EFFECT OF BORE RESISTANCE ON MAXIMUM PRESSURE AND MUZZLE VELOCITY FOR COMPOSITE CHARGE

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1. Introduction

The main problem of internal ballistics is the calculation of maximum pressure and muzzle velocity for given loading conditions in a gun. In the methods of solution of internal ballistic equations the resistance due to the engraving of the band is taken into account either by the use of 'shot-start pressure' or by increasing the rate of burning constant β and the resistance which persists over much of the travel of a projectile is taken into account by an appropriate increase of shot mass. This is a very special form of resistance and gives a rough allowance for the energy lost against friction, but fails to represent the relatively high resistance in the very earliest stages and is probably never a good approximation to the true resistance found in practice. Also it does not enable the shape of the resistance travel curve to be varied in studying a given firing, yet the form changes against one's wishes when the charge is altered.

Corner (1949) has given a method by which the change in muzzle velocity, peak pressure and position of all-burnt can be computed quickly for a bore resistance of any desired form, provided the magnitude of resistance is such that it does not halt the shot for a time. His method is based upon the choice of a standard bore resistance, the determination of its ballistic effects, and the construction of more realistic resistance laws by superposition of standard forms suitably spaced along the travel. The method is analogous to that of finding the weighing factors while calculating the change in range due to perturbations such as head wind, which occur with varying strength at different heights. In this paper the author has found the effect of bore resistance on maximum pressure and muzzle velocity for composite charge on the same lines and assumptions as those of Corner. Also the γ's for the two propellants are taken to be equal, since for most of the propellants γ are practically the same.

If A be the area of cross-section of the bore, then as standard bore resistance we take a force AP_0, constant over a travel Δx near the point x_0, and zero at all other shot positions; it is assumed that Δx is infinitesimal and $P_0 \Delta x$ is finite. The effect on ballistics to the first order in $P_0 \Delta x$ is calculated by subtracting an energy $AP_0 \Delta x$ from the kinetic energy of the projectile as it passes through the point x_0. For travels less than x_0 the solution is the normal solution of the ballistic equations and for travels greater than x_0 the ballistic equations are exactly the same as before except for a perturbing term linear in $P_0 \Delta x$. By taking a specific example Corner has shown that first order theory is adequate to represent the effect of a concentrated bore resistance except when this is sufficient to halt the projectile.

Before proceeding to the solution we must note two limitations of this method. Firstly, if the velocity of the shot V_0 at the point where resistance occurs is zero, the infinitesimal time $\Delta x/V_0$ of the action of resistance does not remain infinitesimal, hence initial resistance to the motion of the shot cannot be represented by this model. Secondly, if the kinetic energy of the shot at the moment the resistance occurs is less than $AP_0\Delta x$ the method breaks down, for it gives the projectile an imaginary velocity after crossing the resistance. In practice the projectile would come to rest and would restart after the pressure had built up behind the shot that is after a finite time which means that $\Delta x/V_0$ is not infinitesimal.

2. The Basic Equations

With the usual notations the basic equations before the occurrence of resistance are as follows:

The energy equation is

$$Z_1C_1R(T_{01}-T)+Z_2C_2R(T_{02}-T) = \frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C_1+C_2}{3}\right)V^2 \quad \ldots \quad (1)$$

where $\bar{\gamma}$ is the value of γ suitably increased to allow for heat loss.

The equation of state of the gas is

$$AP(x+l) = (RTC_1Z_1+RTC_2Z_2)\frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad \ldots \quad (2)$$

where the length l is defined by

$$Al = U-\left(\frac{C_1}{\delta_1}+\frac{C_2}{\delta_2}\right)$$

The form functions are

$$Z_1 = (1-f_1)(1+\theta_1 f_1) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3a)$$

The rate of burning equations are

$$D_1\frac{df_1}{dt} = -\beta_1 P \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$D_2\frac{df_2}{dt} = -\beta_2 P \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4a)$$

The dynamical equation of the shot is

$$\left(W_1+\frac{C_1+C_2}{2}\right)\frac{dv}{dt} = AP \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where W_1 is the effective weight of the projectile accounting for its rotational inertia.

In the above equations C_1, F_1, β_1, D_1, θ_1, f_1, Z_1, T_{01} refer to the first charge and C_2, F_2, β_2, D_2, θ_2, f_2, Z_2, T_{02} refer to the second charge.

3. Simultaneous and Non-Simultaneous Burning of the Charges.

Dividing equations (4) and (4a),

$$\frac{df_1}{df_2} = \frac{\beta_1 D_2}{\beta_2 D_1} = K \text{ (say)}$$

Integrating and applying the conditions that initially $f_1 = f_2 = 1$, we get

$$(1-f_1) = K(1-f_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Now two cases arise:

(i) The two charges may burn out simultaneously.

(ii) The two charges may burn out at different times.

Case (*i*) *:* If the two charges burn out simultaneously then since at all-burnt position $f_1 = f_2 = 0$, we have from (6), $K = 1$, i.e. $\beta_1 D_2 = \beta_2 D_1$, as the condition for simultaneous burning.

Case (*ii*) *:* Let the charge C_1 burn out first. If suffix 'b' refers to this position we have from (6),

$$f_{2b} = 1 - \frac{1}{K}$$

But f_{2b} must be a positive fraction which means $K > 1$.
Thus the condition that the charge C_1 burns out first is that $\beta_1 D_2 > \beta_2 D_1$.
Similarly if charge C_2 burns out first the condition is $\beta_1 D_2 < \beta_2 D_1$.

4. Solution of the Equations

The basic equations (2) to (5) remain true after the resistance and the equation (1) is replaced by

$$Z_1 C_1 R(T_{01} - T) + Z_2 C_2 R(T_{02} - T) = (\bar{\gamma} - 1)\left[\left(W_1 + \frac{C_1 + C_2}{3}\right)\frac{V^2}{2} + AP_0 \Delta x\right] \quad (7)$$

Also just before reaching the resistance the kinetic energy of the charge and projectile is $\frac{1}{2}\left(W_1 + \frac{C_1+C_2}{3}\right)V_0^2$ and immediately afterwards it is $\frac{1}{2}\left(W_1 + \frac{C_1+C_2}{3}\right)V_0^2 - AP_0\Delta x$. If V_1 is the velocity of the shot immediately after the resistance, then, assuming that the total kinetic energy is at all times $\frac{1}{2}\left(W_1 + \frac{C_1+C_2}{3}\right)V^2$, where V is the velocity, we have

$$\frac{1}{2}\left(W_1 + \frac{C_1+C_2}{3}\right)V_1^2 = \frac{1}{2}\left(W_1 + \frac{C_1+C_2}{3}\right)V_0^2 - AP_0\Delta x \quad \ldots \quad (8)$$

Eliminating P from (4) and (5) and integrating, we have

$$V = V_1 + \frac{AD_1(f_{01} - f_1)}{\beta_1\left(W_1 + \frac{C_1+C_2}{2}\right)} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Therefore from equations (7) and (9), we obtain

$$\frac{Z_1 C_1 R(T_{01} - T)}{\bar{\gamma} - 1} + \frac{Z_2 C_2 R(T_{02} - T)}{\bar{\gamma} - 1} = AP_0 \Delta x + \frac{1}{2}\left(W_1 + \frac{C_1+C_2}{3}\right) \times$$
$$\times \left[V_1^2 + \left\{\frac{AD_1(f_{01} - f_1)}{\beta_1\left(W_1 + \frac{C_1+C_2}{2}\right)}\right\}^2 + \frac{2AD_1V_1(f_{01} - f_1)}{\beta_1\left(W_1 + \frac{C_1+C_2}{2}\right)}\right]$$

which by making use of equation (8) reduces to

$$\frac{Z_1 C_1 R(T_{01} - T)}{\bar{\gamma} - 1} + \frac{Z_2 C_2 R(T_{02} - T)}{\bar{\gamma} - 1} = \frac{1}{2}\left(W_1 + \frac{C_1+C_2}{3}\right) \times$$
$$\times \left[V_0^2 + \left\{\frac{AD_1(f_{01} - f_1)}{\beta_1\left(W_1 + \frac{C_1+C_2}{2}\right)}\right\}^2 + \frac{2AD_1V_1(f_{01} - f_1)}{\beta_1\left(W_1 + \frac{C_1+C_2}{2}\right)}\right] \quad \ldots \quad (10)$$

Combining equations (2) and (10), we have

$$Z_1C_1RT_{01}+Z_2C_2RT_{02} = \frac{AP(x+l)\left(1+\frac{C_1+C_2}{3W_1}\right)}{\left(1+\frac{C_1+C_2}{2W_1}\right)}+\frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C_1+C_2}{3}\right)\times$$

$$\times\left[V_0^2+\left\{\frac{AD_1(f_{01}-f_1)}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)}\right\}^2+\frac{2AD_1V_1(f_{01}-f_1)}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)}\right] \quad \text{.. (11)}$$

Up to this point the equations have been exact. Now, keeping only first order terms in (8), (9) and (11):

$$V_1 = V_0-\frac{AP_0\Delta x}{\left(W_1+\frac{C_1+C_2}{3}\right)V_0} \quad \text{.. .. .. (12)}$$

$$V = \frac{AD_1(1-f_1)}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)}-\frac{AP_0\Delta x}{\left(W_1+\frac{C_1+C_2}{3}\right)V_0} \quad \text{.. .. (13)}$$

$$Z_1C_1RT_{01}+Z_2C_2RT_{02} = \frac{AP(x+l)\left(1+\frac{C_1+C_2}{3W_1}\right)}{\left(1+\frac{C_1+C_2}{2W_1}\right)}+\frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C_1+C_2}{3}\right)\times$$

$$\times\left[\left\{\frac{AD_1(1-f_1)}{\beta_1\left(W_1+\frac{C_1+C_2}{2}\right)}\right\}^2-\frac{2AP_0\Delta x(f_{01}-f_1)}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{01})}\right] \quad \text{(14)}$$

The latter equation can be written as

$$(1-f_1)(1+\theta_1f_1)+\frac{C_2RT_{02}}{C_1RT_{01}}(1-f_2)(1+\theta_2f_2) = \frac{AP(x+l)\left(1+\frac{C_1+C_2}{3W_1}\right)}{C_1RT_{01}\left(1+\frac{C_1+C_2}{2W_1}\right)}+$$

$$+\frac{\bar{\gamma}-1}{2}M_1(1-f_1)^2-\frac{(\bar{\gamma}-1)AP_0\Delta x(f_{01}-f_1)}{C_1RT_{01}(1-f_{01})} \quad \text{.. .. (15)}$$

where

$$M_1 = \frac{A^2D_1^2\left(1+\frac{C_1+C_2}{3W_1}\right)}{\beta_1^2W_1C_1RT_{01}\left(1+\frac{C_1+C_2}{2W_1}\right)^2}$$

But from equations (5) and (9)

$$AP = \left[-\frac{A^2D_1^2(1-f_1)}{\beta_1^2\left(W_1+\frac{C_1+C_2}{2}\right)}+\frac{AP_0\Delta x\left(W_1+\frac{C_1+C_2}{2}\right)}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{01})}\right]\frac{df_1}{dx}$$

Thus equation (15) reduces to

$$(1-f_1)(1+\theta_1 f_1)+\frac{C_2RT_{02}}{C_1RT_{01}}(1-f_2)(1+\theta_2 f_2) = -M_1(x+l)(1-f_1)\frac{df_1}{dx}+$$
$$+\frac{AP_0\Delta x(x+l)}{C_1RT_{01}(1-f_{01})}\frac{df_1}{dx}+\frac{\bar{\gamma}-1}{2}M_1(1-f_1)^2-\frac{(\bar{\gamma}-1)AP_0\Delta x(f_{01}-f_1)}{C_1RT_{01}(1-f_{01})}$$

Making use of relation (6) this can be written as

$$(1+\theta_1 f_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}\left\{1+\theta_2\left(1-\frac{1-f_1}{K}\right)\right\}+M_1(x+l)\frac{df_1}{dx}-\frac{\bar{\gamma}-1}{2}M_1(1-f_1)=$$
$$=\frac{AP_0\Delta x}{(1-f_1)(1-f_{01})C_1RT_{01}}\left[(x+l)\frac{df_1}{dx}-(\bar{\gamma}-1)(f_{01}-f_1)\right]$$

or

$$Z+M_1(x+l)\frac{df_1}{dx}=\frac{AP_0\Delta x}{(1-f_1)(1-f_{01})C_1RT_{01}}\left[(x+l)\frac{df_1}{dx}-(\bar{\gamma}-1)(f_{01}-f_1)\right] \quad (16)$$

where

$$Z=\left[1+\frac{C_2RT_{02}}{C_1RT_{01}K}\left\{1+\theta_2\left(1-\frac{1}{K}\right)\right\}-\frac{\bar{\gamma}-1}{2}M_1\right]+\nu f_1 \quad \ldots \quad (17)$$

$$\nu=\theta_1+\frac{\theta_2}{K^2}\frac{C_2RT_{02}}{C_1RT_{01}}+\frac{\bar{\gamma}-1}{2}M_1 \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

Equation (16) shows the first-order effects of the resistance on the equation of unresisted motion

$$Z+M_1(x+l)\frac{df_1}{dx}=0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

and we can obtain a solution correct to terms of order $P_0 \Delta x$ by using (19) on the right of (16). This gives

$$Z+M_1(x+l)\frac{df_1}{dx}=-\frac{AP_0\Delta x}{(1-f)(1-f_{01})C_1RT_{01}}\left[\frac{Z}{M_1}+(\bar{\gamma}-1)(f_{01}-f_1)\right]$$

and so

$$\int\frac{dx}{x+l}=-\int\frac{M_1 df_1}{Z}+\frac{AP_0\Delta x}{(1-f_{01})C_1RT_{01}}\left[\int\frac{df_1}{Z(1-f_1)}+(\bar{\gamma}-1)M_1\int\frac{(f_{01}-f_1)df_1}{Z^2(1-f_1)}\right] \quad (20)$$

Integrating equation (20) and taking the lower limit the point, suffix zero, at which the resistance occurred, we obtain

$$\log\left(\frac{x+l}{l}\right)=\frac{M_1}{\nu}\log\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}{Z}\right]+\frac{AP_0\Delta x}{(1-f_{01})C_1RT_{01}}\times$$
$$\times\left[\frac{1+\theta_1+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\log\left\{\frac{Z(1-f_{01})}{Z_0(1-f_1)}\right\}+\right.$$
$$\left.+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_0}-\frac{1}{Z}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}\right] \quad (21)$$

All the above equations hold good when both the charges are burning.

Case (*i*) : 'Simultaneous burning of charges'. If both the charges burn out simultaneously then all the above equations are true for $K = 1$. If suffix 'B' denotes the position of all-burnt we have

$$\log\left(\frac{x_B+l}{l}\right) = \frac{M_1}{\nu}\log\left[\frac{(1+\theta_1)+\dfrac{C_2RT_{01}}{C_2RT_{02}}(1+\theta_2)}{Z_B}\right] + \frac{AP_0\Delta x}{(1-f_{01})C_1RT_{01}} \times$$

$$\times\left[\frac{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_B(1-f_{01})}{Z_0}\right\}+\right.$$

$$\left.+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}}{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)}\right] \qquad (22)$$

Hence the change in log (x_B+l) is given by

$$\Delta\log(x_B+l) = \frac{AP_0\Delta x}{(1-f_{01})C_1RT_{01}}\left[\frac{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}^2}\times\right.$$

$$\left.\times\log\left\{\frac{Z_B(1-f_{01})}{Z_0}\right\}+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}}{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)}\right] \quad .. \quad (23)$$

Now we proceed on to find the change in muzzle velocity due to the bore resistance in this case. Since after all-burnt the expansion of the gases is adiabatic, the pressure at any instant after 'burnt' is given by

$$P = P_B r^{-\gamma} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (24)$$

where

$$r = \frac{x+l}{x_B+l} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (25)$$

Also the dynamical equation of motion of the shot is

$$\left(W_1+\frac{C_1+C_2}{2}\right)\frac{dV}{dt} = AP \quad .. \quad .. \quad .. \quad (26)$$

Then integrating (26), we have, if suffix 'E' refers to conditions at the muzzle, the muzzle velocity given by

$$V_E^2 = V_B^2 + \frac{AP_B(x_B+l)\Phi}{\left(W_1+\dfrac{C_1+C_2}{2}\right)} \quad .. \quad .. \quad .. \quad (27)$$

where

$$\Phi = \frac{2}{\bar{\gamma}-1}\left(1-r^{1-\bar{\gamma}}\right) \quad .. \quad .. \quad .. \quad .. \quad (28)$$

and

$$r = \frac{x_E+l}{x_B+l} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (29)$$

In equation (27), V_B, P_B (x_B+l) and Φ are all altered by the resistance.
From equation (15),

$$\frac{AP_B(x_B+l)}{\left(1+\dfrac{C_1+C_2}{2W_1}\right)} = \frac{C_1RT_{01}\left[1-\dfrac{\bar{\gamma}-1}{2}M_1\right]+C_2RT_{02}}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} + \frac{(\bar{\gamma}-1)AP_0\Delta x\cdot f_{01}}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)(1-f_{01})} \quad (30)$$

Also,

$$\Delta\Phi = 2r^{-\gamma}\Delta r = -2r^{1-\bar{\gamma}}\Delta\log(x_B+l) \quad .. \quad .. \quad (31)$$

and from equation (13),

$$\Delta\left(V_B^2\right) = -\frac{2AP_0\Delta x}{\left(W_1+\dfrac{C_1+C_2}{3}\right)(1-f_{01})} \quad .. \quad .. \quad .. \quad (32)$$

So that from (23), (27), (30), (31) and (32) we have

$$\Delta\left(V_E^2\right) = -\frac{2AP_0\Delta x(1-f_{01}+f_{01}r^{1-\bar{\gamma}})}{\left(W_1+\dfrac{C_1+C_2}{3}\right)(1-f_{01})} - \frac{2r^{1-\bar{\gamma}}Z_BAP_0\Delta x}{\left(W_1+\dfrac{C_1+C_2}{3}\right)(1-f_{01})} \times$$

$$\times\left[\frac{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_B(1-f_{01})}{Z_0}\right\}+\right.$$

$$\left.+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}}{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)}\right] \quad (33)$$

and finally

$$\frac{\left(W_1+\dfrac{C_1+C_2}{3}\right)V_E(1-f_{01})\Delta V_E}{AP_0\Delta x} = -1+f_{01}-f_{01}r^{1-\bar{\gamma}}$$

$$-r^{1-\bar{\gamma}}Z_B\left[\frac{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_B(1-f_{01})}{Z_0}\right\}+\right.$$

$$\left.+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}}{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)}\right] \quad (34)$$

This gives the change in muzzle velocity ΔV_E due to the bore resistance $AP_0 \Delta x$ occurring at a certain point during the burning of both the charges (and both the charges burn out simultaneously), in terms of the characteristics of the solution without resistance.

If resistance had occurred very early in the travel f_{01} is nearly unity, and

$$\frac{\left(W_1+\frac{C_1+C_2}{3}\right)V_E \Delta V_E}{AP_0 \Delta x} \sim \frac{r^{1-\bar{\gamma}}Z_B}{\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}(1-f_{01})} \times \log\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)}{Z_B(1-f_{01})}\right] \quad .. \quad (35)$$

which shows that if resistance occurs sufficiently early in the travel the muzzle velocity rises. Again when resistance occurs at 'all-burnt', $f_{01} = 0$, and

$$\frac{\left(W_1+\frac{C_1+C_2}{3}\right)V_E \Delta V_E}{AP_0 \Delta x} = -1$$

and the muzzle velocity falls. If resistance occurs after 'all-burnt', i.e. during the adiabatic expansion, the kinetic energy of the shot and the gases at the muzzle are reduced by the work done on the resistance, without having effects on the ballistics (except on the time to travel through the bore). Hence

$$\frac{\left(W_1+\frac{C_1+C_2}{3}\right)V_E \Delta V_E}{AP_0 \Delta x} = -1$$

if the resistance occurs after 'all-burnt'.

Case (ii): 'Non-simultaneous burning of charges'. For the sake of convenience we assume that charge C_1 burns out first and charge C_2 continues to burn. This case we will have to consider in two parts:

(*a*) if the resistance had occurred during the burning of both the charges, and

(*b*) if the resistance occurs when charge C_1 is burnt out but charge C_2 is burning.

Case (a): Now when both the charges are burning the equations from (1) to (21) are true. Since suffix '*b*' denotes the position when the charge C_1 is burnt out, we have

$$\log\left(\frac{x_b+l}{l}\right) = \frac{M_1}{\nu}\log\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}{Z_b}\right] + \frac{AP_0\Delta x}{(1-f_{01})C_1RT_{01}} \times$$

$$\times\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-)M_1(1-f_{01})}{\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_b(1-f_{01})}{Z_0}\right\}+\right.$$

$$\left.+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_b}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}\right]$$

Hence the change in shot-travel up to the position when charge C_1 is burnt out is given by

$$\Delta \log (x_b+l) = \frac{AP_0\Delta x}{(1-f_{01})C_1RT_{01}}\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\times\right.$$

$$\left.\times \log\left\{\frac{Z_b(1-f_{01})}{Z_0}\right\}+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_b}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}\right] \quad (36)$$

Now when the charge C_1 is burnt out and charge C_2 is burning, since $Z_1=1$, the equations corresponding to the equations (13) and (14) will be

$$V=\frac{AD_2(1-f_2)}{\beta_2\left(W_1+\frac{C_1+C_2}{2}\right)}-\frac{AP_0\Delta x}{\left(W_1+\frac{C_1+C_2}{3}\right)V_0} \quad \ldots \quad \ldots \quad (37)$$

and $$C_1RT_{01}+C_2RT_{02}Z_2=\frac{AP(x+l)\left(1+\frac{C_1+C_2}{3W_1}\right)}{\left(1+\frac{C_1+C_2}{2W_1}\right)}+\frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C_1+C_2}{3}\right)\times$$

$$\times\left[\left\{\frac{AD_2(1-f_2)}{\beta_2\left(W_1+\frac{C_1+C_2}{2}\right)}\right\}^2-\frac{2AP_0\Delta x(f_{0,2}-f_2)}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{0,2})}\right] \quad (38)$$

where $f_{0,2}$ is the fraction of D_2 remaining just at the moment the resistance occurs. Also from equations (4*a*), (5), and (37) we have

$$AP=\left[-\frac{A^2D_2^2(1-f_2)}{\beta_2^2\left(W_1+\frac{C_1+C_2}{2}\right)}+\frac{AP_0\Delta x\left(W_1+\frac{C_1+C_2}{2}\right)}{\left(W_1+\frac{C_1+C_2}{3}\right)(1+f_{0,2})}\right]\frac{df_2}{dx}$$

Thus equation (38) reduces to

$$\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2f_2)+M_2(x+l)(1-f_2)\frac{df_2}{dx}-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2$$

$$=\frac{AP_0\Delta x}{(1-f_{0,2})C_2RT_{02}}\left[(x+l)\frac{df_2}{dx}-(\bar{\gamma}-1)(f_{0,2}-f_2)\right] \quad \ldots \quad (39)$$

Equation (39) shows the first order effects on the equation of unresisted motion,

$$\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2f_2)+M_2(x+l)(1-f_2)\frac{df_2}{dx}-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2=0$$

Using this equation on the right of (39) gives

$$\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2f_2)+M_2(x+l)(1-f_2)\frac{df_2}{dx}-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2$$

$$=-\frac{AP_0\Delta x}{(1-f_2)(1-f_{0,2})C_2RT_{02}}\left[\frac{1}{M_2}\left\{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2\right\}+\right.$$

$$\left.+(\bar{\gamma}-1)(1-f_2)(f_{0,2}-f_2)\right]$$

and so

$$\int \frac{dx}{x+l} = -\int \frac{M_2(1-f_2)df_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2}+\frac{AP_0\Delta x}{(1-f_{0,2})C_2RT_{02}}\times$$

$$\times\left[\int \frac{df_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2}+\right.$$

$$\left. M_2(\bar{\gamma}-1)\int \frac{(f_{0,2}-f_2)(1-f_2)}{\left\{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)\right\}^2}\right]$$

$$= -\frac{M_2}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\int \frac{(1-f_2)df_2}{(1-af_2)(1+bf_2)}+\frac{AP_0\Delta x}{(1-f_{0,2})C_2RT_{02}}\times$$

$$\times\left[\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\int\frac{df_2}{(1-af_2)(1+bf_2)}+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\times\right.$$

$$\left.\times\int\frac{(f_{0,2}-f_2)(1-f_2)}{(1-af_2)^2(1+bf_2)^2}\right] \quad \ldots \quad (40)$$

where

$$a-b = \frac{1-\theta_2-(\bar{\gamma}-1)M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2} \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

$$ab = \frac{\theta_2+\frac{\bar{\gamma}-1}{2}M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2} \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

Integrating equation (40) and applying the conditions that $x = x_b$ when $f_{2b} = 1-\frac{1}{\bar{K}}$, we have

$$\log\left(\frac{x+l}{x_b+l}\right) = \frac{M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}\left[\frac{1-a}{a(a+b)}\log\left\{\frac{1-a\left(1-\frac{1}{\bar{K}}\right)}{1-af_2}\right\}+\right.$$

$$\left.+\frac{1+b}{b(a+b)}\log\left\{\frac{1+b\left(1-\frac{1}{\bar{K}}\right)}{1+bf_2}\right\}\right]+\frac{AP_0\Delta x}{(1-f_{0,2})C_2RT_{02}}\left[\frac{1}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}\right.$$

$$\left\{\frac{1}{a+b}\log\frac{1-a\left(1-\frac{1}{\bar{K}}\right)}{1-af_2}-\frac{1}{a+b}\log\frac{1+b\left(1-\frac{1}{\bar{K}}\right)}{1+bf_2}\right\}+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}$$

$$\left\{\frac{\lambda_1}{a}\log\frac{1-a\left(1-\frac{1}{K}\right)}{1-af_2}-\frac{\lambda_2}{a}\left(\frac{1}{1-a\left(1-\frac{1}{K}\right)}-\frac{1}{1-af_2}\right)-\frac{\lambda_3}{b}\log\frac{1+b\left(1-\frac{1}{K}\right)}{1+bf_2}+\right.$$

$$\left.\left.+\frac{\lambda_4}{b}\left(\frac{1}{1+b\left(1-\frac{1}{K}\right)}-\frac{1}{1+bf_2}\right)\right\}\right] \qquad \ldots \quad (43)$$

where

$$\lambda_1=\frac{a[(a-b)(1+f_{0,2})-2(1-abf_{0,2})]}{(a+b)^3}$$

$$\lambda_2=\frac{(1-af_{0,2})(1-a)}{(a+b)^2}$$

$$\lambda_3=\frac{b[(a-b)(1+f_{0,2})-2(1-abf_{0,2})]}{(a+b)^3}$$

$$\lambda_4=\frac{(1+bf_{0,2})(1+b)}{(a+b)^2}$$

At the position of ‘all-burnt’, we have

$$\log\left(\frac{x_B+l}{x_b+l}\right)=\frac{M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}\left[\frac{1-a}{a(a+b)}\log\left\{1-a\left(1-\frac{1}{K}\right)\right\}+\right.$$

$$\left.+\frac{1+b}{b(a+b)}\log\left\{1+b\left(1-\frac{1}{K}\right)\right\}\right]+$$

$$+\frac{AP_0\Delta x}{(1-f_{0,2})C_2RT_{02}}\left[\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)(a+b)}\log\left\{\frac{1-a\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}+\right.$$

$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1}{a}\log\left(1-a\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_2\left(1-\frac{1}{K}\right)}{1-a\left(1-\frac{1}{K}\right)}-\right.$$

$$\left.\left.-\frac{\lambda_3}{b}\log\left(1+b\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_4\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}\right]$$

Hence the change in $\log(x_B+l)$ due to the bore resistance is

$\Delta\log(x_B+l)=$

$$\Delta\log(x_b+l)+\frac{AP_0\Delta x}{(1-f_{0,2})C_2RT_{02}}\left[\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)(a+b)}\log\left\{\frac{1-a\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}\right.$$

$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1}{a}\log\left(1-a\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_2\left(1-\frac{1}{K}\right)}{1-a\left(1-\frac{1}{K}\right)}\right.$$

$$\left.-\frac{\lambda_3}{b}\log\left(1+b\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_4\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}\Bigg]$$

or from (36)

$$\Delta\log(x_B+l)=AP_0\Delta x\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{C_1RT_{01}(1-f_{01})\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\times\right.$$

$$\times\log\left\{\frac{Z_b(1-f_{01})}{Z_0}\right\}$$

$$+\frac{(\bar{\gamma}-1)M_1}{(1-f_{01})C_1RT_{01}\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_0}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}+$$

$$+\frac{1}{C_2RT_{02}(1-f_{0,2})(a+b)\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\log\frac{1-a\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}$$

$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2C_2RT_{02}(1-f_{0,2})}\left\{\frac{\lambda_1}{a}\log\left(1-a\left(1-\frac{1}{K}\right)\right)\right.$$

$$\left.-\frac{\lambda_2\left(1-\frac{1}{K}\right)}{1-a\left(1-\frac{1}{K}\right)}-\frac{\lambda_3}{b}\log\left(1+b\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_4\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}\Bigg] \quad \ldots \quad \ldots \quad (44)$$

Now we proceed to find the change in muzzle velocity in this case. From equations (37) and (38), we have

$$\Delta\left(V_B^2\right)=-\frac{2AP_0\Delta x}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{0,2})} \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

and
$$\frac{AP_B(x_B+l)}{\left(1+\frac{C_1+C_2}{2W_1}\right)}=\frac{C_1RT_{01}+C_2RT_{02}\left[1-\frac{\bar{\gamma}-1}{2}M_2\right]}{\left(1+\frac{C_1+C_2}{3W_1}\right)}+\frac{(\bar{\gamma}-1)AP_0\Delta xf_{0,2}}{\left(1+\frac{C_1+C_2}{3W_1}\right)(1-f_{0,2})} \quad (46)$$

Therefore from equations (27), (28), (29), (44), (45) and (46), we obtain

$$\Delta(\bar{V}_E^2) = -\frac{2AP_0\Delta x(1-f_{0,2}+f_{0,2}r^{1-\bar{\gamma}})}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{0,2})} - 2r^{1-\bar{\gamma}}\frac{C_1RT_{01}+C_2RT_{02}\left\{1-\frac{\bar{\gamma}-1}{2}M_2\right\}}{\left(W_1+\frac{C_1+C_2}{3}\right)} \times$$

$$\times AP_0\Delta x\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{C_1RT_{01}(1-f_{01})\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_b(1-f_{01})}{Z_0}\right\}+\right.$$

$$+\frac{(\bar{\gamma}-1)M_1}{C_1RT_{01}(1-f_{01})\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_b}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}+$$

$$+\frac{1}{C_2RT_{02}(1-f_{0,2})(a+b)\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\log\frac{1-a\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}+$$

$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2C_2RT_{02}(1-f_{0,2})}\left\{\frac{\lambda_1}{a}\log\left(1-a\left(1-\frac{1}{K}\right)\right)-\right.$$

$$\left.\left.-\frac{\lambda_2\left(1-\frac{1}{K}\right)}{1-a\left(1-\frac{1}{K}\right)}-\frac{\lambda_3}{b}\log\left(1+b\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_4\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}\right]$$

or finally

$$\frac{\left(W_1+\frac{C_1+C_2}{3}\right)V_E(1-f_{0,2})\Delta V_E}{AP_0\Delta x} = -1+f_{0,2}-f_{0,2}r^{1-\bar{\gamma}}-2r^{1-\bar{\gamma}}$$

$$\times\left\{C_1RT_{01}+C_2RT_{02}\left(1-\frac{\bar{\gamma}-1}{2}M_2\right)\right\}\times$$

$$\times\left[\frac{(1-f_{0,2})\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})\right\}}{C_1RT_{01}(1-f_{01})\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_b(1-f_{01})}{Z_0}\right\}+\right.$$

$$+\frac{(\bar{\gamma}-1)M_1(1-f_{0,2})}{C_1RT_{01}(1-f_{01})\nu}\left(\frac{1}{Z_0}-\frac{1}{Z_b}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}$$

$$+\frac{1}{C_2RT_{02}(a+b)\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\log\left\{\frac{1-a\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}+$$

$$+\frac{M_2(\bar{\gamma}-1)}{C_2RT_{02}\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1}{a}\log\left(1-a\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_2\left(1-\frac{1}{K}\right)}{1-a\left(1-\frac{1}{K}\right)}-\right.$$

$$\left.\left.-\frac{\lambda_3}{b}\log\left(1+b\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_4\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\}\right] \quad \ldots \quad (47)$$

This gives the change in muzzle velocity due to the bore resistance occurring at a certain point during the burning of both the charges.

Case (*b*): When charge C_1 has burnt out and charge C_2 is burning and resistance also occurs during this stage of burning there will be no change in the solution before the occurrence of resistance, i.e. there will be no change in the solution when both the charges are burning. Now after charge C_1 has burnt out and charge C_2 is burning and resistance has occurred the equations corresponding to equations (13) and (14) will be

$$V=\frac{AD_2(1-f_2)}{\beta_2\left(W_1+\frac{C_1+C_2}{2}\right)}-\frac{AP_0\Delta x}{\left(W_1+\frac{C_1+C_2}{3}\right)V_0} \quad \ldots \quad (48)$$

$$C_1RT_{01}+C_2RT_{02}Z_2=\frac{AP(x+l)\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{2W_1}\right)}+\frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C_1+C_2}{3}\right)\times$$

$$\times\left[\left\{\frac{AD_2(1-f_2)}{\beta_2\left(W_1+\frac{C_1+C_2}{3}\right)}\right\}^2-\frac{2AP_0\Delta x(f_{02}-f_2)}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{02})}\right] \quad \ldots \quad (49)$$

where f_{02} is the fraction of D_2 remaining when resistance occurs

Equation (49) can be written as

$$\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2f_2)+M_2(x+l)(1-f_2)\frac{df_2}{dx}-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2=$$

$$=\frac{AP_0\Delta x}{(1-f_{02})C_2RT_{02}}\left[(x+l)\frac{df_2}{dx}-(\bar{\gamma}-1)(f_{02}-f_2)\right] \quad \ldots \quad (50)$$

Equation (50) shows the first order effects on the equation of unresisted motion,

$$\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2f_2)+M_2(x+l)(1-f_2)\frac{df_2}{dx}-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2=0.$$

Using this equation on the right of (50) gives

$$\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2+M_2(x+l)(1-f_2)\frac{df_2}{dx}=$$

$$=-\frac{AP_0\Delta x}{(1-f_2)(1-f_{02})C_2RT_{02}}\left[\frac{1}{M_2}\left\{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2\right\}+\right.$$

$$\left.+(\bar{\gamma}-1)(1-f_2)(f_{02}-f_2)\right]$$

and so

$$\int\frac{dx}{x+l}=-\int\frac{M_2(1-f_2)\,df_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+(1+f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2}+\frac{AP_0\Delta x}{C_2RT_{02}(1-f_{02})}\times$$

$$\times\left[\int\frac{df_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2}+M_2(\bar{\gamma}-1)\right.$$

$$\left.\int\frac{(f_{02}-f_2)(1-f_2)\,df_2}{\left\{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2\right\}^2}\right]$$

$$=-\frac{M_2}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\int\frac{(1-f_2)\,df_2}{(1-af_2)(1+bf_2)}+\frac{AP_0\Delta x}{C_2RT_{02}(1-f_{02})}\times$$

$$\times\left[\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\int\frac{df_2}{(1-af_2)(1+bf_2)}+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\times\right.$$

$$\left.\times\int\frac{(f_{02}-f_2)(1-f_2)\,df_2}{(1-af_2)^2(1+bf_2)^2}\right] \quad .. \quad (51)$$

where

$$a-b=\frac{1-\theta_2-(\bar{\gamma}-1)M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}$$

$$ab=\frac{\theta_2+\frac{\bar{\gamma}-1}{2}M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}$$

Integrating (51) and taking the lower limit the point, suffix zero, where resistance occurs, we get

$$\log\left(\frac{x+l}{x_0+l}\right)=\frac{M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}\left[\frac{1-a}{a(a+b)}\log\left\{\frac{1-a\left(1-\frac{1}{K}\right)}{1-af_2}\right\}+\right.$$

$$+\frac{1+b}{b(a+b)}\log\left\{\frac{1+b\left(1-\frac{1}{K}\right)}{1+bf_2}\right\}\Bigg]+$$

$$+\frac{AP_0\Delta x}{C_2RT_{02}(1-f_{02})}\left[\frac{1}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}\left\{\frac{1}{a+b}\log\frac{1-af_{02}}{1-af_2}-\frac{1}{a+b}\log\frac{1+bf_{02}}{1+bf_2}\right\}+\right.$$

$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1'}{a}\log\frac{1-af_{02}}{1-af_2}-\frac{\lambda_2'}{a}\left(\frac{1}{1-af_{02}}-\frac{1}{1-af_2}\right)-\right.$$

$$\left.\left.-\frac{\lambda_3'}{b}\log\frac{1+bf_{02}}{1+bf_2}+\frac{\lambda_4'}{b}\left(\frac{1}{1+bf_{02}}-\frac{1}{1+bf_2}\right)\right\}\right] \quad \ldots \quad (52)$$

where

$$\lambda_1'=\frac{a[(a-b)(1+f_{02})-2(1-abf_{02})]}{(a+b)^3}$$

$$\lambda_2'=\frac{(1-af_{02})(1-a)}{(a+b)^2}$$

$$\lambda_3'=\frac{b[(a-b)(1+f_{02})-2(1-abf_{02})]}{(a+b)^3}$$

$$\lambda_4'=\frac{(1+bf_{02})(1+b)}{(a+b)^2}$$

At the position of all-burnt we have

$$\log\left(\frac{x_B+l}{x_b+l}\right)=\frac{M_2}{\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2}\left[\frac{1-a}{a(a+b)}\log\left\{1-a\left(1-\frac{1}{K}\right)\right\}+\right.$$

$$\left.+\frac{1+b}{b(a+b)}\log\left\{1+b\left(1-\frac{1}{K}\right)\right\}\right]+$$

$$+\frac{AP_0\Delta x}{C_2RT_{02}(1-f_{02})}\left[\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)(a+b)}\log\left\{\frac{1-af_{02}}{1+bf_{02}}\right\}+\right.$$

$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1'}{a}\log(1-af_{02})-\frac{\lambda_2'f_{02}}{1-af_{02}}-\right.$$

$$\left.\left.-\frac{\lambda_3'}{b}\log(1+bf_{02})-\frac{\lambda_4'f_{02}}{1+bf_{02}}\right\}\right].$$

Hence the change in log (x_B+l) due to the bore resistance if it acts during the burning of charge C_2 is

$$\Delta \log (x_B+l) = \frac{AP_0\Delta x}{C_2RT_{02}(1-f_{02})}\left[\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)(a+b)}\log\left\{\frac{1-af_{02}}{1+bf_{02}}\right\}+\right.$$
$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1'}{a}\log(1-af_{02})-\frac{\lambda_2'f_{02}}{1-af_{02}}-\right.$$
$$\left.\left.-\frac{\lambda_3'}{b}\log(1+bf_{02})-\frac{\lambda_4'f_{02}}{1+bf_{02}}\right\}\right] \quad \ldots \quad (53)$$

Now we find the change in muzzle velocity in this case. From equations (48) and (49) we have

$$\Delta(V_B^2) = -\frac{2AP_0\Delta x}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{02})} \quad \ldots \quad \ldots \quad \ldots \quad (54)$$

$$\frac{AP_B(x_B+l)}{\left(1+\frac{C_1+C_2}{2W_1}\right)} = \frac{C_1RT_{01}+C_2RT_{02}\left(1-\frac{\bar{\gamma}-1}{2}M_2\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)}+\frac{(\bar{\gamma}-1)AP_0\Delta x f_{02}}{\left(1+\frac{C_1+C_2}{3W_1}\right)(1-f_{02})} \quad (55)$$

Therefore from (27), (28), (29), (53), (54) and (55) we obtain

$$\Delta(V_E^2) = -\frac{2AP_0\Delta x(1-f_{02}+f_{02}r^{1-\bar{\gamma}})}{\left(W_1+\frac{C_1+C_2}{3}\right)(1-f_{02})}-$$
$$-2r^{1-\bar{\gamma}}\frac{C_1RT_{01}+C_2RT_{02}\left(1-\frac{\bar{\gamma}-1}{2}M_2\right)AP_0\Delta x}{(1-f_{02})\left(W_1+\frac{C_1+C_2}{3}\right)C_2RT_{02}}\times$$
$$\times\left[\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)(a+b)}\log\left\{\frac{1-af_{02}}{1+bf_{02}}\right\}+\right.$$
$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1'}{a}\log(1-af_{02})-\frac{\lambda_2'f_{02}}{1-af_{02}}-\right.$$
$$\left.\left.-\frac{\lambda_3'}{b}\log(1+bf_{02})-\frac{\lambda_4'f_{02}}{1+bf_{02}}\right\}\right]$$

or finally

$$\frac{\left(W_1+\frac{C_1+C_2}{3}\right)V_E(1-f_{02})\Delta V_E}{AP_0\Delta x}=-1+f_{02}-f_{02}\,r^{1-\bar{\gamma}}$$

$$-2r^{1-\bar{\gamma}}\left[\frac{1}{a+b}\log\left\{\frac{1-af_{02}}{1+bf_{02}}\right\}+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\left\{\frac{\lambda_1'}{a}\log(1-af_{02})\right.\right.$$

$$\left.\left.-\frac{\lambda_2'f_{02}}{1-af_{02}}-\frac{\lambda_3'}{b}\log(1+bf_{02})-\frac{\lambda_4'f_{02}}{1+bf_{02}}\right\}\right] \quad \ldots \quad \ldots \quad (56)$$

This gives the change in muzzle velocity ΔV_E due to the bore resistance $AP_0\,\Delta\,x$ occurring at a certain point during the burning of charge C_2, in terms of the characteristics of the solution without resistance.

5. Maximum Pressure

Maximum pressure may occur when

(I) Both the charges are burning

(II) C_1 has burnt out and C_2 is burning and

(III) At the position of ' all-burnt '.

Case (*i*) : When both the charges are burning

$$P=\frac{C_1RT_{01}(1-f_1)(1+\theta_1f_1)+\frac{C_2RT_{02}}{K}(1-f_1)\left\{1+\theta_2\left(1-\frac{1}{K}\right)+\frac{\theta_2}{K}f_1\right\}}{A(x+l)}\times$$

$$\times\frac{-\frac{\bar{\gamma}-1}{2}M_1C_1RT_{01}(1-f_1)^2+\frac{(\bar{\gamma}-1)AP_0\Delta x(f_{01}-f_1)}{(1-f_{01})}}{}\times\frac{\left(1+\frac{C_1+C_2}{2W_1}\right)}{\left(1+\frac{C_1+C_2}{3W_1}\right)} \quad \ldots \quad (57)$$

where $(x+l)$ is given by (21)

For pressure to the maximum $d\,P=0$. Therefore differentiating (57) and simplifying, we obtain

$$f_{1m}=\frac{\bar{\gamma}M_1+\theta_1-1+\frac{C_2RT_{02}}{C_1RT_{02}K}\left(\frac{2\theta_2}{K}-1-\theta_2\right)}{2\theta_1+\frac{2\theta_2}{K^2}\frac{C_2RT_{02}}{C_1RT_{01}}+\bar{\gamma}M_1}-$$

$$-\frac{(\bar{\gamma}-1)\,AP_0\Delta x}{C_1RT_{01}(1-f_{01})\left\{2\theta_1+\frac{2\theta_2}{K^2}\frac{C_2RT_{02}}{C_1RT_{01}}+\bar{\gamma}M_1\right\}}$$

To find the change in P_m to the first order in $AP_0\,\Delta\,x$ we put in (57),

$$f_{1m}=\frac{\bar{\gamma}M_1+\theta_1-1+\frac{C_2RT_{02}}{C_1RT_{01}K}\left(\frac{2\theta_2}{K}-1-\theta_2\right)}{2\theta_1+\frac{2\theta_2}{K^2}\frac{C_2RT_{02}}{C_1RT_{01}}+\bar{\gamma}M_1}$$

Thus

$$\frac{C_1RT_{01}(1-f_{01})\Delta P_m}{AP_0\Delta x\cdot P_m}=\frac{(\bar{\gamma}-1)(f_{01}-f_{1m})}{(1-f_{1m})Z_m}+$$

$$+\frac{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_0(1-f_{1m})}{Z_m(1-f_{01})}\right\}+$$

$$+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_m}-\frac{1}{Z_0}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)} \quad (58)$$

This holds only if

$$f_{01}\geqslant\frac{\bar{\gamma}M_1+\theta_1-1+\dfrac{C_2RT_{02}}{C_1RT_{01}K}\left(\dfrac{2\theta_2}{K}-1-\theta_2\right)}{2\theta_1+\dfrac{2\theta_2}{K^2}\dfrac{C_2RT_{02}}{C_1RT_{01}}+\bar{\gamma}M_1}\geqslant 0.$$

If

$$f_{01}<\frac{\bar{\gamma}M_1+\theta_1-1+\dfrac{C_2RT_{02}}{C_1RT_{01}K}\left(\dfrac{2\theta_2}{K}-1-\theta_2\right)}{2\theta_1+\dfrac{2\theta_2}{K^2}\dfrac{C_2RT_{02}}{C_1RT_{01}}+\bar{\gamma}M_1}\geqslant 0.$$

then there is no change in the maximum pressure.

Case (ii) :

This case we will have to consider into two parts:

(*a*) if resistance occurs when both the charges are burning,

and (*b*) if resistance occurs when charge C_1 has burnt out and charge C_2 is burning.

Case (ii a) :

When charge C_1 is burnt out and charge C_2 is burning and resistance occurs during the burning of both the charges,

$$P=\frac{C_1RT_{01}+C_2RT_{02}(1-f_2)(1+\theta_2f_2)-\dfrac{\bar{\gamma}-1}{2}M_2C_2RT_{02}(1-f_2)^2+}{A(x+l)}\times$$

$$\begin{matrix}+\\ \times\end{matrix}\frac{\dfrac{(\bar{\gamma}-1)AP_0\Delta x(f_{0,2}-f_2)}{(1-f_{02})}}{}\times\frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)}\quad\ldots \quad (59)$$

where $(x+l)$ is given by (43)

For pressure to be maximum $dP=0$. Therefore differentiating (59) and simplifying, we obtain

$$f_{2m}=\frac{\bar{\gamma}M_2+\theta_2-1}{\bar{\gamma}M_2+2\theta_2}-\frac{\bar{\gamma}AP_0\Delta x}{C_2RT_{02}(1-f_{0,2})(\bar{\gamma}M_2+2\theta_2)}.$$

The change in P_m to the first order in $AP_0\,\Delta\,x$ is obtained by writing in (59)

$$f_{2m}=\frac{\bar{\gamma}M_2+\theta_2-1}{\bar{\gamma}M_2+2\theta_2}$$

Hence

$$\frac{C_2RT_{02}(1-f_{0,2})\Delta P_m}{AP_0\Delta x\cdot P_m} = \frac{(\bar\gamma-1)(f_{0,2}-f_{2m})}{\dfrac{C_1RT_{01}}{C_2RT_{02}}+(1-f_{2m})(1+\theta_2 f_{2m})-\dfrac{\bar\gamma-1}{2}M_2(1-f_{2m})^2}$$

$$+\frac{C_2RT_{02}(1-f_{0,2})}{C_1RT_{01}(1-f_{01})}\left[\frac{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar\gamma-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_0}{Z_b(1-f_{01})}\right\}\right.$$

$$+\frac{(\bar\gamma-1)M_1}{\nu}\left(\frac{1}{Z_b}-\frac{1}{Z_0}\right)\frac{(1+\theta_1)-\nu(1-f_{01})+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}{(1+\theta_1)+\dfrac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}$$

$$+\frac{1}{\left(\dfrac{C_1RT_{01}}{C_2RT_{02}}+1-\dfrac{\bar\gamma-1}{2}M_2\right)}\left\{\frac{1}{(a+b)}\log\frac{1-af_{2m}}{1-a\left(1-\dfrac{1}{K}\right)}-\frac{1}{(a+b)}\log\frac{1+bf_{2m}}{1+b\left(1-\dfrac{1}{K}\right)}\right\}+$$

$$+\frac{M_2(\bar\gamma-1)}{\left(\dfrac{C_1RT_{01}}{C_2RT_{02}}+1-\dfrac{\bar\gamma-1}{2}M_2\right)^2}\left\{\frac{\lambda_1}{a}\log\frac{1-af_{2m}}{1-a\left(1-\dfrac{1}{K}\right)}-\frac{\lambda_2}{a}\left(\frac{1}{1-af_{2m}}-\frac{1}{1-a\left(1-\dfrac{1}{K}\right)}\right)\right.$$

$$\left.\left.-\frac{\lambda_3}{b}\log\frac{1+bf_{2m}}{1+b\left(1-\dfrac{1}{K}\right)}+\frac{\lambda_4}{b}\left(\frac{1}{1+bf_{2m}}-\frac{1}{1+b\left(1-\dfrac{1}{K}\right)}\right)\right\}\right] \tag{60}$$

Case (ii b) :

When charge C_1 is burnt out and charge C_2 is burning and resistance occurs during the burning of charge C_2,

$$P=\frac{C_1RT_{01}+C_2RT_{02}(1-f_2)(1+\theta_2 f_2)-\dfrac{\bar\gamma-1}{2}M_2C_2RT_{02}(1-f_2)^2+}{A(x+l)}\times$$

$$+\frac{(\bar\gamma-1)AP_0\Delta x(f_{02}-f_2)}{(1-f_{02})}\times\frac{\left(1+\dfrac{C_1+C_2}{2W_1}\right)}{\left(1+\dfrac{C_1+C_2}{3W_1}\right)} \quad .. \tag{61}$$

where $(x+l)$ is given by (52)

For pressure to be maximum $dP=0$. Therefore differentiating (61) and simplifying, we get

$$f_{2m}=\frac{\bar\gamma M_2+\theta_2-1}{\bar\gamma M_2+2\theta_2}-\frac{\bar\gamma AP_0\Delta x}{C_2RT_{02}(1-f_{02})(\bar\gamma M_2+2\theta_2)}$$

To find the change in P_m to the first order in $AP_0\,\Delta\,x$ we substitute in (61)

$$f_{2m}=\frac{\bar\gamma M_2+\theta_2-1}{\bar\gamma M_2+2\theta_2}\,.$$

Hence

$$\frac{C_2RT_{02}(1-f_{02})\Delta P_m}{AP_0\Delta x\,.\,P_m}=\frac{(\bar{\gamma}-1)(f_{02}-f_{2m})}{\frac{C_1RT_{01}}{C_2RT_{02}}+(1-f_{2m})(1+\theta_2f_{2m})-\frac{\bar{\gamma}-1}{2}M_2(1-f_{2m})^2}+$$
$$+\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}\left\{\frac{1}{(a+b)}\log\frac{1-af_{2m}}{1-af_{02}}-\frac{1}{(a+b)}\log\frac{1+bf_{2m}}{1+bf_{02}}\right\}+$$
$$+\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1'}{a}\log\frac{1-af_{2m}}{1-af_{02}}-\frac{\lambda_2'}{a}\left(\frac{1}{1-af_{2m}}-\frac{1}{1-af_{02}}\right)-\right.$$
$$\left.-\frac{\lambda_3'}{b}\log\frac{1+bf_{2m}}{1+bf_{02}}+\frac{\lambda_4'}{b}\left(\frac{1}{1+bf_{2m}}-\frac{1}{1+bf_{02}}\right)\right\}\Big]\quad\ldots\quad(62)$$

This is true only if

$$1-\frac{1}{K}\geqslant f_{02}\geqslant\frac{\bar{\gamma}M_2+2\theta_2-1}{\bar{\gamma}M_2+2\theta_2}\geqslant 0.$$

If

$$1-\frac{1}{K}>f_{02}<\frac{\bar{\gamma}M_2+\theta_2-1}{\bar{\gamma}M_2+2\theta_2}\geqslant 0$$

then there is no change in the maximum pressure.

Case (iii) :

(*a*) If the two charges burn out simultaneously, then from (23) and (30), we get

$$\frac{C_1RT_{01}(1-f_{01})\Delta(P_B)}{AP_0\Delta x\,.\,P_B}=\frac{(\bar{\gamma}-1)f_{01}}{\left(\frac{C_2RT_{02}}{C_1RT_{01}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}+$$
$$+\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_0}{Z_B(1-f_{01})}\right\}+$$
$$+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_B}-\frac{1}{Z_0}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}}(1+\theta_2)}\quad\ldots\quad(63)$$

as the change in maximum pressure.

(*b*) If charge C_1 burns out first and resistance occurs during the burning of both the charges, then we have from (44) and (46),

$$\frac{C_2RT_{02}(1-f_{0,2})\Delta P_B}{AP_0\Delta x\,.\,P_B}=\frac{(\bar{\gamma}-1)f_{0,2}}{\frac{C_1RT_{01}}{C_2RT_{02}}+\left(1-\frac{\bar{\gamma}-1}{2}M_2\right)}$$
$$+\frac{C_2RT_{02}(1-f_{0,2})}{C_1RT_{01}(1-f_{01})}\left[\frac{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)-(\bar{\gamma}-1)M_1(1-f_{01})}{\left\{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}^2}\log\left\{\frac{Z_0}{Z_b(1-f_{01})}\right\}+\right.$$

$$+\frac{(\bar{\gamma}-1)M_1}{\nu}\left(\frac{1}{Z_b}-\frac{1}{Z_0}\right)\frac{\left\{(1+\theta_1)-\nu(1-f_{01})+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)\right\}}{(1+\theta_1)+\frac{C_2RT_{02}}{C_1RT_{01}K}(1+\theta_2)}\Bigg]+$$

$$+\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)(a+b)}\log\left\{\frac{1+b\left(1-\frac{1}{K}\right)}{1-a\left(1-\frac{1}{K}\right)}\right\}-$$

$$-\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1}{a}\log\left(1-a\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_2\left(1-\frac{1}{K}\right)}{1-a\left(1-\frac{1}{K}\right)}-\right.$$

$$\left.-\frac{\lambda_3}{b}\log\left(1+b\left(1-\frac{1}{K}\right)\right)-\frac{\lambda_4\left(1-\frac{1}{K}\right)}{1+b\left(1-\frac{1}{K}\right)}\right\} \quad (64)$$

as the change in maximum pressure.

(c) If charge C_1 burns out first but resistance occurs during the burning of charge C_2, then from (53) and (61) the change in maximum pressure is

$$\frac{C_2RT_{02}(1-f_{02})\Delta P_B}{AP_0\Delta x\,.\,P_B}=\frac{(\bar{\gamma}-1)f_{02}}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)}+$$

$$+\frac{1}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)(a+b)}\log\left\{\frac{1+bf_{02}}{1-af_{02}}\right\}-$$

$$-\frac{M_2(\bar{\gamma}-1)}{\left(\frac{C_1RT_{01}}{C_2RT_{02}}+1-\frac{\bar{\gamma}-1}{2}M_2\right)^2}\left\{\frac{\lambda_1'}{a}\log(1-af_{02})-\frac{\lambda_2'f_{02}}{1-af_{02}}-\right.$$

$$\left.-\frac{\lambda_3'}{b}\log(1+bf_{02})-\frac{\lambda_4'f_{02}}{1+bf_{02}}\right\} \quad .. \quad (65)$$

6. Calculation of the Effect of Long Stretches of Bore Resistance

The effect of bore resistance extending over a large part of the travel and varying in any manner, can be represented as an integral over suitably chosen resistances of our standard type. Thus since the exact solution is known from all-burnt to muzzle, we consider only the part up to all-burnt. Hence

$$\Delta V_E=\int\frac{\Delta V_E}{\Delta x}\frac{dx}{df_1}df_1$$

if the stretch of resistance lies between $f_1 = 0, 1$. Thus

$$\Delta V_E = -\int \frac{M_1(x+l)}{Z} \frac{\Delta V_E}{\Delta x} df_1$$

$$= -\frac{M_1 A}{\left(W_1+\frac{C_1+C_2}{3}\right)V_E} \int \frac{(x+l)P_0}{(1-f_1)Z} \left[\frac{(1-f_1)\left(W_1+\frac{C_1+C_2}{3}\right)V_E \Delta V_E}{AP_0 \Delta x}\right] df_1 \quad (66)$$

This integral can be easily evaluated. P_0 is a known function of x, which is itself a simple function of f_1. Also Z is a function of f_1 and the factor within brackets comes from (34) or (47) according as the charges burn out simultaneously or at different times respectively.

Again if some part of the stretch lies between $f_1 = (0, 1)$ and the rest between $f_2 = \left(1-\frac{1}{K}, 0\right)$, then we have

$$\Delta V_E = \int \frac{\Delta V_E}{\Delta x} \frac{dx}{df_1} df_1 + \int \frac{\Delta V_E}{\Delta x} \frac{dx}{df_2} df_2 =$$

$$= -\frac{M_1 A}{\left(W_1+\frac{C_1+C_2}{3}\right)V_E} \int \frac{(x+l)P_0}{Z(1-f_1)} \left[\frac{(1-f_1)\left(W_1+\frac{C_1+C_2}{3}\right)V_E \Delta V_E}{AP_0 \Delta x}\right] df_1 -$$

$$-\frac{M_2 A}{\left(W_1+\frac{C_1+C_2}{3}\right)V_E} \int \frac{(x+l)P_0}{\frac{C_1 RT_{01}}{C_2 RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2} \times$$

$$\times \left[\frac{(1-f_2)\left(W_1+\frac{C_1+C_2}{3}\right)V_E \Delta V_E}{AP_0 \Delta x}\right] df_2 \quad .. \quad (67)$$

These two integrals can be evaluated easily. In this case the factor within brackets in the first integral comes from (47) and the factor within brackets in the second integral comes from (56).

There is another possibility, viz. the stretch of resistance may wholly lie between $f_2 = \left(1-\frac{1}{K}, 0\right)$. In that case,

$$\Delta V_E = \int \frac{\Delta V_E}{\Delta x} \frac{dx}{df_2} df_2$$

$$= -\frac{M_2 A}{\left(W_1+\frac{C_1 C_2}{3}\right)V_E} \int \frac{(x+l)P_0}{\frac{C_1 RT_{01}}{C_2 RT_{02}}+(1-f_2)(1+\theta_2 f_2)-\frac{\bar{\gamma}-1}{2}M_2(1-f_2)^2} \times$$

$$\times \left[\frac{(1-f_2)\left(W_1+\frac{C_1+C_2}{3}\right)V_E \Delta V_E}{AP_0 \Delta x}\right] df_2 \quad .. \quad (68)$$

the factor within brackets comes from (56).

Similarly we can find the total change in maximum pressure for long stretches of bore resistance.

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ABSTRACT

Using Coopock's method of solution and neglecting B the covolume, Corner has found, for a single charge, the effects on maximum pressure and muzzle velocity due to a standard bore resistance. In the present paper the author has found, on the same lines and assumptions as those of Corner, the effects on maximum pressure and muzzle velocity for composite charge. It is assumed that (i) $\gamma_1 = \gamma_2 = \gamma$ since for most of the propellants γ are practically the same and (ii) the rate of burning of the charges is linear, i.e. $r = \beta P$.

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