ON THE EQUILIBRIUM CONFIGURATIONS OF OBLATE FLUID SPHEROIDS UNDER THE INFLUENCE OF MAGNETIC FIELD

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1. Introduction

Recently Miss G. Gjellestad (1954) discussed the equilibrium configurations of gravitating incompressible fluid spheroids (homogeneous, non-rotating, inviscid, and infinitely conducting) subject to a uniform magnetic field H inside and a dipole field outside. However, because of an error in the evaluation of the external magnetic energy (e.g. the sign of the first integral in equation (65) of her paper should be negative and not positive), the final results obtained in the paper need a revision. This is done in the present paper.

Under the influence of an internal uniform magnetic field and external dipole field it is shown in this paper that a sequence of oblate spheroids of equilibrium exists. Further we shall show that for values of magnetic field greater than that given by

$$\frac{H}{4\pi\rho R\sqrt{G}} = 0{\cdot}188 \qquad \ldots \qquad (1)$$

(where ρ, R, G denote the density, the radius of a sphere of volume equal to that of the spheroid, and the gravitational constant), the oblate spheroid is not a possible form of equilibrium. On the other hand there are two spheroidal forms possible, one more eccentric than the other, corresponding to every value of magnetic field H less than the critical magnetic field given by the equation (1).

The spheroids are assumed to be homogeneous, inviscid, incompressible, infinitely conducting and *non-rotating.*

2. Formulation of the Problem

The method adopted is similar to Gjellestad's. We investigate the stability of an oblate spheroid of boundary given by

$$\zeta = E \qquad \ldots \qquad (2)$$

by subjecting it to a general P_n-deformation, so that its boundary changes to one given by

$$\zeta = E + \epsilon \frac{1+E^2}{E^2+\mu^2} P_n(\mu) \quad (n > 0) \qquad \ldots \qquad (3)$$

where ϵ is a non-dimensional constant. The change, $\Delta \mathfrak{M}$, in the magnetic energy and the change, $\Delta\Omega$, in the gravitational potential energy are evaluated. The condition, $\Delta \mathfrak{M} + \Delta\Omega = 0$, is then used to define the equilibrium spheroids for a P_2-deformation. The change in the total magnetic energy, $\Delta \mathfrak{M}$, consists of two parts, (i) the change, $\Delta \mathfrak{M}^{(i)}$, in the internal magnetic energy of the spheroid and the change, $\Delta \mathfrak{M}^{(e)}$, in the external magnetic energy. The change in the gravitational potential energy, $\Delta\Omega$, and the change in the internal magnetic energy, $\Delta \mathfrak{M}^{(i)}$, have been quoted here as calculated by G. Gjellestad.

3. Change in the Total Magnetic Energy, $\Delta \mathfrak{M}$

The components $^\circ H_\zeta^{(i)}$ and $^\circ H_\theta^{(i)}$ of the uniform field H in the z-direction inside the spheroid are

$$^\circ H_\zeta^{(i)} = H\left(\frac{1+\zeta^2}{\zeta^2+\mu^2}\right)^{\frac{1}{2}}\mu, \qquad \ldots \quad (4)$$

and

$$^\circ H_\theta^{(i)} = -H\left(\frac{1-\mu^2}{\zeta^2+\mu^2}\right)^{\frac{1}{2}}\zeta$$

The change in the internal magnetic energy, $\Delta \mathfrak{M}^{(i)}$, as calculated by Gjellestad, is given by

$$\Delta \mathfrak{M}^{(i)} = 0 \qquad (n \text{ odd})$$

and

$$\Delta \mathfrak{M}_{2n}^{(i)} = -\frac{H^2C^3E\,(1+E^2)^{3/2}}{P_{2n}^1\,(iE)}\,\epsilon_{2n} \qquad \ldots \quad (5)$$

The external magnetic field $\underline{H}^{(e)}$, corresponding to equations (4), must satisfy the equations

$$\text{curl}\ \underline{H}^{(e)} = 0\,, \quad \text{and} \quad \text{div.}\ \underline{H}^{(e)} = 0 \qquad \ldots \quad (6)$$

The boundary condition to be satisfied is

$$H_\zeta^{(e)} = H_\zeta^{(i)} \text{ at } \zeta = E \qquad \ldots \quad (7)$$

The components $^\circ H_\zeta^{(e)}$ and $^\circ H_\theta^{(e)}$ of $\underline{H}^{(e)}$ satisfying equations (6) and (7) are

$$^\circ H_\zeta^{(e)} = \frac{H\,(1+E^2)^{\frac{1}{2}}}{Q_1^1\,(iE)}\,\frac{1}{(\zeta^2+\mu^2)^{\frac{1}{2}}}\,Q_1^1\,(i\zeta)\,P_1\,(\mu), \qquad \ldots \quad (8)$$

and

$$^\circ H_\theta^{(e)} = -\frac{H\,(1+E^2)^{\frac{1}{2}}}{Q_1^1\,(iE)}\,\frac{1}{(\zeta^2+\mu^2)^{\frac{1}{2}}}\,Q_1\,(i\zeta)\,P_1^1\,(\mu).$$

The functions $Q_l\,(i\zeta)$ denote the Legendre functions of second kind, so that

$$Q_1\,(i\zeta) = \zeta\cot^{-1}\zeta - 1$$

$$Q_1^1\,(i\zeta) = (1+\zeta^2)^{\frac{1}{2}}\,\frac{dQ_1\,(i\zeta)}{d\zeta} = (1+\zeta^2)^{\frac{1}{2}}\left[\cot^{-1}\zeta - \frac{\zeta}{1+\zeta^2}\right] \qquad \ldots \quad (9)$$

where the function $\cot^{-1}\zeta$ is defined in the range $0 \leqslant \cot^{-1}\zeta \leqslant \pi$.

The change, $\delta H^{(e)}$, in the external magnetic field due to the P_n-deformation, must satisfy a set of equations similar to equation (6), and hence can be written down as

$$\delta H_\zeta^{(e)} = \frac{\epsilon H}{(\zeta^2+\mu^2)^{\frac{1}{2}}}\sum_{l=1}^{\infty} D_l\,Q_l^1\,(i\zeta)\,P_l\,(\mu) \qquad \ldots \quad (10)$$

and

$$\delta H_\theta^{(e)} = -\frac{\epsilon H}{(\zeta^2+\mu^2)^{\frac{1}{2}}}\sum_{l=1}^{\infty} D_l\,Q_l\,(i\zeta)\,P_l^1\,(\mu)$$

The constants, D_l, are determined from the continuity of the normal component of the magnetic field at the deformed boundary (3). Since the normal component A_N, of a vector $\underline{A}$ at the boundary (3) in the meridian plane, is given by

$$A_N = A_\zeta + A_\theta \left(\frac{1-\mu^2}{1+E^2}\right)^{\frac{1}{2}} \frac{\partial}{\partial\mu}\left[\frac{1+E^2}{E^2+\mu^2}\,\epsilon\, P_n(\mu)\right], \quad \ldots \quad \ldots \quad (11)$$

correct to the first order of ϵ, the boundary condition for the normal component therefore gives

$$\left[H_\zeta^{(i)} - H_\zeta^{(e)}\right]_{\zeta'} + \left[H_\theta^{(i)} - H_\theta^{(e)}\right]_E \left(\frac{1-\mu^2}{1+E^2}\right)^{\frac{1}{2}} \frac{\partial}{\partial\mu}\left[\frac{1+E^2}{E^2+\mu^2}\,\epsilon\, P_n(\mu)\right] + \left(\delta H_\zeta^{(i)} - \delta H_\zeta^{(e)}\right)_E = 0 \quad (12)$$

where ζ' is used for $\left[E+\dfrac{(1+E^2)\epsilon P_n(\mu)}{E^2+\mu^2}\right]$ for convenience.

Substituting the respective expressions and simplifying we get

$$D_m(n) = \frac{2m+1}{2}\,\frac{1}{Q_m^1(iE)\,Q_1^1(iE)}\left\{\frac{m(m+1)}{2m+1}\int_{-1}^{+1}\frac{P_{m-1}(\mu)-P_{m+1}(\mu)}{E^2+\mu^2}P_n(\mu)\,d\mu \right.$$
$$\left. + \frac{i(1+E^2)^{\frac{1}{2}}Q_1^1(iE)}{P_n^1(iE)}\sum_l^{n-1}{}'(2l+1)P_l^1(iE)\int_{-1}^{+1}P_l(\mu)\,P_m(\mu)\,d\mu\right\} \quad \ldots \quad (13)$$

where $\sum_l^{n-1}{}'$ denotes the summation over l starting from $l=(n-1)$, and going through the values $(n-3)$, $(n-5)$....and ending with $l=1$, or 0, depending upon whether n is even or odd. It can be shown that the integral in the first part of equation (13) vanishes if $(m+n)$ is even.

The change, $\Delta\mathfrak{M}^{(e)}$, in the external magnetic energy can be written as

$$\Delta\mathfrak{M}^{(e)} = \mathfrak{M}^{(e)} - {}^0\mathfrak{M}^{(e)} = \frac{1}{8\pi}\int_{\zeta'}^{\infty}\int_{-1}^{+1}\int_0^{2\pi}\left\{\left[H_\zeta^{(e)}\right]^2+\left[H_\theta^{(e)}\right]^2\right\}c^3(\zeta^2+\mu^2)\,d\zeta\,d\mu\,d\phi$$
$$-\frac{1}{8\pi}\int_E^{\infty}\int_{-1}^{+1}\int_0^{2\pi}\left\{\left[{}^0H_\zeta^{(e)}\right]^2+\left[{}^0H_\theta^{(e)}\right]^2\right\}c^3(\zeta^2+\mu^2)\,d\zeta\,d\mu\,d\phi \quad \ldots \quad (14)$$

where $H_\zeta^{(e)} = {}^0H_\zeta^{(e)} + \delta H_\zeta^{(e)}$,

and $H_\theta^{(e)} = {}^0H_\theta^{(e)} + \delta H_\theta^{(e)}$ .. .. (15)

Substituting equations (15) into equations (14), we obtain

$$\Delta\mathfrak{M}^{(e)} = -\frac{1}{8\pi}\int_E^{\zeta'}\int_{-1}^{+1}\int_0^{2\pi}\left\{\left[{}^0H_\zeta^{(e)}\right]^2+\left[{}^0H_\theta^{(e)}\right]^2\right\}c^3(\zeta^2+\mu^2)\,d\zeta\,d\mu\,d\phi$$
$$+\frac{1}{8\pi}\int_E^{\infty}\int_{-1}^{+1}\int_0^{2\pi}2\left[{}^0H_\zeta^{(e)}.\,\delta H_\zeta^{(e)} + {}^0H_\theta^{(e)}\,\delta H_\theta^{(e)}\right]c^3(\zeta^2+\mu^2)\,d\zeta\,d\mu\,d\phi \quad \ldots \quad (16)$$

Using equations (8) and (10) we get

$$\Delta \mathfrak{M}^{(e)} = -\frac{H^2(1+E^2)c^3}{4\left[Q_1^1(iE)\right]^2}\int_E^{\zeta'}\int_{-1}^{+1}\left\{\left[Q_1^1(i\zeta)\right]^2\mu^2+\left[Q_1(i\zeta)\right]^2(1-\mu^2)\right\}d\mu\, d\zeta$$

$$+\frac{\epsilon H^2(1+E^2)^{\frac{1}{2}}c^3}{2Q_1^1(iE)}\int_E^{\infty}\int_{-1}^{+1}\left\{Q_1^1(i\zeta)\mu\sum_{l=1}^{\infty}D_lQ_l^1(i\zeta)P_l(\mu)\right.$$

$$\left.+Q_1(i\zeta)P_1^1(\mu)\sum_{l=1}^{\infty}D_lQ_l(i\zeta)P_l^1(\mu)\right\}d\zeta\, d\mu \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Because of the orthogonality property of Legendre functions in the interval -1 to $+1$, the first integral in equation (17) simplifies to, (after some reductions),

$$\frac{H^2(1+E^2)^2c^3\epsilon}{4\left[Q_1^1(iE)\right]^2}\left\{2Q_1(iE)+(1+E^2)^{-1}\right\}\int_{-1}^{+1}\frac{P_n(\mu)}{E^2+\mu^2}\,d\mu, \quad \ldots \quad (18)$$

and the second integral in equation (17) vanishes for all values of l except $l = 1$. The second integral reduces to, (after some simplifications),

$$-\frac{H^3\epsilon c^3(1+E^2)}{3}D_1(n)\,Q_1(iE) \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

Thus substitution of expressions (18) and (19) into equation (17) gives for the change in the external magnetic energy,

$$\Delta \mathfrak{M}^{(e)} = \frac{H^2(1+E^2)^2\epsilon c^3}{4\left[Q_1^1(iE)\right]^2}\left\{2Q_1(iE)+\frac{1}{1+E^2}\right\}\int_{-1}^{+1}\frac{P_n(\mu)}{E^2+\mu^2}\,d\mu$$

$$-\frac{H^2\epsilon c^3}{3}D_1(n)\,(1+E^2)\,Q_1(iE) \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

Now the integrals in the first part of equation (20) and that in the equation (13) can be shown to vanish for n odd.

Hence $$\Delta \mathfrak{M}^{(e)} = 0 \quad (n \text{ odd}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

and for a general P_{2n}-deformation

$$\Delta \mathfrak{M}_{2n}^{(e)} = \frac{H^2(1+E^2)^2\epsilon_{2n}c^3}{4\left[Q_1^1(iE)\right]^2}\left\{2Q_1(iE)+\frac{1}{1+E^2}\right\}\int_{-1}^{+1}\frac{P_{2n}(\mu)}{E^2+\mu^2}\,d\mu$$

$$-\frac{H^2\epsilon_{2n}c^3}{3}D_1(2n)\,(1+E^2)Q_1(iE) \quad \ldots \quad (22)$$

By putting $m = 1$ in equation (13), for a general P_{2n}-deformation one can write

$$D_1(2n) = \frac{(1+E^2)}{\left[Q_1^1(iE)\right]^2}\left\{\frac{3}{2}\int_{-1}^{+1}\frac{P_{2n}(\mu)}{E^2+\mu^2}\,d\mu-\frac{3Q_1^1(iE)}{P_{2n}^1(iE)}\right\} \quad \ldots \quad (23)$$

Substituting equation (23) into equation (22), we get

$$\Delta \mathfrak{M}^{(e)} = 0 \quad (n \text{ odd})$$

and

$$\Delta \mathfrak{M}_{2n}^{(e)} = \frac{H^2 \epsilon_{2n} c^3 (1+E^2)^2}{4\left[Q_1^1(iE)\right]^2} \left\{ \frac{1}{1+E^2} \int_{-1}^{+1} \frac{P_{2n}(\mu)}{E^2+\mu^2} d\mu + \frac{4Q_1^1(iE) Q_1(iE)}{P_{2n}^1(iE)} \right\} \tag{24}$$

The total change, $\Delta \mathfrak{M}$, in magnetic energy is, by equations (5) and (24), given by

$$\Delta \mathfrak{M} = \Delta \mathfrak{M}^{(i)} + \Delta \mathfrak{M}^{(e)}$$

$$= 0 \quad (n \text{ odd}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{25}$$

and

$$\Delta \mathfrak{M}_{2n} = -\frac{H^2 c^3 (1+E^2) \epsilon_{2n}}{Q_1^1(iE)} \left\{ \frac{1}{P_{2n}^1(iE)} - \frac{1}{4\left[Q_1^1(iE)\right]} \int_{-1}^{+1} \frac{P_{2n}(\mu)}{E^2+\mu^2} d\mu \right\} \tag{26}$$

where equation (9) is made use of in simplification.

4. The Change in the Gravitational Potential Energy, $\Delta\Omega$

The change in the gravitational potential energy, correct to the first order in ϵ, for a spheroid has been worked out by Gjellestad, and the result is as follows,

$$\Delta\Omega = -\frac{3}{10} \frac{M^2 G}{c} \epsilon_2 \left[\frac{3E^2+1}{E} \cot^{-1} E - 3 \right], \quad \text{for } n = 2$$

and

$$= 0 \quad \text{for } n \neq 2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{27}$$

where M denotes the mass of the spheroid,

$$M = \frac{4\pi}{3} \rho c^3 E (1+E^2). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{28}$$

5. The Condition for Stability and Equilibrium Spheroids

Thus while the change in the gravitational potential energy, $\Delta\Omega$, of the spheroid is of the first order in ϵ only for a P_2-deformation, and of higher order for all higher order deformations; the change in the magnetic energy, $\Delta \mathfrak{M}$, is of the order ϵ for all even P_n-deformations.

For equilibrium the total change in energy, ΔE must satisfy the equation

$$\Delta E = \Delta \mathfrak{M} + \Delta\Omega = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{29}$$

Thus using equations (26) and (27), equation (29) becomes

$$\Delta E = -\frac{3}{10} \frac{M^2 G}{c} \epsilon_2 \left[\frac{3E^2+1}{E} \cot^{-1} E - 3 \right] - \frac{H^2 C^3 (1+E^2)}{Q_1^1(iE)} \sum_{n=1} \epsilon_{2n} \left\{ \frac{1}{P_{2n}^1(iE)} - \frac{1}{4Q_1^1(iE)} \int_{-1}^{+1} \frac{P_{2n}(\mu)}{E^2+\mu^2} d\mu \right\}$$

$$= 0 \quad (\text{for equilibrium}) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \tag{30}$$

On introducing e, the eccentricity of the spheroid defined by

$$e = (1+E^2)^{-\frac{1}{2}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

and the radius R, of a sphere of volume equal to that of the spheroid, defined by

$$R^3 = a^3\,(1-c^2)^{\frac{1}{2}} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

(where a is the major half-axis of the spheroid), the equation (30) can be put as

$$\Delta E = -\frac{8}{15}\,\rho^2\pi^2 R^5 G\,\frac{(1-e^2)^{\frac{1}{2}}}{e}\,f(e)\epsilon_2 + H^2R^3\cdot\frac{e}{(1-e)^{\frac{1}{2}}}\sum_{n=1} F_{2n}(e)\,\epsilon_{2n}$$

$$= 0 \quad \text{(for equilibrium)} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

$$\text{where } f(e) = \left[\frac{3-2e^2}{e(1-e^2)^{\frac{1}{2}}}\cot^{-1}\left(\frac{1-e^2}{e^2}\right)^{\frac{1}{2}} - 3\right] \qquad \ldots \quad \ldots \quad \ldots \quad (34)$$

and

$$F_{2n}(e) = -\left[\frac{1}{P^1_{2n}(iE)Q^1_1(iE)} - \frac{1}{4\left[Q^1_1(iE)\right]^2}\int_{-1}^{+1}\frac{P_{2n}(\mu)}{E^2+\mu^2}\,d\mu\right] \qquad \ldots \quad (35)$$

Since for equilibrium the total change in the energy due to deformation must vanish, and for a single deformation higher than P_2 the expression for the change in the gravitational potential energy does not contain (to the first order in ϵ) any term to balance the magnetic term in expression for ΔE, the spheroid shall, therefore, in general, not be stable for higher order deformations $(2n > 2)$. We shall discuss the equilibrium spheroids for a P_2-deformation in the eccentricity interval $0 < e \leqslant 1$.

For a P_2-deformation, the total change in energy can be written from equation (33) as

$$\Delta E_2 = \left[-\frac{8}{15}\rho^2\pi^2R^5G\,\frac{[1-e^2]^{\frac{1}{2}}}{e}\,f(e) + H^2R^3\cdot\frac{e}{(1-e^2)^{\frac{1}{2}}}\,F_2(e)\right]\epsilon_2$$

$$= 0 \quad \text{(for equilibrium)} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

Thus the condition for a stable configuration under a P_2-deformation is

$$\frac{H}{4\pi\rho R\sqrt{G}} = (1-e^2)^{\frac{1}{2}}\left[\frac{f(e)}{30\,e^2F_2(e)}\right]^{\frac{1}{2}} \qquad \ldots \quad \ldots \quad \ldots \quad (37)$$

where $f(e)$ is as defined by equation (34), and

$$F_2(e) = -\left[\frac{1}{P^1_2(iE)Q^1_1(iE)} - \frac{1}{4\left[Q^1_1(iE)\right]^2}\int_{-1}^{+1}\frac{P_2(\mu)}{E^2+\mu^2}\,d\mu\right]$$

$$= \frac{e^2}{3(1-e^2)^{\frac{1}{2}}\cdot Q^1_1(iE)} + \frac{1}{4\left[Q^1_1(iE)\right]^2}\cdot\left\{3 - \frac{3-2e^2}{e(1-e^2)^{\frac{1}{2}}}\cot^{-1}E\right\}$$

$$= \frac{e^2}{3(1-e^2)^{\frac{1}{2}}\cdot Q^1_1(iE)} - \frac{1}{4\left[Q^1_1(iE)\right]^2}\cdot f(e) \qquad \ldots \quad \ldots \quad \ldots \quad (38)$$

The function $H/4\pi\rho R\sqrt{G}$ is plotted against e in Figure 1. We find that if $H/4\pi\rho R\sqrt{G} > 0{\cdot}188$, the oblate spheroid is not a possible form of equilibrium, but if $H/4\pi\rho R\sqrt{G} < 0{\cdot}188$, there are two spheroidal forms of equilibrium possible, for there are two values e_1 and e_2 of the abscissa corresponding to every value of the ordinate less than 0·188. When there are two real values e_1, e_2 of the eccentricity, one is greater and the other less than 0·97. Let $e_2 > e_1$, then as $H/4\pi\rho R\sqrt{G}$ is diminished, we find from the figure that e_1 decreases and e_2 increases. As the ratio of the major to minor axis of the spheroid is given by $(1-e^2)^{-\frac{1}{2}}$, the larger value ($=e_2$) of the eccentricity always represents a much flattened (disc-like) spheroid. The smaller we take $H/4\pi\rho R\sqrt{G}$, the flatter does the equilibrium spheroid become that corresponds to the value e_2. On the other hand with decrease of $H/4\pi\rho R\sqrt{G}$, the value ($=e_1$) corresponding to the lower value of the eccentricity of the equilibrium spheroid shall decrease. Further for each value of the eccentricity, there is a unique and finite value of the ordinate (i.e. magnetic field) required to make the spheroid a stable configuration for P_2-deformation.

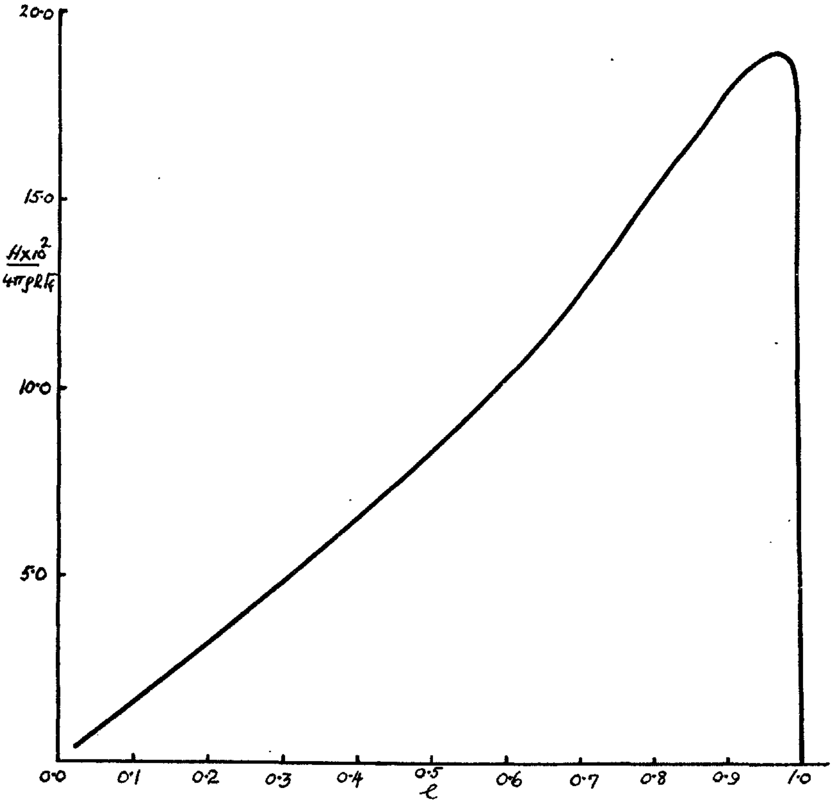

FIG. 1

Thus we conclude that under the influence of an internal uniform magnetic field and external dipole field—

(i) Every oblate spheroid is a possible form of equilibrium provided the magnetic field is of proper value.

(ii) The maximum possible value of the uniform magnetic field is given by $H/4\pi\rho R\sqrt{G} = 0{\cdot}188$ and occurs for $e = 0{\cdot}97$, and the ratio of major to minor axis being 4·12. For values of magnetic field greater than this, no oblate spheroidal forms shall exist.

(iii) For each value of magnetic field less than that given by equation (1), there are two spheroidal forms possible, one more eccentric than the other.

(iv) Each spheroid requires a unique and finite magnetic field to make it a stable configuration for P_2-deformation.

The problem of the stability of spheroids under a uniform external magnetic field is deferred to a later paper.

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Summary

The equilibrium configurations of gravitating, incompressible oblate fluid spheroids (homogeneous, inviscid, infinitely conducting and *non-rotating*) subject to a uniform magnetic field inside and dipole field outside are discussed. It is shown that a sequence of oblate spheroids of equilibrium exists. Further the maximum possible value of the uniform magnetic field inside for the spheroidal form to exist is given by $\left(\frac{H}{4\pi\rho R\sqrt{G}}\right) = 0{\cdot}188$. For every value of field less than this maximum, it is shown that there are two spheroids of equilibrium possible, one more eccentric than the other.

Reference

Gjellestad, G. (1954). Equilibrium Configurations of Oblate Spheroids under a Magnetic Field. *Astrophys. Jour.*, **119**, 14.

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