THE EFFECT OF A CORE ON THE EQUILIBRIUM CONFIGURATION OF MAGNETIC GRAVITATING SPHERE

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1. INTRODUCTION

In a recent paper Chandrasekhar and Fermi (1953) (later referred to as C.F.) have considered the problem of the flattening of a gravitating fluid sphere subject to the influence of a magnetic field due to the surface currents. The magnetic field is assumed to be uniform inside the sphere and dipole outside it. Also they assumed the fluid to be incompressible, inviscid and infinitely conducting. They showed that the equilibrium configuration is an oblate spheroid. Gjellestad (1954), following C.F., has considered the case in which a uniform field equal and opposite to that of the constant field inside the sphere is superimposed over the field in the C.F. problem. This results in a zero field inside the sphere and a constant field at large distances from it. She found that the equilibrium configuration is a prolate spheroid of ellipticity

$$\frac{\epsilon}{R_0} = \frac{5}{8} \cdot \frac{H_0^2 R_0^4}{M^2 G} \, {}^*$$

where the ratio of the axis of symmetry to the equatorial axis is $\left(1+3\,\frac{\epsilon}{R_0}\right)^{\frac{1}{2}}$, M is the mass of the spheroid, H_0 is the constant magnetic field at large distances from the sphere and G is the gravitational constant.

Ferraro (1954) considered the problem of a rotating star of incompressible and inviscid fluid subject to the influence of a magnetic field generated by the electric currents inside the star, restricting the discussion to the case of symmetry about the axis of rotation. He showed that the surface of the fluid will be an oblate spheroid of ellipticity $\frac{\epsilon}{R_0}$ given by

$$\frac{\epsilon}{R_0} = -\frac{25}{12}\frac{H_P^2 R_0^4}{M^2 G}$$

where H_p is the field at the pole of the star.

In the present paper the problem of the equilibrium of a rotating fluid sphere having a central core is discussed. In part A, we have considered the case where the electric currents are assumed to flow, only, inside the core. It is found that the equilibrium configuration is an oblate spheroid and the expression for the ellipticity reduces to that of Ferraro when the radius of the core approaches the radius of the sphere. In part B, we have treated the case where the electric currents are assumed to flow around the axis of rotation and are confined to the shell formed by the surface of the core and the sphere. These currents give rise to a constant

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* This expression for ellipticity refers to the corrected value as pointed out by Kothari and Auluck (1955).

magnetic field inside the core and a dipole field outside the sphere. In this case the surface of the fluid comes out to be an oblate spheroid. In the last section we have considered the case in which a constant magnetic field equal and opposite to the field inside the core is superimposed throughout the space. In this case of non-magnetic core the equilibrium configuration comes out to be a prolate spheroid and the expression for ellipticity in this case reduces to that of Gjellestad when the radius of the core approaches the radius of the sphere.

2. Equilibrium condition for a rotating fluid sphere

We consider a finite mass of ionized fluid of density s rotating as a rigid body with constant angular velocity ω about the axis of symmetry, which is taken as Z-axis of the cylindrical co-ordinate system (z, ϕ, ρ). If H be the magnetic field developed by the electric currents of density j prevalent inside the fluid, then in the steady state the field equations are given by

$$\operatorname{div} H = 0 \qquad (1)$$

$$\operatorname{curl} H = 4\pi j \qquad (2)$$

In spherical co-ordinates $\rho = r \sin\theta$, where θ is the angle made by the radius vector with the Z-direction. The components of H are

$$H_r = -\frac{1}{r^2 \sin\theta}\frac{\partial U}{\partial \theta}$$

$$H_\theta = \frac{1}{r\sin\theta}\frac{\partial U}{\partial r} \qquad (3)$$

$$H_\phi = 0$$

where U is the Stokes stream function.
The equation of mechanical equilibrium of the liquid is

$$s\omega \times (\omega \times r) = -\operatorname{grad} P + j \times H \qquad (4)$$

where $P = p + s\Omega$, p is the hydrostatic pressure of the liquid and Ω is the gravitational potential.
Resolving (4) into its components we get

$$-s\omega^2 r \sin^2\theta = -\frac{\partial P}{\partial r} - JH_\theta \qquad (5)$$

$$-s\omega^2 r \sin\theta\cos\theta = -\frac{1}{r}\frac{\partial P}{\partial \theta} + JH_r \qquad (6)$$

$$j_r = j_\theta = 0 \text{ and } j_\phi = J = -\frac{\Delta U}{4\pi r \sin\theta} \qquad (7)$$

where

$$\Delta U = \frac{\partial^2 U}{\partial r^2} + \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial U}{\partial r}\right) \qquad (8)$$

From (5) and (6) the condition for the equilibrium becomes

$$\frac{\partial}{\partial r}\left(\frac{J}{r\sin\theta}\right)\frac{\partial U}{\partial\theta} - \frac{\partial}{\partial\theta}\left(\frac{J}{r\sin\theta}\right)\frac{\partial U}{\partial r} = 0 \qquad (9)$$

This is similar to the condition of equilibrium found by Stokes for steady rotational motion of an incompressible fluid.

Equation (9) is satisfied if $\Delta U = r^2 \sin^2 \theta f(U)$, where $f(U)$ is an arbitrary function of U.

Considering the simplest case of Ferraro where he takes $f(U) = k$, we have

$$\Delta U = kr^2 \sin^2 \theta \quad \ldots \quad (10)$$

This means that the currents are concentrated near the surface.

Part A

3. Equilibrium configuration of a rotating fluid sphere having a central core when the sphere is subjected to the influence of a magnetic field generated by the electric currents inside the core

Stream functions

Let U, U' be the stream functions associated with the magnetic field developed inside and outside the core respectively. They must satisfy the equations

$$\Delta U = kr^2 \sin^2 \theta \qquad r \leq r_0 \quad \ldots \quad (11a)$$

and

$$\Delta U' = 0 \qquad r \geqslant r_0 \quad \ldots \quad (11b)$$

where r_1 is the radius of the core.

The stream functions satisfying these equations are

$$U = \left(\alpha + \frac{k}{10} r^2\right) r^2 \sin^2 \theta \qquad r \leqslant r_1 \quad \ldots \quad (12a)$$

$$U' = \frac{\beta \sin^2 \theta}{r} \qquad r \geqslant r_1 \quad \ldots \quad (12b)$$

To evaluate the constants of equations (12), we assume that the surface of the core is nearly spherical and of mean radius r_0 and the magnetic field is continuous at the surface of the core, i.e. we have

$$\left.\frac{\partial U}{\partial r}\right|_{r_0} = \left.\frac{\partial U'}{\partial r}\right|_{r_0}; \quad \left.\frac{\partial U}{\partial \theta}\right|_{r_0} = \left.\frac{\partial U'}{\partial \theta}\right|_{r_0} \quad \ldots \quad (13)$$

From equations (12) and (13) we get the values of the constants as

$$\alpha = -\frac{1}{6} kr_0^2$$

and

$$\beta = -\frac{1}{15} kr_0^5$$

Hence the required stream functions are

$$U = k\left(\frac{1}{10} r^2 - \frac{1}{6} r_0^2\right) r^2 \sin^2 \theta \qquad r \leqslant r_0 \quad \ldots \quad (14a)$$

$$U' = -\frac{kr_0^5}{15r} \sin^2 \theta \qquad r \geqslant r_0 \quad \ldots \quad (14b)$$

The magnetic field at the pole of the sphere, i.e. at $r = R_0$ and $\theta = 0$, is found to be

$$H_p = \frac{2}{15} k \frac{r_0^5}{R_0^3} \qquad (15)$$

Evaluation of ellipticity

From equations (5), (6) and (7) we find that the equation of the free surface is

$$s\,\Omega_s + \frac{k}{4\pi} U_s^{r_0} = \frac{1}{2} s\omega^2 \rho_s^2 + \text{constant} \qquad (16)$$

because $p = 0$ at the surface of the sphere. Here Ω_s is the gravitational potential at the surface of the sphere and $U_s^{r_0}$ is the value of the stream function at the surface of the core, given by

$$U_s^{r_0} = -\frac{k}{15} \frac{r_0^4}{R_0^2} \rho_s^2 \qquad (17)$$

Substituting the expression for $U_s^{r_0}$ in equation (16) we find that the effect of the magnetic field is the same as that of rotation. Thus the sphere will be an oblate spheroid of ellipticity $\frac{\epsilon}{R_0}$ (say).

The gravitational potential of the uniform spheroidal mass in terms of ellipticity is

$$\Omega_s = s\pi G \left[\left(\frac{2}{3} + \frac{2}{5} \frac{\epsilon}{R_0} \right) \rho_s^2 + \left(\frac{2}{3} - \frac{4}{5} \frac{\epsilon}{R_0} \right) z^2 \right] \qquad (18)$$

where $\frac{\epsilon}{R_0}$ is small compared to unity.

From equations (16), (17) and (18) we get

$$s^2 \pi G \left[\left(\frac{2}{3} + \frac{2}{5} \frac{\epsilon}{R_0} \right) \rho_s^2 + \left(\frac{2}{3} - \frac{4}{5} \frac{\epsilon}{R_0} \right) z^2 \right] - \frac{k^2}{60\pi} \frac{r_0^4}{R_0^2} \rho_s^2 - \tfrac{1}{2} s\omega^2 \rho_s^2 = \text{constant}.$$

Since this represents a spheroid of ellipticity $\frac{\epsilon}{R_0}$, it follows that the coefficients of ρ_s^2 and z^2 are in the ratio $\left(1 + 3\frac{\epsilon}{R_0}\right)$. Hence we find that

$$\frac{\epsilon}{R_0} = -\frac{k^2}{48\pi^2 sG} \frac{r_0^4}{R_0^2} - \frac{5}{8} \frac{\omega^2}{\pi s G}$$

$$= -\frac{k^2}{27} \frac{r_0^4 R_0^4}{M^2 G} - \frac{5}{8} \frac{\omega^2}{\pi s G} \qquad (19)$$

when $r_0 \to R_0$, then this expression reduces to

$$\frac{\epsilon}{R_0} = -\frac{k^2}{27} \frac{R_0^8}{M^2 G} - \frac{5}{8} \frac{\omega^2}{\pi s G}$$

$$= -\frac{25}{12} \frac{H_P^2 R_0^4}{M^2 G} - \frac{5}{8} \frac{\omega^2}{\pi s G}$$

where $$H_p = \frac{2}{15} kR_0^2$$

This expression for $\frac{\epsilon}{R_0}$ is exactly the same as that obtained by Ferraro.

PART B

4. EQUILIBRIUM CONFIGURATION OF A ROTATING FLUID SPHERE HAVING A CENTRAL CORE SUBJECT TO THE INFLUENCE OF A MAGNETIC FIELD GENERATED BY THE ELECTRIC CURRENTS CONFINED TO THE SHELL FORMED BY THE SURFACE OF THE SPHERE AND THE CORE

The expressions for stream functions

Let U_1, U_2 and U_3 be the stream functions associated with the magnetic fields developed inside the core, the shell and outside the sphere respectively. These functions satisfy the following equations

$$\Delta U_1 = 0 \qquad r \leqslant r_1 \quad \ldots \quad (20a)$$

$$\Delta U_2 = kr^2 \sin^2\theta \qquad r_1 \leqslant r \leqslant R \quad \ldots \quad (20b)$$

$$\Delta U_3 = 0 \qquad r \geqslant R \quad \ldots \quad (20c)$$

where r_1 and R are the radii of the core and the sphere respectively.
Solving these equations we get

$$U_1 = Ar^2 \sin^2\theta \qquad r \leqslant r_1 \quad \ldots \quad (21a)$$

$$U_2 = \left(\frac{B}{r} + cr^2 + \frac{k}{10} r^4\right) \sin^2\theta \qquad r_1 \leqslant r \leqslant R \quad \ldots \quad (21b)$$

$$U_3 = \frac{D}{r} \sin^2\theta \qquad r \geqslant R \quad \ldots \quad (21c)$$

To evaluate the constants we assume that the surface of the core and the sphere are nearly spherical and of mean radii r_0 and R_0 respectively and the magnetic field is continuous at both the surfaces, i.e. we have

$$\left.\begin{array}{l} \left.\frac{\partial U_1}{\partial r}\right|_{r_0} = \left.\frac{\partial U_2}{\partial r}\right|_{r_0}; \quad \left.\frac{\partial U_1}{\partial \theta}\right|_{r_0} = \left.\frac{\partial U_2}{\partial \theta}\right|_{r_0} \\ \left.\frac{\partial U_2}{\partial r}\right|_{R_0} = \left.\frac{\partial U_3}{\partial r}\right|_{R_0}; \quad \left.\frac{\partial U_2}{\partial \theta}\right|_{R_0} = \left.\frac{\partial U_3}{\partial \theta}\right|_{R_0} \end{array}\right\} \quad \ldots \quad (22)$$

From equations (21) and (22) we get the values of the constants as

$$\left.\begin{array}{l} A = -\frac{k}{6}(R_0^2 - r_0^2) \\ B = \frac{k}{15} r_0^5 \\ C = -\frac{k}{6} R_0^2 \\ D = -\frac{k}{15}(R_0^5 - r_0^5) \end{array}\right\} \quad \ldots \quad (23)$$

Thus the stream functions are

$$U_1 = -\frac{k}{6}\left(R_0^2 - r_0^2\right) r^2 \sin^2\theta \qquad r \leqslant r_0 \quad \ldots \quad \ldots \quad (24a)$$

$$U_2 = \left(\frac{1}{15}\frac{r_0^5}{r^3} - \frac{1}{6}R_0^2 + \frac{1}{10}r^2\right) kr^2 \sin^2\theta \qquad r_1 \leqslant r \leqslant R_0 \quad \ldots \quad \ldots \quad (24b)$$

$$U_3 = -\frac{k}{15}\left(R_0^5 - r_0^5\right)\frac{\sin^2\theta}{r} \qquad r \geqslant R_0 \quad \ldots \quad \ldots \quad (24c)$$

Evaluation of ellipticity

The equation of the free surface for the equilibrium configuration is obtained from equations (5), (6) and (7) and is given by

$$s\Omega_s + \frac{k}{4\pi}\left(U_s - U_s^{r_0}\right) = \tfrac{1}{2} s\omega^2 \rho_s^2 + \text{constant} \quad \ldots \quad \ldots \quad (25)$$

U_s and $U_s^{r_0}$ are the values of the functions at the surface of the sphere and the surface of the core respectively.
From equation (24*a*) it follows that the magnetic field inside the core has the components $H_x = H_y = 0$ and $H_z = -\frac{k}{3}\left(R_0^2 - r_0^2\right)$ in the cartesion co-ordinates.
From equations (24*a*) and (24*b*) we get

$$\frac{k}{4\pi}\left(U_s - U_{r_0}\right) = \frac{9H_z^2}{4\pi}\left[\frac{\frac{1}{6}\left(R_0^2 - r_0^2\right)\frac{r_0^2}{R_0^2} - \frac{1}{15}\frac{\left(R_0^5 - r_0^5\right)}{R_0^3}}{\left(R_0^2 - r_0^2\right)^2}\right]\rho_s^2 \quad \ldots \quad (26)$$

This expression is negative for all values of r_0 lying between zero and R_0. Thus we conclude that the effect of the magnetic field is the same as that of the constant angular velocity, i.e. the surface of the fluid will nearly be an oblate spheroid of ellipticity $\frac{\epsilon}{R_0}$ (say).

From equations (18), (25) and (26) we get

$$\frac{2\pi}{3} s^2 G \left\{\left(1 + \frac{3}{5}\frac{\epsilon}{R_0}\right)\rho_s^2 + \left(1 - \frac{6}{5}\frac{\epsilon}{R_0}\right) z^2\right\} - \frac{1}{2} s\omega^2 \rho_s^2 -$$

$$- \frac{9H_z^2}{4\pi}\left[\frac{\frac{1}{15}\frac{\left(R_0^5 - r_0^5\right)}{R_0^3} - \frac{1}{6}\left(R_0^2 - r_0^2\right)\frac{r_0^2}{R_0^2}}{\left(R_0^2 - r_0^2\right)^2}\right]\rho_s^2 = \text{constant}.$$

Since the equilibrium configuration is an oblate spheroid, therefore the ratio of the coefficients of ρ_s^2 and z^2 must be given by $\left(1 + 3\frac{\epsilon}{R_0}\right)$.

Thus we have

$$1+3\frac{\epsilon}{R_0}=\frac{1+3\dfrac{\epsilon}{R_0}-\dfrac{3\omega^2}{4\pi sG}-\dfrac{27H_z^2}{8\pi^2s^2G}\left[\dfrac{\dfrac{1}{15}\dfrac{(R_0^5-r_0^5)}{R_0^3}-\dfrac{1}{6}(R_0^2-r_0^2)\dfrac{r_0^2}{R_0^2}}{(R_0^2-r_0^2)^2}\right]}{1-\dfrac{6}{5}\dfrac{\epsilon}{R_0}}$$

which after simplification gives the value of $\frac{\epsilon}{R_0}$ as

$$\frac{\epsilon}{R_0}=-\frac{5}{8}\frac{\omega^2}{\pi sG}-\frac{45H_z^2}{16\pi^2s^2G}\left[\frac{\dfrac{1}{15}\dfrac{(R_0^5-r_0^5)}{R_0^3}-\dfrac{1}{6}(R_0^2-r_0^2)\dfrac{r_0^2}{R_0^2}}{(R_0^2-r_0^2)^2}\right] \quad \ldots \quad (27)$$

when $r_0=0$, we get

$$\frac{\epsilon}{R_0}=-\frac{5}{8}\frac{\omega^2}{\pi sG}-\frac{45H_z^2}{16\pi^2s^2G}\cdot\frac{1}{15R_0^2}$$

where
$$H_z\,(\text{at } r_0=0)=-\frac{k}{3}R_0^2.$$

The magnetic field at the pole, i.e. at $r=R_0$ and $\theta=0$, is

$$H_P\,(\text{at } r_0=0)=\frac{2k}{15}R_0^2=-\frac{2}{5}H_z.$$

Therefore the expression for ellipticity in terms of H_P is

$$\frac{\epsilon}{R_0}=-\frac{5}{8}\frac{\omega^2}{\pi sG}-\frac{25}{12}\frac{H_P^2R_0^4}{M^2G}. \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

This value is the same as that obtained by Ferraro.

Similarly the expression for $\frac{\epsilon}{R_0}$ when $r_0 \to R_0$ comes out to be

$$\frac{\epsilon}{R_0}=-\frac{5}{8}\frac{\omega^2}{\pi sG}-\frac{5}{24}\frac{H_z^2R_0^4}{M^2G} \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

here H_z is the constant magnetic field inside the sphere.

Equilibrium configuration of the sphere with non-magnetic core

Here we consider a case which is related to Ferraro's problem as Gjellestad's is related to the Chandrasekhar and Fermi problem.

From equation (24*a*) we find that the magnetic field inside the core is constant and parallel to the *Z*-axis. If we superpose throughout the space a uniform field equal and opposite to H_z then the new fields in the different regions are described by the Stokes functions

$$U'=0 \qquad r\leqslant r_0 \quad \ldots \quad \ldots \quad (30a)$$

$$U''=\left[\frac{1}{15}\frac{r_0^5}{r^3}+\frac{1}{10}r^2-\frac{1}{6}r_0^2\right]kr^2\sin^2\theta \qquad r_0\leqslant r\leqslant R_0 \quad \ldots \quad \ldots \quad (30b)$$

$$U''' = \left[\frac{1}{6}\left(R_0^2-r_0^2\right) - \frac{1}{15}\frac{R_0^5-r_0^5}{r^3}\right] kr^2 \sin^2\theta \quad r_0 \geqslant R_0 \quad \ldots \quad \ldots (30c)$$

Substituting the values of U_s, $U_s^{r_0}$ and K in terms of H_z in $\frac{k}{4\pi}\left(U_s - U_s^{r_0}\right)$ we find that it is a positive quantity. This means that the effect of the magnetic field is opposite to that of the constant angular velocity. Thus from equation (25) we conclude that the equilibrium configuration will be an oblate spheroid or prolate spheroid according as the value of the rotational term is greater than or smaller than that of the magnetic term. From equations (18), (25) and (30) we get

$$s^2\pi G\left[\left(\frac{2}{3}+\frac{2}{5}\frac{\epsilon}{R_0}\right)\rho_s^2+\left(\frac{2}{3}-\frac{4}{5}\frac{\epsilon}{R_0}\right)z^2\right] - \frac{1}{2}s\omega^2\rho_s^2 + \\ +\frac{9H_z^2\rho_s^2}{4\pi(R_0^2-r_0^2)^2}\left[\frac{1}{6}\left(R_0^2-r_0^2\right) - \frac{1}{15}\frac{R_0^5-r_0^5}{R_0^3}\right] = \text{const.}$$

comparing the coefficients of ρ_s^2 and Z^2 we obtain

$$1+3\frac{\epsilon}{R_0} = \frac{\left(\frac{2}{3}+\frac{2}{5}\frac{\epsilon}{R_0}\right)+\frac{9H_z^2}{4\pi^2s^2G}\left[\frac{\frac{1}{6}\left(R_0^2-r_0^2\right) - \frac{1}{15}\frac{R_0^5-r_0^5}{R_0^3}}{\left(R_0^2-r_0^2\right)^2}\right] - \frac{\omega^2}{2\pi sG}}{\frac{2}{3}-\frac{4}{5}\frac{\epsilon}{R_0}}$$

which on simplification gives

$$\frac{\epsilon}{R_0} = \frac{45H_z^2}{16\pi^2s^2G(R_0^2-r_0^2)^2}\left[\frac{1}{6}\left(R_0^2-r_0^2\right) - \frac{1}{15}\frac{R_0^5-r_0^5}{R_0^3}\right] - \frac{5}{8}\frac{\omega^2}{\pi sG} \quad \ldots (31)$$

When $r_0 \rightarrow 0$, then equation (31) reduces to

$$\frac{\epsilon}{R_0} = \frac{1}{2}\frac{H_z^2R_0^4}{M^2G} - \frac{5}{8}\frac{\omega^2}{\pi sG} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (32)$$

The value of $\frac{\epsilon}{R_0}$ for $\omega = 0$ is exactly the same as that obtained by Kothari and Auluck. In this case the magnetic field at large distances from the sphere is constant $\left(=\frac{k}{3}R_0^2\right)$ and zero at the centre of the sphere.

Similarly when $r_0 \rightarrow R_0$, equation (31) becomes

$$\frac{\epsilon}{R_0} = \frac{5}{8}\frac{H_z^2R_0^4}{M^2G} - \frac{5}{8}\frac{\omega^2}{\pi sG}$$

For $\omega = 0$

$$\frac{\epsilon}{R_0} = \frac{5}{8}\frac{H_z^2R_0^4}{M^2G}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (33)$$

This expression is the same as that obtained by Gjellestad for non-magnetic gravitating fluid sphere by energy method of C.F. In this case $H_z = \frac{k}{3}\left(R_0^2-r_0^2\right)$ is a constant field at large distances from the sphere.

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Summary

In this paper the equilibrium configuration of a magnetic gravitating sphere of incompressible, inviscid and infinitely conducting fluid having a central core has been discussed in two parts. In part A, the magnetic field is assumed to be generated by the electric currents flowing inside the core and in part B, the field is assumed to be due to the flow of electric currents inside the shell formed by the surface of the core and the sphere. In both the cases the effect of the magnetic field is found to be the same as that of rotation, i.e. the equilibrium configuration is an oblate spheroid. However, in a special case where a constant magnetic field equal and opposite to the field inside the core is superposed throughout the space, the equilibrium configuration of the sphere comes out to be a prolate spheroid.

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