A NOTE ON THE RADIATIVE CORRECTIONS TO COMPTON SCATTERING

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(Communicated by S. N. Bose, F.N.I.)

(*Received November 28, 1955 ; read March 2, 1956*)

Introduction

The development of Quantum electrodynamics in recent years has enabled one to evaluate the radiative correction to any desired order of approximation and for any process participated by free electrons, positrons and photons. So far as free particles are concerned it seems that the Dyson (1949) approach is the best for treatment of such problems. The analyses of both the Schwinger (1949) and Feynman (1949) theories are best appreciated by the method given by Dyson. A fourth order radiative correction to Compton scattering was first worked out by Schafroth (1949) using the method of Schwinger and the same calculation was done by Brown and Feynman (1952) using the technique of the latter. The two results, however, disagree. Here we shall reconsider this very problem from the point of view of Dyson. In this present paper, however, we shall not go into the details of the calculation of the formidable integrals which occur in both the above-mentioned papers, but shall simply go to see whether they used the right number of terms in working out the matrix elements.

Matrix Element

In the interaction representation the S matrix expansion in powers of α is

$$S = \sum_{n=0}^{\infty} (-i\alpha)^n \frac{1}{n!} \int_{-\infty}^{\infty} d^4x_1 \dots \int_{-\infty}^{\infty} d^4x_n \, P\left(H_i(x_1)\dots H_i(x_n)\right)$$

where P is the well-known Dyson's P bracket (1949). For processes in which no new particles are created or destroyed the coefficients of odd powers of α vanish. In our case, i.e. in the case of Compton scattering, the matrix coefficients of α^2 leads to the usual Klein-Nishina cross-section; all other higher powers of α lead to the radiative corrections after proper mass and charge renormalization. In our case we shall consider the first radiative correction term. However, in considering graphs attached to the various types of matrix elements we shall not consider the renormalization elements. In other words for the sake of understanding all possible graphs we shall take in the interaction representation the interaction Hamiltonian to be

$$H_i = \psi^*\gamma_\mu A_\mu \psi \text{ instead of}$$

$$H_i = \psi^*\gamma_\mu A_\mu \psi - \delta m \psi^* \beta \psi.$$

The fourth order matrix element now contains the term

$$\psi^*(x_1)\gamma_\mu A_\mu(x_1)\psi(x_1)\psi^*(x_2)\gamma_2 A_2(x_2)\psi(x_2)\psi^*(x_3)\gamma_\sigma A_\sigma(x_3)\psi(x_3)\psi^*(x_4)\gamma_\rho A_\rho(x_4)\psi(x_4)\,.. \quad (1)$$

and all possible permutations of (1234). In our case one must annihilate the electron in the initial state and one must create an electron in the final state and similarly for the A's.

The various types of graphs for various types of pairing of (1) are exhibited below, in which we have purposely dropped the γ's and have written ψ_i for $\psi(x_i)$.

I. $\psi_4^* A_4 \psi_1 A_1 (\psi_2 \psi_1^*)(\psi_4 \psi_3^*)(\psi_3 \psi_2^*)(A_3 A_2)$

II. $\psi_3^* A_4 \psi_2 A_1 (\psi_4 \psi_1^*)(\psi_1 \psi_2^*)(\psi_3 \psi_4^*)(A_2 A_3)$

III. $\psi_4^* A_4 \psi_2 A_1 (\psi_3 \psi_2^*)(\psi_1 \psi_3^*)(\psi_4 \psi_1^*)(A_2 A_3)$

IV. $\psi_3^* A_4 \psi_1 A_1 (\psi_3 \psi_2^*)(\psi_2 \psi_4^*)(\psi_4 \psi_1^*)(A_2 A_3)$

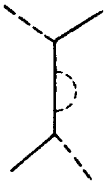
V. $\psi_4^* A_4 \psi_2 A_1 (\psi_4 \psi_3^*)(\psi_3 \psi_1^*)(\psi_1 \psi_2^*)(A_2 A_3)$

VI. $\psi_3^* A_4 \psi_1 A_1 (\psi_3 \psi_4^*)(\psi_4 \psi_2^*)(\psi_2 \psi_1^*)(A_2 A_3)$

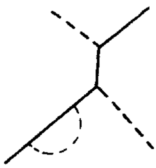
VII. $\psi_4^* A_4 \psi_1 A_2 (\psi_3^* \psi_2)(\psi_2^* \psi_3)(\psi_4 \psi_1^*)(A_1 A_3)$

VIII. $\psi_4^* A_3 \psi_1 A_1 (\psi_4 \psi_1^*)(\psi_3^* \psi_2)(\psi_2^* \psi_3)(A_2 A_4)$

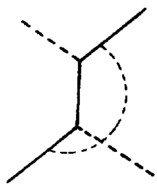
IX. $\psi_4^* A_2 \psi_4 A_1 (\psi_3 \psi_1^*)(\psi_1 \psi_2^*)(\psi_2 \psi_3^*)(A_4 A_3)$



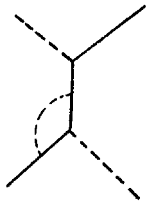
I



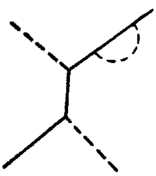
III



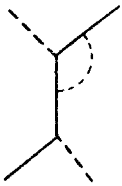
II



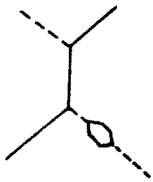
V



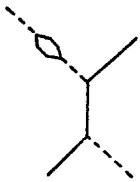
IV



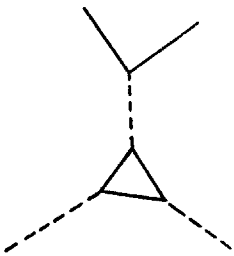
VI



VII



VIII



IX

Amongst all these graphs I, III and IV have got self energy parts. However, contributions from graphs I, III, IV, V and VI taken together give small and finite contribution after mass renormalization. The contribution from graphs VII and VIII can be shown to be equal to zero by a frank appeal to the gauge invariance of the expression. These, however, represent the effects of vacuum polarization as is obvious from the figure. The graph IX however vanishes according to the well-known Furry theorem (1937), which is effectively due to charge conjugation.

The graphs can easily be identified with the different terms of equation (34) of Schafroth (1949). For proper understanding one needs, however, transform the equation (34) of Schafroth in the momentum space integrals. The terms (I), (II), (III), (IV), (V) of (34) of Schafroth with their complex conjugate are the (II), (V, VI), (I), (III, IV), (VII, VIII) of ours respectively. In the calculation of Brown and Feynman (1952), however, their (J), (K'), (K''), (L), (M'), (M''), (N'), (N'') graphs are the (II), (V), (VI), (I), (III), (IV), (VII), (VIII) of ours respectively.

Conclusion

The various non-vanishing valued diagrams that came in the calculation of the matrix element from the Dyson analysis have properly been taken by both Schafroth and Brown and Feynman. The diagram IX was considered by Schafroth but it was omitted by Brown according to Feynman theory as it should give zero contribution. Our analysis thus shows that the difference in the final results of the two authors are due to the differences in the actual evaluation of the various elementary parametric integrals in which the final calculation was reduced.

Abstract

In this paper, the author has made a study of the various matrix elements that occur in the calculation of the Compton scattering cross-section. Further, following Dyson he has also identified the various Feynman graphs with the analytic expressions of Schafroth.

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Issued September 17, 1956.