THE EQUIVALENT CHARGE METHOD IN THE GENERAL THEORY OF COMPOSITE CHARGES

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1. INTRODUCTION

The general theory of composite charges, consisting of two component charges, has been discussed by Clemmow (1920, 1951) and by Corner (1950). Clemmow, in his discussion, takes the two component charges of the same composition, but of different shapes and sizes. In Corner's discussion, the restriction of the same composition has been removed, but he replaces the composite charge by a charge of a single composition by taking suitable weighted averages of the parameters defining the composition of the component charges. However, his choice of the weighted averages is more or less intuitive.

In general, there would be two distinct stages of burning. In the first, both the component charges burn, while in the second only one of the charges burns, the other having been burnt out completely in the first stage. Both Clemmow and Corner determine the form function for the two stages for the 'equivalent' charge by using the principle that the mass of the gas produced at any instant by the equivalent charge is the same as that produced by the composite charge at that instant. To make their reduction to an equivalent charge complete, they next try to replace the form functions for the two stages of burning by a single form function of the standard form

$$z = (1-f)(1+\theta f) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

which should be applicable to both the stages alike. Clemmow and Corner, however, use different considerations for obtaining a suitable value of the equivalent form-factor θ. Clemmow claims that 'a fairly good approximation throughout is obtained by ensuring agreement at the intermediate point', i.e. he estimates the value of θ so that (1) passes through the common point of the $z-f$ curves for the two stages of burning. Corner, on the other hand, dealing with the case $\theta_1 = \theta_2 = 0$ adopts that value of θ which gives the area under (1) between $f = 1$ and $f = 0$ same as the area under the $z-f$ curves for the two stages of burning.

It is obvious that both the methods are arbitrary and would, in general, give different results. Moreover, Clemmow's method will fail if the composite charge consists of more than two component charges; since there would be in this case a number of 'intermediate' points and curve (1), consisting of only one unknown parameter θ, cannot, in general, be made to pass through all of them. Corner's method suffers from the defect that it will not allow the determination of parameters θ and ψ, if we have reason to assume that

$$z = (1-f)(1+\theta f+\psi f^2) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

gives a better fit; for Corner's method imposes only one condition on the form function which will allow the determination of only one unknown parameter.

In the present paper, a composite charge consisting of n component charges with different sizes, force constants, rate of burning constants, but with the same pressure index, has been considered. The assumption of the same pressure index has also been made by both Clemmow and Corner. In addition they took the form functions for the component charges in the standard form (1). The present treatment allows for the possibility when the component charges can have their form functions in the general form

$$z = \phi(f) \quad \ldots \quad (3)$$

which, of course, includes both (1) and (2) as particular cases.

An equivalent charge is here obtained from the consideration that the Ballistic equations should be the same throughout for the composite and equivalent charges. This would imply that the total energy produced at any instant during burning by the equivalent charge is the same as that produced by the composite charge at that instant and that, at the position of all-burnt, the total mass of the gas produced by the equivalent charge is the same as that produced by the composite charge. In the particular case when the force constants of the various component charges are equal or when a weighted mean of these force constants is assumed to give the force constant for the equivalent charge, the principle of 'equivalence of energy' during burning reduces to the principle of 'equivalence of mass' used by Clemmow and Corner. From the more general considerations used here, the composition of the equivalent charge is deduced and it may be noted that this procedure provides a justification for the assumptions made by Corner regarding the composition of the equivalent charge.

A method, similar to the method of least squares used in Statistics, has been used, in the present paper, to obtain a single form function for all the stages of burning. This method ensures the best fit of the form function for the equivalent charge to the different form functions for the various stages of burning and is applicable to those cases also where the earlier two methods fail.

In Section 7, some of the cases, in which a composite charge consisting of more than two component charges is likely to be useful, have been pointed out.

It may also be pointed out here that recently two more methods of dealing with the general theory of composite charges have been given: one is the 'direct' method by Venkatesan and Patni (1953) and the other is the 'equivalent form function' method by the present author. Venkatesan and Patni have extended the Hunts-Hind method for a single charge to a composite charge consisting of two component charges. Their method, proceeding as it does from first principles, requires a separate investigation of all problems of Internal Ballistics for single and for composite charges. In the equivalent form function method, the form function for the equivalent charge is obtained for each of the stages of burning once for all and once a problem of Internal Ballistics has been solved for a single charge for the general quadratic form function, the solution of the same problem for composite charges follows at once. Both these methods avoid the approximation inherent in the estimation of θ in the equivalent form-factor method of Corner and Clemmow; but both on the other hand suffer from the following defects:

(1) For a composite charge consisting of n component charges, separate calculations have to be done for the n stages of burning and thus the calculations are much more laborious.

(2) It is not possible to make use of tables which are available for the form function $z = (1-f)\ (1+\theta f)$. So far no tables for the actual form functions derived here for the n stages of burning have been prepared. Until these tables are constructed, we may have to neglect co-volume correction terms and this approximation may be of the same order as the approximation in the estimation of θ.

Also with the equivalent form-factor method, the composite charge is reduced to a single equivalent charge with its form function in the standard form (1) and all the methods of Internal Ballistics for a single charge can be directly applied to composite charges.

2. The Form Function for the Equivalent Charge

We consider a composite charge consisting of n component charges. For the i th component charge ($i = 1, 2, 3, \ldots\ldots n$), let

F_i denote the force constant
C_i denote the charge mass
D_i denote the Ballistic size
β_i denote the rate of burning constant
δ_i denote the density of the propellant
b_i denote the co-volume
z_i denote the fraction of the charge mass burnt at any time
f_i denote the fraction of the web-size remaining at any time
$z_i = \phi_i(f)$ denote the form function
γ_i denote the ratio of the specific heat
α denote the pressure index

(assumed to be the same for all component charges)

Without loss of generality, we can take

$$\frac{D_1}{\beta_1} \leqslant \frac{D_2}{\beta_2} \leqslant \frac{D_3}{\beta_3} \leqslant \cdots \leqslant \frac{D_n}{\beta_n}. \qquad (4)$$

Also let $F, C, D, \beta, \delta, b, z, f, z = \phi(f), \gamma, \alpha$ denote the corresponding quantities for the equivalent charge.

From the definition of the equivalent charge, the rate of production of energy by the composite and the equivalent charges is the same. Therefore

$$\frac{F_1C_1}{\gamma_1-1}\frac{dz_1}{dt}+\frac{F_2C_2}{\gamma_2-1}\frac{dz_2}{dt}+\cdots+\frac{F_nC_n}{\gamma_n-1}\frac{dz_n}{dt}=\frac{FC}{\gamma-1}\frac{dz}{dt} \qquad (5)$$

Integrating and remembering that initially

$$z_1 = z_2 = z_3 = \cdots = z_n = 0$$

we get

$$\frac{F_1C_1}{\gamma_1-1}z_1+\frac{F_2C_2}{\gamma_2-1}z_2+\cdots+\frac{F_nC_n}{\gamma_n-1}z_n=\frac{FC}{\gamma-1}z \qquad (6)$$

or defining

$$\frac{\dfrac{F_iC_i}{\gamma_i-1}}{\dfrac{FC}{\gamma-1}} = \lambda_i \qquad (7)$$

(6) becomes

$$z = \lambda_1 z_1 + \lambda_2 z_2 + \cdots + \lambda_n z_n \qquad (8)$$

But

$$z_i = \phi_i(f_i)$$

$\therefore$

$$z = \sum_{i=1}^{n} \lambda_i \phi_i(f_i) \qquad (9)$$

Now from the rate of burning equations

$$\frac{D_1}{\beta_1}\frac{df_1}{dt}=\frac{D_2}{\beta_2}\frac{df_2}{dt}=\cdots=\frac{D_n}{\beta_n}\frac{df_n}{dt}=\frac{D}{\beta}\frac{df}{dt}=-p^{\alpha} \qquad (10)$$

Integrating and remembering that initially

$$f_1 = f_2 = \ldots\ldots = f_n = 1$$

we get

$$\frac{1-f_1}{\frac{\beta_1}{D_1}} = \frac{1-f_2}{\frac{\beta_2}{D_2}} = \ldots\ldots = \frac{1-f_n}{\frac{\beta_n}{D_n}} = \frac{1-f}{\frac{\beta}{D}}$$

or

$$\frac{1-f_1}{\beta_1'} = \frac{1-f_2}{\beta_2'} = \ldots\ldots = \frac{1-f_n}{\beta_n'} = \frac{1-f}{\beta'} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

where

$$\beta_i' = \frac{\beta_i}{D_i}\,;\ \beta' = \frac{\beta}{D} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

From (4) and (12)

$$\beta_1' \geqslant \beta_2' \geqslant \cdots\cdots \geqslant \beta_n'$$

$\therefore$ from (11)

$$1-f_1 \geqslant 1-f_2 \geqslant 1-f_3 \geqslant \cdots\cdots \geqslant 1-f_n$$

or

$$f_1 \leqslant f_2 \leqslant f_3 \leqslant \cdots\cdots \leqslant f_n$$

$\therefore f_1$ vanishes first, then f_2, then f_3, and the last to vanish is f_n.

If $\qquad \beta_1' > \beta_2' > \beta_3' > \cdots\cdots > \beta_n',$

then $\qquad f_1 < f_2 < f_3 < \cdots\cdots < f_n$

so that in this case there are n distinct stages of burning. At the end of the first stage, the first charge is burnt out and in the r th stage only the r th, $(r+1)$ th, ... n th charges burn, the first $(r-1)$ charges having been burnt out earlier.

At the end of the r th stage $f_r = 0$, and therefore from (11)

$$1-f = \frac{\beta'}{\beta_r'} = \frac{1}{k_r}$$

where

$$k_i = \frac{\beta_i'}{\beta'}\ (i = 1, 2, \cdots\cdots n) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

or

$$f = 1 - \frac{1}{k_r}$$

so that at the ends of the various stages of burning f takes the values:

$$1-\frac{1}{k_1},\ 1-\frac{1}{k_2}\ \ 1-\frac{1}{k_3},\ \cdots\cdots,\ 1-\frac{1}{k_r},\ \cdots\cdots 1-\frac{1}{k_n} \quad \ldots \quad \ldots \quad (14)$$

However, at the end of the n th stage, $f_n = 0$. Also since the equivalent charge is also burnt out at this stage $f = 0$

$\therefore$ from (11)

$$\beta_n' = \beta'$$

or

$$\frac{D_n}{\beta_n} = \frac{D}{\beta} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

If $\beta_1' = \beta_2'$, then $f_1 = f_2$ and the first two charges burnt out simultaneously. The number of distinct stages is reduced to $n-1$. Similarly if p of the quantities $\frac{\beta_i}{D_i}$ are equal, the corresponding charges burn out simultaneously and the number of distinct stages of burning is reduced to $n-p+1$

If
$$\beta_1' = \beta_2' = \beta_3' = \cdots\cdots = \beta_n'$$
i.e. if
$$k_1 = k_2 = k_3 = \cdots = k_n,$$
then all the charges burn out simultaneously and there is only one distinct stage of burning.

Now we proceed to determine the form functions for the equivalent charge for the various stages of burning.

For the first stage, when all the charges are burning, we have, from (11)

$$1-f_i = \frac{\beta_i'}{\beta'}(1-f)$$

or

$$1-f_i = k_i(1-f)$$

so that (9) becomes

$$z = \sum_{i=1}^{n} \lambda_i \phi_i [1-k_i(1-f)] \quad \ldots \quad (16)$$

For the r th stage of burning, when the first $(r-1)$ charges have been consumed, (9) and (11) become

$$z = \lambda_1 + \lambda_2 + \ldots + \lambda_{r-1} + \sum_{i=r}^{n} \lambda_i \phi_i(f_i) \quad \ldots \quad (17)$$

and

$$\frac{1-f_r}{\beta_r'} = \frac{1-f_{r+1}}{\beta_{r+1}'} = \ldots = \frac{1-f}{\beta'} \quad \ldots \quad (18)$$

Substituting in (17) from (18), the form function for the r th stage is

$$z = \lambda_1 + \lambda_2 + \ldots + \lambda_{r-1} + \sum_{i=r}^{n} \lambda_i \phi_i [1-k_i(1-f)] \quad \ldots \quad (19)$$

The equation (19) would hold during the r th stage of burning, i.e. when from (14), f lies between $1-\frac{1}{k_{r-1}}$ and $1-\frac{1}{k_r}$. Since at all-burnt

$$z_1 = z_2 = \ldots = z_n = z = 1$$

$\therefore$ from (8)

$$1 = \lambda_1 + \lambda_2 + \ldots + \lambda_n$$

$\therefore$ from (7)

$$\sum_{i=1}^{n} \frac{F_i C_i}{\lambda_i - 1} = \frac{FC}{\lambda - 1} \quad \ldots \quad (20)$$

expressing that the total energy available in the equivalent charge is equal to the sum of energies available in the component charges.

Further since the total mass of the gas at all-burnt is the same for the equivalent and the composite charges,

$$C = C_1 + C_2 + \ldots + C_n \quad \ldots \quad (21)$$

(15), (20), and (21) determine respectively $\frac{D}{B}$, $\frac{F}{\gamma-1}$ and C for the equivalent charge, (14) determines the various stages of burning and (19) the form function for the equivalent charge for the various stages. The density of the solid propellant can be obtained from

$$\frac{1}{\delta} = \frac{\frac{C_1}{\delta_1} + \frac{C_2}{\delta_2} + \ldots + \frac{C_n}{\delta_n}}{C_1 + C_2 + \ldots + C_n} \quad \ldots \quad (22)$$

while determination of b, the co-volume, and of γ, the ratio of specific heat for the equivalent charge, are problems of Thermodynamics rather than those of Internal Ballistics. But their values are the same for most of the propellants used in practice. Thus the reduction to equivalent charge is complete. If we could find a single form function

$$z = (1-f)(1+\theta f)$$

to fit all the stages exactly, reduction could be made to a single charge with the standard form function. But this, in general, is not possible.

3. Particular Case when the Component Charges have Cubic Form Functions

In equation (19) we have obtained the form function for the r th stage of burning for the equivalent charge when the component charges have general form functions. In practice the form

$$z = (1-f)(1+\theta f + \psi f^2)$$

covers all the cases that are likely to arise. We, therefore, consider the case when the i th component charge has the form function

$$z_i = (1-f_i)(1+\theta_i f_i + \psi_i f_i^2) \quad \ldots \quad (23)$$

This will itself include, as a particular case, the more important case when the i th component charge has the form function

$$z_i = (1-f_i)(1+\theta_i f_i) \quad \ldots \quad (24)$$

From (23) and (19), the form function for the r th stage is

$$z = \lambda_1 + \lambda_2 + \ldots + \lambda_{r-1} + \sum_{i=r}^{n} \lambda_i [k_i(1-f)][1 + \theta_i(1-k_i\overline{1-f}) + \psi_i(1-k_i\overline{1-f})]$$

or

$$z = \lambda_1 + \lambda_2 + \ldots + \lambda_{r-1} + (1-f)\left[\sum_{i=r}^{n} \lambda_i k_i(1+\theta_i+\psi_r) - (1-f)\sum_{i=r}^{n} \lambda_i k_i^2(\theta_i + 2\psi_i) + (1-f)^2 \sum_{i=r}^{n} \lambda_i k_i^3 \psi_i\right]$$

or

$$z = A_r + B_r(1-f) - C_r(1-f)^2 + D_r(1-f)^3, \quad \ldots \quad \ldots \quad (25)$$

where

$$\left.\begin{aligned} A_r &= \sum_{i=1}^{r-1} \lambda_i = 1 - \sum_{i=r}^{n} \lambda_i \\ B_r &= \sum_{i=r}^{n} \lambda_i k_i (1+\theta_i+\psi_i) \\ C_r &= \sum_{i=r}^{n} \lambda_i k_i^2 (\theta_i + 2\psi_i) \\ D_r &= \sum_{i=r}^{n} \lambda_i k_i^3 \psi_i. \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

If $\psi_i = 0$ for all i, $D_r = 0$ and the form function for each stage reduces to a quadratic. z will be continuous at the end of the r th stage, if

$$A_r + \frac{B_r}{k_r} - \frac{C_r}{k_r^2} + \frac{D_r}{k_r^3} = A_{r+1} + \frac{B_{r+1}}{k_r} - \frac{C_{r+1}}{k_r^2} + \frac{D_{r+1}}{k_r^3}$$

i.e. if $\quad (A_{r+1} - A_r) + \frac{1}{k_r}(B_{r+1} - B_r) - \frac{1}{k_r^2}(C_{r+1} - C_r) + \frac{1}{k_r^3}(D_{r+1} - D_r) = 0$

i.e. if $\quad \lambda_r + \frac{1}{k_r}[-\lambda_r k_r (1+\theta_r+\psi_r)] - \frac{1}{k_r^2}[-\lambda_r k_r^2(\theta_r + 2\psi_r)] + \frac{1}{k_r^3}[-\lambda_r k_r^3 \psi_r] = 0$

i.e. if $\quad \lambda_r[1 - 1 - \theta_r - \psi_r + \theta_r + 2\psi_r - \psi_r] = 0$

which is satisfied.
Therefore the form function is continuous at the end of each stage and since it is continuous throughout each stage, z is a continuous function of f.

From (22), in the r th stage

$$\frac{dz}{df} = -B_r + 2C_r(1-f) - 3D_r(1-f)^2$$

will be continuous at the end of the r th stage if

$$-B_r + \frac{2C_r}{k_r} - \frac{3D_r}{k_r^2} = -B_{r+1} + \frac{2C_{r+1}}{k_r} - \frac{3D_{r+1}}{k_r^2}$$

i.e. if

$$(B_{r+1} - B_r) + \frac{2}{k_r}(C_r - C_{r+1}) - \frac{3}{k_r^2}(D_r - D_{r+1}) = 0$$

i.e. if

$$-\lambda_r k_r [1+\theta_r+\psi_r] + \frac{2}{k_r}\lambda_r k_r^2(\theta_r + 2\psi_r) - \frac{3}{k_r^2}[\lambda_r \psi_r k_r^3] = 0$$

i.e. if

$$-1 + \theta_r - \psi_r = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

In general $\frac{dz}{df}$ will be discontinuous at the end of each stage of burning.

If all the component charges are in cord form so that $\theta_r = 1, \psi_r = 0$, $\frac{dz}{df}$ will be throughout continuous.

Let us now investigate the particular case when

$$\frac{D_1}{\beta_1} = \frac{D_2}{\beta_2} = \cdots\cdots\cdots = \frac{D_n}{\beta_n}$$

so that

$$k_1 = k_2 = \cdots\cdots\cdots = k_n = 1$$

so that there is only one stage of burning.

From (25) and (26) the form function for the equivalent charge is

$$z = A_1 + B_1(1-f) - C_1(1-f)^2 + D_1(1-f)^3 \quad .. \quad .. \quad (28)$$

where

$$A_1 = 0$$

$$B_1 = \sum_{i=1}^{n} \lambda i + \sum_{i=1}^{n} \lambda i \theta i + \sum_{i=1}^{n} \lambda i \psi i$$

$$C_1 = \sum_{i=1}^{n} \lambda i \theta i + 2 \sum_{i=1}^{n} \lambda i \psi i$$

$$D_1 = \sum_{i=1}^{n} \lambda i \psi i \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (29)$$

Substituting from (29) in (28) and simplifying, the form function for the equivalent charge becomes

$$z = (1-f)\left(1 + f \sum_{i=1}^{n} \lambda_i \theta_i + f^2 \sum_{i=1}^{n} \lambda_i \psi i\right) \quad .. \quad .. \quad (30)$$

so that in this case, the composite charge has been reduced to a single charge with form function of the form

$$z = (1-f)(1 + \theta f + \psi f^2)$$

where θ is the weighted average of θ_i*'s* and ψ is the weighted average of ψ_i*'s*, the weights in each being λ_i*'s*.

4. Corner's and Clemmow's Methods of Fitting

Before proceeding with the different methods of dealing with the problem, we prove two lemmas which will be found useful later.

LEMMA 1:

$$\sum_{r=1}^{n} \left[\left(\sum_{i=r}^{n} xi\right)(y_r - y_{r-1})\right] = \sum_{i=1}^{n} x_i y_i \text{, if } y_0 = 0 \quad .. \quad .. \quad (31)$$

Proof:

$$L.H.S. = \sum_{r=1}^{n} \left[(x_r + x_{r+1} + \cdots\cdots + x_n)\,(y_r - y_{r-1}) \right]$$

$$= (x_1+x_2+\cdots\cdot+x_n)\,(y_1-y_0)+(x_2+x_3+\cdots\cdot+x_n)\,(y_2-y_1)$$

$$+\cdots\cdots\cdots+x_n(y_n-y_{n-1})$$

$$= x_1(y_1-y_0)+x_2(y_2-y_0)+\cdots\cdot+x_n(y_n-y_0)$$

$$= \sum_{i=1}^{n} x_i y_i \text{ as } y_0 = 0$$

$$= R.H.S.$$

LEMMA 2:

$$\sum_{r=1}^{n} \left[\left(\sum_{i=1}^{r-1} x_i \right) (y_r - y_{r-1}) \right] = 1 - \sum_{i=1}^{n} x_i y_i \qquad \ldots \quad \ldots \quad (32)$$

if

$$\sum_{i=1}^{n} x_i = 1,\ y_n = 1,\ y_0 = 0.$$

Proof: Using Lemma 1

$$L.H.S. = \sum_{r=1}^{n} \left[\left(1 - \sum_{i=r}^{n} x_i \right) (y_r - y_{r-1}) \right] = \sum_{r=1}^{n} (y_r - y_{r-1}) - \sum_{i=1}^{n} x_i y_i$$

$$= 1 - \sum_{i=1}^{n} x_i y_i$$

$$= R.H.S.$$

CORNER'S METHOD:

According to this method, we should choose θ so that the area under the curve

$$z = (1-f)(1+\theta f)$$

should be the same as under the $z-f$ curve for the equivalent charge, so that

$$\int_{1}^{0} (1-f)(1+\theta f)df = \sum_{r=1}^{n} \int_{1-\frac{1}{k_{r-1}}}^{1-\frac{1}{k_r}} [A_r + B_r(1-f) - C_r(1-f)^2 + D_r(1-f)^3]df$$

or

$$-\frac{1}{2} - \frac{\theta}{6} = \sum_{r=1}^{n} \left[-A_r \left(\frac{1}{k_r} - \frac{1}{k_{r-1}} \right) - \frac{B_r}{2} \left(\frac{1}{k_r^2} - \frac{1}{k_{r-1}^2} \right) + \frac{C_r}{3} \left(\frac{1}{k_r^3} - \frac{1}{k_{r-1}^3} \right) - \frac{D_r}{4} \left(\frac{1}{k_r^4} - \frac{1}{k_{r-1}^4} \right) \right]$$

using the lemmas and remembering that $\frac{1}{k_0} = 0, \frac{1}{k_n} = 1$,

$$-\frac{1}{2} - \frac{\theta}{6} = -1 + \sum_{i=1}^{n} \frac{\lambda i}{ki} - \frac{1}{2} \sum_{i=1}^{n} \frac{\lambda_i(1+\theta_i+\psi_i)}{k_i} + \frac{1}{3} \sum_{i=1}^{n} \frac{\lambda_i(\theta_i+2\psi_i)}{k_i} - \frac{1}{4} \sum_{i=1}^{n} \frac{\lambda_i\psi_i}{k_i}$$

$$= -1 + \frac{1}{2} \sum_{i=1}^{n} \frac{\lambda_i}{k_i} - \frac{1}{6} \sum_{i=1}^{n} \frac{\lambda_i\theta_i}{k_i} + \frac{5}{12} \sum_{i=1}^{n} \frac{\lambda_i\psi_i}{k_i}$$

$$\therefore \theta = 3 - 3 \sum_{i=1}^{n} \frac{\lambda_i}{k_i} + \sum_{i=1}^{n} \frac{\lambda_i\theta_i}{k_i} - \frac{5}{2} \sum_{i=1}^{n} \frac{\lambda_i\psi_i}{k_i} \tag{33}$$

Particular cases

Case (I):

Let $\frac{D_1}{\beta_1} = \frac{D_2}{\beta_2} = \cdots\cdots = \frac{D_n}{\beta_n}$

so that $k_1 = k_2 = \cdots\cdots = k_n = 1$

and there is only one distinct stage of burning.

$$\theta = 3 - 3 \sum_{i=1}^{n} \lambda i + \sum_{i=1}^{n} \lambda i\theta i - \frac{5}{2} \sum_{i=1}^{n} \lambda i\psi i$$

or

$$\theta = \sum_{i=1}^{n} \lambda i\theta i - \frac{5}{2} \sum_{i=1}^{n} \lambda i\psi i \quad \ldots \ldots \ldots \ldots \ldots \tag{34}$$

Case (II):

Let $k_i = k_2 = \ldots\ldots\ldots = k_n = 1; \psi_1 = \psi_2 = \ldots\ldots\ldots = \psi_n = 0$

so that all the component charges have their form functions in the standard form

$$z = (1-f)(1+\theta f)$$

From (34)

$$\theta = \sum_{i=1}^{n} \lambda_i\theta_i \quad \ldots \ldots \ldots \ldots \tag{35}$$

so that in this case θ is the weighted mean of θ_i's, the weights being λ_i's.

Case (III):

Let $\psi_1 = \psi_2 = \cdots = \psi_n = 0$

In this case from (33)

$$\theta = 3 - 3 \sum_{i=1}^{n} \frac{\lambda_i}{k_i} + \sum_{i=1}^{n} \frac{\lambda_i\theta_i}{k_i} \quad \ldots \ldots \ldots \tag{36}$$

Case (IV):

$n = 2$; $\psi_1 = \psi_2 = 0$, then from (33) or (36)

$$\theta = 3-3\left(\frac{\lambda_1}{k_1}+\frac{\lambda_2}{k_2}\right)+\left(\frac{\lambda_1\theta_1}{k_1}+\frac{\lambda_2\theta_2}{k_2}\right)$$

But $$k_2 = 1, \quad \lambda_1+\lambda_2 = 1$$

$\therefore$ $$\theta = 3-\frac{3\lambda_1}{k_1}-3\,\frac{(1-\lambda_1)}{1}+\frac{\lambda_1\theta_1}{k_1}+(1-\lambda_1)\theta_2$$

or $$\theta = 3\lambda_1\left(1-\frac{1}{k_1}\right)+\frac{\lambda_1\theta_1}{k_1}+(1-\lambda_1)\theta_2 \qquad \ldots \quad \ldots \quad \ldots \quad (37)$$

which agrees with Corner's result when $\theta_1 = \theta_2 = 0$.

Case (V):

$n = 1$, then from (36)

$\theta = 3-3\cdot\frac{1}{1}+1.\frac{\theta_1}{1} = \theta_1$, as it should be.

In all the above cases and others, the value of θ obtained can be used in a normal Ballistic method only if $\theta \leqslant 1$. In the cases when $\theta > 1$, which will, in practice, occur only exceptionally, i.e. when the composite charge is more degressive than a single cord, a form function more complicated than the standard one should be used. The obvious choice is

$$z = (1-f)(1+\theta f+\psi f^2)$$

which would cover all strengths of degression up to and including spheres or cubes ($\theta = \psi = 1$). Corner has indicated the necessity for use of this form in exceptional cases, but has not indicated a method of estimating θ and ψ. Obviously his method would fail in such a case. We shall see later that the method of least squares would be of help.

Clemmow's Method:

The method requires that we should choose θ so that $z = (1-f)\ (1+\theta f)$ passes through the common point. Obviously it is applicable only when $n = 2$. It fails also when

$$\frac{D_1}{\beta_1} = \frac{D_2}{\beta_2}$$

The common point of the form functions of the two stages of burning is

$$f = 1-\frac{1}{k_1}$$

$$z = \lambda_1+\frac{\lambda_2(1+\theta_2+\psi_2)}{k_1}-\frac{\lambda_2(\theta_2+2\psi_2)}{k_1^2}+\frac{\lambda_2\psi_2}{k_1^3}$$

$$= \lambda_1+\frac{1-\lambda_1}{k_1}\left[1+\theta_2+\psi_2-\frac{\theta_2+2\psi_2}{k_1}+\frac{\psi_2}{k_1^2}\right]$$

Substituting these values in $z = (1-f)\ (1+\theta f)$

and solving for θ, we get

$$\theta = \lambda_1 k_1 + (1-\lambda_1)\theta_2 + (1-\lambda_1)\left(1-\frac{1}{k_1}\right)\psi_2 \quad \cdots \quad (38)$$

which agrees with Clemmow's result in the particular case $\psi_2 = 0$.

Incidentally we note that the value of θ is independent of both θ_1 and ψ_1.

If the value of θ given by (38) is greater than 1, we have to fit the more degressive form

$$z = (1-f)(1+\theta f+\psi f^2) \quad \cdots \quad (39)$$

but again the method would fail to determine more than one parameter.

When the composite charge consists of three components, this form can be fitted. We can easily determine θ and ψ by making (39) pass through the two common points of the three form functions of the three stages of burning.

5. Method of Least Squares

According to the principle of least squares, we should choose θ so as to minimize the quantity

$$\sum_{r=1}^{n}\left\{\int_{1-\frac{1}{k_{r-1}}}^{1-\frac{1}{k_r}} [(1-f)(1+\theta f)-(A_r+B_r(1-f)-C_r(1-f)^2+D_r(1-f)^3)]^2\, df\right\}$$

Differentiating under the sign of integration, we get the following equation for determining θ:

$$\sum_{r=1}^{n}\left\{\int_{1-\frac{1}{k_{r-1}}}^{1-\frac{1}{k_r}} f(1-f)\left[(1-f)(1+\theta f)-(A_r+B_r(1-f)-C_r(1-f)^2+D_r(1-f)^3)\right] df\right\} = 0.$$

Let $1-f = y$, then

$$\sum_{r=1}^{n}\left\{\int_{\frac{1}{k_{r-1}}}^{\frac{1}{k_r}} y(1-y)\left[y(1+\theta-\theta y)-(A_r+B_r y-C_r y^2+D_r y^3)\right] dy\right\} = 0$$

or

$$\sum_{r=1}^{n}\left\{\int_{\frac{1}{k_{r-1}}}^{\frac{1}{k_r}} \left[-A_r y+y^2(A_r+1+\theta-B_r)+y^3(-\theta+C_r-1-\theta+B_r) +y^4(-D_r+\theta-C_r)+D_r y^5\right] dy\right\} = 0$$

or

$$-\tfrac{1}{2}\sum_{r=1}^{n} A_r\left(\frac{1}{k_r^2}-\frac{1}{k_{r-1}^2}\right)+\tfrac{1}{3}\sum_{r=1}^{n}(A_r+1+\theta-B_r)\left(\frac{1}{k_r^3}-\frac{1}{k_{r-1}^3}\right)$$
$$+\tfrac{1}{4}\sum_{r=1}^{n}(-1+C_r+B_r-2\theta)\left(\frac{1}{k_r^4}-\frac{1}{k_{r-1}^4}\right)$$
$$+\tfrac{1}{5}\sum_{r=1}^{n}(-D_r+\theta-C_r)\left(\frac{1}{k_r^5}-\frac{1}{k_{r-1}^5}\right)$$
$$+\tfrac{1}{6}\sum_{r=1}^{n}D_r\left(\frac{1}{k_r^6}-\frac{1}{k_{r-1}^6}\right)=0.$$

Using the lemmas, we get

$$-\tfrac{1}{2}\left[1-\sum_{i=1}^{n}\frac{\lambda_i}{k_i^2}\right]+\tfrac{1}{3}(1+\theta)+\tfrac{1}{3}\left(1-\sum_{i=1}^{n}\frac{\lambda_i}{k_i^3}\right)$$
$$-\tfrac{1}{3}\left[\sum_{i=1}^{n}\frac{\lambda_i}{k_i^2}+\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^2}+\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^2}\right]+\tfrac{1}{4}(-1-2\theta)$$
$$+\tfrac{1}{4}\left[\sum_{i=1}^{n}\frac{\lambda_i}{k_i^3}+\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^3}+\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^3}\right]$$
$$+\tfrac{1}{4}\left[\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^2}+2\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^2}\right]+\tfrac{1}{5}\,\theta$$
$$-\tfrac{1}{5}\left[\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^3}+2\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^3}\right]-\tfrac{1}{5}\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^2}$$
$$+\tfrac{1}{6}\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^3}=0.$$

Simplifying, we get

$$\theta=\tfrac{1}{2}\left[5-10\sum_{i=1}^{n}\frac{\lambda_i}{k_i^2}+5\sum_{i=1}^{n}\frac{\lambda_i}{k_i^3}+5\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^2}-3\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^3}+2\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^2}-\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^3}\right]. \quad .. \quad (40)$$

If the value of θ obtained from (40) is greater than unity, we have to fit the more degressive form (39). Therefore we have to minimize for variations in θ and ψ, the quantity

$$\sum_{i=1}^{n}\left\{\int_{1-\frac{1}{k_{r-1}}}^{1-\frac{1}{k_r}}\left[(1-f)(1+\theta f+\psi f^2)-(A_r+B_r(1-f)-C_r(1-f)^2+D_r(1-f)^3)\right]^2 df\right\}$$

Differentiating under the sign of integration with respect to θ and ψ, we get the two equations for the determination of θ and ψ.

$$\sum_{r=1}^{n}\left\{\int_{1-\frac{1}{k_{r-1}}}^{1-\frac{1}{k_r}} f(1-f)\left[(1-f)(1+\theta f+\psi f^2)-(A_r+B_r(1-f)-C_r(1-f)^2+D_r(1-f)^3)\right]df\right\}=0$$

and

$$\sum_{r=1}^{n}\left\{\int_{1-\frac{1}{k_{r-1}}}^{1-\frac{1}{k_r}} f^2(1-f)\left[(1-f)(1+\theta f+\psi f^2)-(A_r+B_r(1-f)-C_r(1-f)^2+D_r(1-f)^3)\right]df\right\}=0.$$

Making the substituting $1-f=y$, integrating, using the lemmas, we get after a little simplification

$$2\theta+\psi=5-10\sum_{i=1}^{n}\frac{\lambda_i}{k_i^2}+5\sum_{i=1}^{n}\frac{\lambda_i}{k_i^3}+5\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^2}-3\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^3}+2\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^2}-\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^3} \quad \ldots \quad (41)$$

and

$$7\theta+4\psi=21-70\sum_{i=1}^{n}\frac{\lambda_i}{k_i^2}+70\sum_{i=1}^{n}\frac{\lambda_i}{k_i^3}-21\sum_{i=1}^{n}\frac{\lambda_i}{k_i^4}+35\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^2}-42\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^3}+14\sum_{i=1}^{n}\frac{\lambda_i\theta_i}{k_i^4}+14\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^2}-14\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^3}+4\sum_{i=1}^{n}\frac{\lambda_i\psi_i}{k_i^4} \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

From (41) and (42) we can easily get estimated values of θ and ψ. These equations can easily be simplified when $\psi_i=0$ for all i.

PARTICULAR CASES:

Case (I): $k_1 = k_2 = \dots\dots\dots\dots = k_n = 1$

then (41) and (42) give

$$2\theta+\psi = 5-10+5+5\sum_{i=1}^{n}\lambda_i\theta_i-3\sum_{i=1}^{n}\lambda_i\theta_i+2\sum_{i=1}^{n}\lambda_i\psi_i-\sum_{i=1}^{n}\lambda_i\psi_i$$

$$= 2\sum_{i=1}^{n}\lambda_i\theta_i+\sum_{i=1}^{n}\lambda_i\psi_i \quad \dots \quad (43)$$

$$7\theta+4\psi = 21-70+70-21+35\sum_{i=1}^{n}\lambda_i\theta_i-42\sum_{i=1}^{n}\lambda_i\theta_i+14\sum_{i=1}^{n}\lambda_i\theta_i$$

$$+14\sum_{i=1}^{n}\lambda_i\psi_i-14\sum_{i=1}^{n}\lambda_i\psi_i+4\sum_{i=1}^{n}\lambda_i\psi_i$$

$$= 7\sum_{i=1}^{n}\lambda_i\theta_i+4\sum_{i=1}^{n}\lambda_i\psi_i \quad \dots \quad (44)$$

From (43) and (44)

$$\theta = \sum_{i=1}^{n}\lambda_i\theta_i$$

$$\psi = \sum_{i=1}^{n}\lambda_i\psi_i$$

which agrees with the result of equation (30) verifying that in the case when there is only one stage of burning, the composite charge behaves in exactly the same way as a simple charge with form function

$$z = (1-f)\left(1+f\sum_{i=1}^{n}\lambda_i\theta_i+f^2\sum_{i=1}^{n}\lambda_i\psi_i\right).$$

Case (II): $n = 2$, $\lambda_1+\lambda_2 = 1$, $k_2 = 1$, then from (40)

$$\theta = \tfrac{1}{2}\Big[5-10\frac{\lambda_1}{k_1^2}-\frac{10(1-\lambda_1)}{1}+\frac{5\lambda_1}{k_1^3}+5(1-\lambda_1)$$

$$+\frac{5\lambda_1\theta_1}{k_1^2}+5(1-\lambda_1)\theta_2-\frac{3\lambda_1\theta_1}{k_1^3}-3(1-\lambda_1)\theta_2$$

$$+\frac{2\lambda_1\psi_1}{k_1^2}+2(1-\lambda_1)\psi_2-\frac{\lambda\psi_1}{k_1^3}-(1-\lambda_1)\psi_2\Big] \quad \dots \quad (45)$$

If $\psi_1 = 0$, $\psi_2 = 0$, $\lambda_1 = \lambda$, $k_1 = k$

$$\theta = \frac{1}{2}\left[5\lambda-\frac{10\lambda}{k^2}+\frac{5\lambda}{k^3}+\lambda\theta_1\left(\frac{5}{k^2}-\frac{3}{k^3}\right)+2(1-\lambda)\theta_2\right] \quad \dots \quad (46)$$

6. Comparison of the Three Values of θ

For the particular case $n = 2$, $\psi_1 = \psi_2 = 0$, we have omitting the suffix 1,

$$\left.\begin{aligned}\theta &= 3\left(1-\frac{1}{k}\right)\lambda+\frac{\lambda\theta_1}{k}+(1-\lambda)\theta_2 \text{ [CORNER]}\\ \theta &= k\lambda+(1-\lambda)\theta_2 \text{ [CLEMMOW]}\\ \theta &= \frac{5}{2}\lambda-\frac{5\lambda}{k^2}+\frac{5}{2}\frac{\lambda}{k^3}+\frac{1}{2}\lambda\theta_1\left(\frac{5}{k^2}-\frac{3}{k^3}\right)+(1-\lambda)\theta_2 \text{ [LEAST SQUARES]}\end{aligned}\right\} \quad (47)$$

We note the following points:

(i) The value of θ in the second method is independent of θ_1 which is rather undesirable.

(ii) The value of θ in each case depends on λ corresponding to the ratio of the energies of the two component charges, k giving the effect of the ballistic sizes and rates of burning constants and θ_1, θ_2 depending on the shapes of the component charges.

(iii) The terms containing θ_2 is the same in all the three methods. In general, the terms containing θ_n will always be the same in both Corner's and Least Square methods; for from (33) the coefficient of θ_n is λ_n as $k_n = 1$ and from (40) the coefficient of θ_n is $\frac{1}{5}(5\lambda_n-5\lambda_n)=\lambda_n$

(iv) The ratios of the constant terms and the ratios of the coefficients of θ_1 are independent of λ and depend on k only. Thus agreement will depend more on the value of k and θ_1 than on the values of λ or θ_2

(v) *Corner's and Least Squares methods:*

The condition that the constant terms are the same is that

$$3-\frac{3}{k}=\frac{5}{2}-\frac{5}{k_2}+\frac{5}{2k^3}$$

or
$$k^3-6k^2+10k-5=0$$

or
$$k=1 \text{ or nearly } \frac{7}{2} \text{ or } \frac{3}{2}$$

Coefficient of θ_1 will agree if

$$\frac{2}{k}=\frac{5}{k^2}-\frac{3}{k^3}$$

giving
$$k=1 \text{ or } \frac{3}{2}$$

For $k = 1$, the agreement is perfect as it should be; for $k = \frac{3}{2}$ the agreement is very good. For other values, the agreement is not so good

(vi) *Corner's and Clemmow's methods:*

(a) The two values of θ will agree if

$$\theta_2(1-\lambda)+k\lambda = 3\lambda\left(1-\frac{1}{k}\right)+\frac{\lambda\theta_1}{k}+(1-\lambda)\theta_2$$

i.e. if
$$k^2-3k+3-\theta_1 = 0$$

i.e. if
$$k = \frac{3\pm\sqrt{4\theta_1-3}}{2}$$

If $\theta_1 < \frac{3}{4}$, the two values of θ are bound to be different. If $\theta_1 = \frac{3}{4}$, the two values agree if $k = \frac{3}{2}$. In this case all the three values of θ are in good agreement.

If $\theta_1 = 1$, the two values agree if $k = 1$ or 2.

(b) If $\theta_1 = \theta_2 = 0$, i.e. if both the charges are tubular, Clemmow's values will always be greater than Corner's if

$$k > 3\left(1-\frac{1}{k}\right)$$

i.e. if
$$\left(k-\frac{3}{2}\right)^2 > -\frac{3}{4}$$

which is always true.

Thus in this case, Clemmow's method will always give a greater value than Corner's method. In this case, as has been shown by Clemmow, his method always over-estimates.

(vii) *All three methods:*

We consider the following important cases:

CASE	θ_1	θ_2	λ	k	θ CORNER	θ CLEMMOW	θ LEAST SQUARES
1	0	0	$\frac{1}{2}$	2	$\frac{3}{4}$	1	$\frac{25}{32}$
2	1	0	$\frac{1}{2}$	2	1	1	1
3	0	0	$\frac{1}{3}$	3	$\frac{2}{5}$	1	$\frac{55}{81}$
4	1	0	$\frac{1}{3}$	3	$\frac{7}{9}$	1	$\frac{61}{81}$
5	1	1	1	2	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$

In the last case, all the three values of θ are inadmissible and we have to use the method of least squares to estimate θ and ψ for the form $z = (1-f)(1+\theta f+\psi f^2)$ we get

$$2\theta+\psi = 3$$

$$7\theta+4\psi = 11 \text{ (nearly)}$$

$\therefore$
$$\theta = 1,\ \psi = 1 \text{ (approximately)}$$

and the equivalent charge has the form function

$$z = (1-f)(1+f+f^2) = 1-f^3$$

i.e. the form function is same as that for a single sphere or cube.

From the values we have calculated above and other similar calculations, we find that, in general, the agreement between the estimates given by Corner's and Least Squares method is very good, and since Least Squares estimate is theoretically more plausible we can consider both the estimates as reliable. On the other hand, estimates given by Clemmow's method may give a sufficiently biased estimate for θ.

7. Some Cases where Composite Charges with more than two Component Charges are Likely to be Useful

Case (I): Let $k_1 = k_2 = k_3 = 1$; $\psi_1 = \psi_2 = \psi_3 = 0$; $\gamma_1 = \gamma_2 = \gamma_3$ then we have

From (21) $C = C_1+C_2+C_3$ .. .. (48)

From (20) $CF = C_1F_1+C_2F_2+C_3F_3$ .. .. (49)

From (30) $CF\theta = C_1F_1\theta_1+C_2F_2\theta_2+C_3F_3\theta_3$ .. .. (50)

If we are given three component charges with same $\frac{D_i}{\beta_i}$, we can choose masses C_1, C_2, C_3 from the above three equations to give a composite charge which would behave as a single charge with given mass C, given force constant F and a given form-factor θ.

Case (II): Let $k_1 = k_2 = k_3 = k_4 = 1$; $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4$ then we have

From (21) $C = C_1+C_2+C_3+C_4$ .. .. (51)

From (20) $CF = C_1F_1+C_2F_2+C_3F_3+C_4F_4$ .. .. (52)

From (30) $CF\theta = F_1C_1\theta_1+F_2C_2\theta_2+F_3C_3\theta_3+F_4C_4\theta_4$ .. .. .. (53)

From (30) $CF\psi = C_1F_1\psi_1+F_2C_2\psi_2+F_3C_3\psi_3+F_4C_4\psi_4$ .. .. .. (54)

From these we can determine the charge masses C_1, C_2, C_3, C_4, so that the composite charge behaves as a single charge with given mass C, given force constant F and with given form function

$$z = (1-f)(1+\theta f+\psi f^2)$$

provided the component charges have same values for $\frac{D_i}{\beta_i}$

Case (III): Let $n = 3$ and let $k_1 > k_2 > 1$, so that there are three stages of burning, then the equation (50) is replaced by (using (40))

$$CF\theta = \frac{1}{2}\left[5CF-10\left(\frac{C_1F_1}{k_1^2}+\frac{C_2F_2}{k_2^2}+\frac{C_3F_3}{k_3^2}\right)+5\left(\frac{C_1F_1}{k_1^3}+\frac{C_2F_2}{k_2^3}+\frac{C_3F_3}{k_3^3}\right)\right.$$
$$+5\left(\frac{C_1F_1\theta_1}{k_1^2}+\frac{C_2F_2\theta_2}{k_2^2}+\frac{C_3F_3\theta_3}{k_3^2}\right)$$
$$\left.-3\left(\frac{C_1F_1\theta_1}{k_1^3}+\frac{C_2F_2\theta_2}{k_2^3}+\frac{C_3F_3\theta_3}{k_3^3}\right)\right] \quad \text{.. .. .. (55)}$$

From (48), (49) and (55) we can solve for charge masses C_1, C_2, C_3 so that the composite charge will behave as a single charge with given mass C, given force constant F, and with form-factor approximately equal to θ.

Case (IV): Let $n = 4, k_1 > k_2 > k_3 > 1$

If we want the composite charge to behave approximately as a single charge with form function

$$z = (1-f)(1+\theta f+\psi f^2)$$

we use the equations (51), (52) and (41), (42) to solve for C_1, C_2, C_3, C_4.

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9. Abstract

In this paper, the general theory of composite charges as given by Corner and Clemmow has been extended in the following directions:

(i) The composite charge may consist of n component charges instead of two in their theory.

(ii) The force constants of the component charges can be different. Actually the force constant for the equivalent charge is deduced from our definition for it.

(iii) Up to the stage of all-burnt at least, even $\gamma_1, \gamma_2, \ldots, \gamma_n$ can be different, provided any possible variations in γ during the burning period can be neglected; otherwise the analysis becomes more complicated and it is better to assume $\gamma_1 = \gamma_2 = \cdots = \gamma_n$.

(iv) The component charges can have general form functions of the form $z = \phi(f)$. The theory has, however, been illustrated when the component charges have the cubic form function of the form $z = (1-f)(1+\theta f+\psi f^2)$.

(v) The methods of fitting as proposed by Clemmow and Corner are shown to fail under certain conditions and a theoretically more satisfactory method based on the principle of least squares has been suggested and worked out.

(vi) It has been shown that in the case $\frac{D_1}{\beta_1} = \frac{D_2}{\beta_2} = \ldots = \frac{D_n}{\beta_n}$ the composite charge behaves in all respects as a single charge.

In the introduction, the method of Clemmow and Corner has been compared with the other two methods of dealing with the general theory of composite charges given recently, and in section 7, we have pointed out certain situations in which the theory for $n > 2$ is likely to be useful. The solution of the equations of Internal Ballistics for the equivalent charge [form function method] has been obtained and will be published separately.

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