}+\frac{(1+\alpha)}{2\alpha}\,\frac{1}{\left(f_2+\frac{\alpha}{2}\right)^2+\left(\frac{\sqrt{3}\alpha}{2}\right)^2}\right]df_2 \quad \ldots \quad (33)$$

Integrating (33) and applying the conditions that $x=x_b$ when $f_{2b}=1-\frac{1}{k}$ we get

$$\log\frac{(x+l)}{(x_b+l)}=M_2\left[\frac{\alpha-1}{3\alpha^2}\log\left\{\frac{\alpha-\left(1-\frac{1}{k}\right)}{\alpha-f_2}\right\}-\frac{\alpha-1}{6\alpha^2}\log\left\{\frac{\alpha^2+\alpha\left(1-\frac{1}{K}\right)+\left(1-\frac{1}{K}\right)^2}{\alpha^2+\alpha f_2+f_2^2}\right\}\right.$$

$$\left.-\frac{1+\alpha}{\sqrt{3}\alpha^2}\tan^{-1}\left\{\frac{\sqrt{3}\alpha\left(f_2-\left(1-\frac{1}{K}\right)\right)}{2\alpha^2+f_2\left(2\left(1-\frac{1}{K}\right)+\alpha\right)+\alpha\left(1-\frac{1}{K}\right)}\right\}\right] \quad \ldots \quad (34)$$

where (x_b+l) is given by (24)
and from (26) we have

$$P=\frac{F_1C_1+F_2C_2(1-f_2^3)}{A(x+l)}\,\frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots \quad (35)$$

where $(x+l)$ is given by (34)

Thus we know x, V and P at any instant during the burning of charge C_2.

Values at all-burnt in this case:—

$$V_B=\frac{AD_2}{\beta_2\left(W_1+\frac{C_1+C_2}{2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

$$\log\left(\frac{x_B+l}{x_b+l}\right)=M_2\left[\frac{\alpha-1}{3\alpha^2}\log\left\{\alpha-\left(1-\frac{1}{K}\right)\right\}-\frac{\alpha-1}{6\alpha^2}\log\left\{\frac{\alpha^2+\alpha\left(1-\frac{1}{K}\right)+\left(1-\frac{1}{K}\right)^2}{\alpha^2}\right\}\right.$$

$$\left.+\frac{1+\alpha}{\sqrt{3}\alpha^2}\tan^{-1}\left\{\frac{\sqrt{3}\left(1-\frac{1}{K}\right)}{2\alpha+\left(1-\frac{1}{K}\right)}\right\}\right] \quad \ldots \quad (37)$$

and
$$P_B = \frac{F_1C_1+F_2C_2}{A(x_B+l)} \frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

where (x_B+l) is given by equation (37).

Solution after all-burnt in this case :—

Let
$$r = \frac{x+l}{x_B+l} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

Since after all-burnt the expansion of the gas is adiabatic, the pressure at any travel $x > x_B$ is given by
$$P = P_B r^{-\gamma} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

Also the equation of the motion of the shot is
$$\left(W_1+\frac{C_1+C_2}{2}\right)\frac{dV}{dt} = AP \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

Therefore the velocity is given by
$$V^2 - V_B^2 = \frac{2A}{\left(W_1+\frac{C_1C_2}{2}\right)}\int_{x_B}^{x} Pdx$$
$$= \frac{(F_1C_1+F_2C_2)\Phi}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

where
$$\Phi = \frac{2}{\gamma-1}\left(1-r^{1-\gamma}\right)$$

But from (36)
$$V_B^2 = \frac{F_2C_2M_2}{\left(W_1+\frac{C_1+C_2}{3}\right)}$$

Thus the velocity at any travel after all-burnt is given by
$$V^2 = \frac{F_1C_1\Phi+F_2C_2(M_2+\Phi)}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (43)$$

and the muzzle velocity is given by
$$V_E^2 = \frac{F_1C_1\Phi_E+F_2C_2(M_2+\Phi_E)}{\left(W_1+\frac{C_1+C_2}{3}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

where
$$\Phi_E = \frac{2}{\gamma-1}\left[1-\left(\frac{x_E+l}{x_B+l}\right)^{1-\gamma}\right]$$

5. Maximum Pressure

Maximum pressure may occur when:

(a) both the charges are burning,

(b) C_1 has burnt out but C_2 is burning, and

(c) at the position of all-burnt.

Case (a):—When both the charges are burning, the pressure is given by

$$P=\frac{F_1C_1(1-f_1^3)+F_2C_2(1-f_2^3)}{A(x+l)}\frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} \quad .. \quad .. \quad (45)$$

If pressure is to be maximum $dP=0$. So differentiating (45) and simplifying with the help of equation (7), we get the equation giving the value of f_1 at maximum pressure as

$$3f_1^2\left(F_1C_1+\frac{F_2C_2}{K^3}\right)+f_1\left\{F_1C_1M_1+\frac{6F_2C_2}{K^2}\left(1-\frac{1}{K}\right)\right\}-$$

$$-\left\{F_1C_1M_1-\frac{3F_2C_2}{K}\left(1-\frac{1}{K}\right)^2\right\}=0$$

Now in order that this equation has a real root we must have $F_1C_1M_1>\dfrac{3F_2C_2}{K}\left(1-\dfrac{1}{K}\right)^2$ and thus its positive root is

$$f_{1m}=\frac{\sqrt{F_1^2C_1^2M_1(M_1+12)+\dfrac{12F_1C_1F_2C_2}{K^2}\left\{M_1-3K\left(1-\dfrac{1}{K}\right)^2\right\}}-\left\{F_1C_1M_1+\dfrac{6F_2C_2}{K^2}\left(1-\dfrac{1}{K}\right)\right\}}{6\left(F_1C_1+\dfrac{F_2C_2}{K^3}\right)} \quad .. \quad (46)$$

So from equation (45) we have the maximum pressure as

$$P_m=\frac{F_1C_1(1-f_{1m}^3)+\dfrac{F_2C_2}{K}(1-f_{1m})\left\{3\left(1-\dfrac{1}{K}\right)+\dfrac{1}{K^2}+\dfrac{f_{1m}}{K}\left(3-\dfrac{2}{K}\right)+\dfrac{f_{1m}^2}{K^2}\right\}}{A(x_m+l)}\frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} \quad .. \quad (47)$$

where from equation (10)

$$\log\left(\frac{x_m+l}{l}\right)=\frac{2M_1}{\sqrt{(4ac-b^2)}}\tan^{-1}\left[\frac{\sqrt{(4ac-b^2)}\,(1-f_m)}{(2a+b)+f_{1m}(2c+b)}\right] \quad .. \quad (48)$$

Case (b) :—When charge C_1 has burnt out but C_2 is burning

$$P=\frac{F_1C_1+F_2C_2(1-f_2^3)}{A(x+l)}\frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} \quad .. \quad .. \quad (49)$$

If pressure is to be maximum, $dP = 0$. So differentiating (49) and simplifying with the help of equation (31), we get the equation giving the value of f_2 at maximum pressure as

$$3f_2^2+M_2f_2-M_2 = 0.$$

Its positive root is given by

$$f_{2m} = \frac{\sqrt{M_2^2+12M_2}-M_2}{6} \qquad \ldots \quad \ldots \quad \ldots \quad (50)$$

So from equation (49) we have the maximum pressure

$$P_m = \frac{F_1C_1+F_2C_2(1-f_{2m}^3)}{A(x_m+l)} \frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} \qquad \ldots \quad \ldots \quad (51)$$

where from equation (34)

$$\log\left(\frac{x_m+l}{x_b+l}\right) = M_2\left[\frac{\alpha-1}{3\alpha^2}\log\left\{\frac{\alpha-\left(1-\frac{1}{K}\right)}{\alpha-fx_m}\right\} - \frac{\alpha-1}{6\alpha^2}\log\left\{\frac{\alpha^2+\alpha\left(1-\frac{1}{K}\right)+\left(1-\frac{1}{K}\right)^2}{\alpha^2+\alpha f_{2m}+f_{2m}^2}\right\}\right.$$

$$\left. -\frac{1+\alpha}{\sqrt{3}\alpha^2}\tan^{-1}\left\{\frac{\sqrt{3}\alpha\left(f_{2m}-\left(1-\frac{1}{K}\right)\right)}{2\alpha^2+f_{2m}\left(2\left(1-\frac{1}{K}\right)+\alpha\right)+\alpha\left(1-\frac{1}{K}\right)}\right\}\right] \qquad \ldots \quad (52)$$

Case (c) :—This case has already been discussed.

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ABSTRACT

In this communication the author has solved the problem of internal ballistics of composite charges when they burn according to the geometric form functions for spheres, using the 'R. D. 38' method which is based upon the usual isothermal model. It is assumed that $\gamma_1 = \gamma_2 = \gamma$ (say) since γ is practically the same for most propellants. Also the linear rates of burning have been assumed.

REFERENCES

Corner, J (1950). Theory of Interior Ballistics of Guns, New York.

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