ON INTERNAL BALLISTICS OF A TAPERED BORE GUN USING COMPOSITE CHARGES

by V. K. JAIN, *Defence Science Laboratory, New Delhi*

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INTRODUCTION

Corner (1950) has set up the basic equations of Internal Ballistics of a tapered bore gun by the method of R.D. 38 and has suggested a method for the numerical integration of these equations. In an earlier paper (under communication) Jain and Sodha have set up the equations of Internal Ballistics of a tapered bore gun on the non-isothermal modal using non-dimensional variables and have suggested a method for solving them. In this paper the author has suggested a step by step numerical solution of basic equations of the Internal Ballistics of a tapered bore gun using composite charges, considering the same power law of burning, and taking into account the covolume, heat transfer and bore resistance. An analytical solution has also been presented assuming a linear rate of burning and neglecting covolume, heat transfer and bore resistance. An example illustrating the method has also been solved.

BASIC EQUATIONS

The cross-section of the barrel of a tapered bore gun is a section of a converging cone and its area A is given by :

$$A = A_0\left(1-\frac{ax}{l}\right)^2$$

where A_0 is the cross-sectional area at the initial position of the shot ($x = 0$) and $\frac{a}{l}=\frac{\tan}{r_0}$, α being the semi-vertical angle of the cone and r_0 the radius of the bore at the initial position of the shot.

Assuming the same power law of burning for the two propellants, the basic equations of Internal Ballistics of a tapered bore gun using composite charges are :

$$Z_1 = (1-f_1)(1+\theta_1 f_1) \quad \ldots \quad (1a)$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2) \quad \ldots \quad (1b)$$

$$D_1\frac{df_1}{dt} = -\beta_1 P^{\alpha} \quad \ldots \quad (2a)$$

$$D_2\frac{df_2}{dt} = -\beta_2 P^{\alpha} \quad \ldots \quad (2b)$$

$$W_1 V\frac{dV}{dx} = AP-R \quad \ldots \quad (3)$$

$$\text{and } F_1C_1Z_1+F_2C_2Z_2 = P\left\{\int_0^x A\,dx - C_1Z_1(b_1-1/\delta_1)-C_2Z_2(b_2-1/\delta_2)+A_0l\right\}$$
$$+\frac{1}{2}W'V^2(\gamma-1)+(\gamma-1)\left\{\int H\,dx+\int R\,dx\right\} \quad \cdots \quad \cdots \quad (4)$$

where suffix 1 is referring to the first charge,
suffix 2 is referring to the second charge,

- Z is the fraction of the charge burnt at any instant,
- D is the web size of the propellant,
- f is the fraction of D remaining unburnt at any instant,
- θ is the form factor of the propellant,
- P is the mean pressure of the propellant gases,
- α is the index of burning,
- β is the rate of burning coefficient
- $W' = 1{\cdot}01\ W+\frac{C_1}{3}+\frac{C_2}{3}$, C_1 and C_2 being the charge weight and W the shot weight.
- x is the shot travel at any instant,
- V is the shot velocity at any instant,
- l is the equivalent chamber length,
- b is the volume of propellant gas,
- δ is the density of the propellant,
- γ is the ratio of the specific heats at constant pressure and at constant volume,
- F is the force constant of the propellant,
- H is the heat transferred per unit length of shot travel,

and R is the force acting on the projectile at any instant due to friction and swaging of flanges.

Putting

$$\xi = 1+\frac{x}{l},$$

$$B_1 = (b_1-1/\delta_1)C_1/A_0l,$$

$$B_2 = (b_2-1/\delta_2)C_2/A_0l,$$

$$\zeta_1 = \frac{PA_0l}{F_1C_1},$$

$$\zeta_2 = \frac{PA_0l}{F_2C_2},$$

$$\eta_1 = \frac{VA_0D_1}{F_1C_1\beta_1}\left(\frac{F_1C_1}{A_0l}\right)^{1-\alpha},$$

$$\eta_2 = \frac{VA_0D_2}{F_2C_2\beta_2}\left(\frac{F_2C_2}{A_0l}\right)^{1-\alpha},$$

$$M_1 = \frac{A_0^2D_1^2}{F_1C_1\beta_2^1W'}\left(\frac{F_1C_1}{A_0l}\right)^{2(1-\alpha)},$$

$$M_2 = \frac{A_0^2D_2^2}{F_2C_2\beta_2^2W'}\left(\frac{F_2C_2}{A_0l}\right)^{2(1-\alpha)},$$

$$\rho_1 = \frac{Rl}{F_1C_1}$$

$$\rho_2 = \frac{Rl}{F_2C_2}$$

and
$$h = \frac{Hl}{F_2C_2}$$

in equations (2a), (2b), (3a), (3b) and (4) we obtain

$$\eta_1 \frac{df_1}{d\xi} = -\zeta_1^{\alpha} \quad \ldots \quad (5a)$$

$$\eta_2 \frac{df_2}{d\xi} = -\zeta_2^{\alpha} \quad \ldots \quad (5b)$$

$$\eta_1 \frac{d\eta_1}{d\xi} = M_1\zeta_1[1-a(\xi-1)]^2 - M_1\rho_1 \quad \ldots \quad (6a)$$

$$\eta_2 \frac{d\eta_2}{d\xi} = M_2\zeta_2[1-a(\xi-1)]^2 - M_2\rho_2 \quad \ldots \quad (6b)$$

$$\frac{F_1C_1}{F_2C_2}Z_1 + Z_2 = \zeta_2\left[\xi - a(\xi-1)^2 + \frac{a^2}{3}(\xi-1)^3 - B_1Z_1 - B_2Z_2\right] + (\gamma-1)\left\{\frac{\eta_2^2}{2M_2} + \int \rho_2 d\xi + \int h d\xi\right\} \quad \ldots \quad (7)$$

Step by Step Numerical Solution

It is easy to see (Venkatesan and Patni, 1953) that if

(1) $\beta_1 D_2 f_{20} > \beta_2 D_1 f_{10}$ then f_2 cannot become zero before f_1, hence charge C_1 must burn out first.

(2) $\beta_1 D_2 f_{20} < \beta_2 D_1 f_{10}$ then f_1 cannot become zero before f_2, hence charge C_2 must burn out first.

(3) $\beta_1 D_2 f_{20} = \beta_2 D_1 f_{10}$ then both the charges will have to be burnt out simultaneously.

We shall be taking up the case when both the charges burn out at different times and for the sake of definiteness we shall assume that f_1 burns out first.

From (5a) and (5b) and using the boundary condition that initially $f_1 = f_{10}$ and $f_2 = f_{20}$
we have

$$f_1 = f_2 \frac{\beta_1 D_2}{\beta_2 D_1} + f_{10} - f_{20}\frac{\beta_1 D_2}{\beta_2 D_1} \quad \ldots \quad (8)$$

The ballistic equations (1a), (1b), (5a), (5b), (6a), (6b) and (7) can be solved numerically by a step by step process with the help of equation (8) and the most convenient independent variable for the purpose is f_2.

Let ′ denote values of variables at the beginning of a step and ″ the values of the end of a step. Let Δ denote the difference between end and beginning of a step and let — denote the arithmetic mean in the step so that for example

$$\Delta\eta_2 = \eta_2'' - \eta_2'$$

$$\bar{\eta}_2 = \tfrac{1}{2}(\eta_2'' + \eta_2')$$

From equations (5*b*) and (6*b*) we have

$$d\eta_2 = -M_2[\zeta_2^{1-\alpha}\{1-a(\xi-1)\}^2-\rho_2\zeta_2^{-\alpha}]df_2$$

and in the small interval Δf_2 this becomes

$$\Delta\eta_2 = -M_2[\bar{\zeta}_2^{(1-\alpha)}\{1-a(\bar{\xi}-1)\}^2-\bar{\rho}_2\bar{\zeta}_2^{(-\alpha)}]\Delta f_2 \quad .. \quad .. \quad (9)$$

Equation (6*b*) gives

$$\Delta\xi = \frac{\bar{\eta}_2\Delta\eta_2}{M_2\bar{\zeta}[1-a(\bar{\xi}-1)]^2-M_2\bar{\rho}_2} \quad .. \quad .. \quad .. \quad (10)$$

The process consists in estimating $\Delta\zeta_2$ and so obtaining a tentative value of $\bar{\zeta}_2$ from

$$\bar{\zeta}_2 = \zeta_2' + \frac{\Delta\zeta_2}{2}$$

Tentative values of $\Delta\eta_2$ and $\Delta\xi$ and therefore of η_2'' and ξ'' are obtained from equations (9) and (10) taking $\bar{\xi} = \xi' + \frac{\Delta\xi}{2}$. The value of Z_2'' is determined directly from (1*b*) since f_2'' is known. The value of Z_1'' is determined with the help of (1*a*) and (8). Then from equation (7)

$$\zeta_2'' = \frac{Z_2'' + \frac{F_1C_1}{F_2C_2}Z_1'' - (\gamma-1)\left[\eta_2''\Big/2M_2 + \int(\rho_2+h)d\xi\right]}{\xi''-a(\xi''-1)^2+a^2(\xi''-1)^3/3-B_1Z_1''-B_2Z_2''} \quad .. \quad .. \quad (11)$$

and a value of ζ_2'' is determined. Thence a more accurate value of $\Delta\zeta_2 = \zeta_2''-\zeta_2'$ is obtained. The process is repeated with this new value of $\Delta\zeta_2$ and $\bar{\xi} = \xi' + \frac{\Delta\xi}{2}$ until the estimated and this new value of $\Delta\zeta_2$ agree.

A good estimate of $\Delta\zeta_2$ is given by

$$\Delta\zeta_2 = \left[\frac{\frac{F_1C_1}{F_2C_2}\dot{Z}_1 + \dot{Z}_2 - \gamma\zeta_2'^{(1-\alpha)}\dot{\eta}_2}{\xi'}\right]\Delta f_2 \quad .. \quad .. \quad (12)$$

which is obtained by differentiating (7) *w.r.t.* f_2, neglecting B_1, B_2, ρ_2 and a. The dots represent differentiation with respect to f_2.

The above set of ballistic equations hold so long as both the charges are burning when the first charge has burnt out, the equations of Internal Ballistics become

$$Z_2 = (1-f_2)(1+\theta_2 f_2)$$

$$\eta_2\frac{df_2}{d\xi} = -\zeta_2^{\alpha}$$

$$\eta_2\frac{d\eta_2}{d\xi} = M_2\zeta_2[1-a(\xi-1)]^2-M_2\rho_2$$

and

$$\frac{F_1C_1}{F_2C_2}+Z_2 = M_2\zeta_2\left[\xi-a(\xi-1)^2+\frac{a^2}{3}(\xi-1)^3-B_1-B_2Z_2\right] + (\gamma-1)\left[\eta_2^2\Big/2M_2+\int\rho_2 d\xi+\int h d\xi\right] \quad .. \quad .. \quad .. \quad (7a)$$

The equations (9), (10), (11) and (12) still hold with the difference that wherever Z_1 occurs replace it by 1.

An Analytical Solution

Assuming

(*i*) a linear rate of burning

(*ii*) $b_1 = 1/\delta_1$, $b_2 = 1/\delta_2$ or $B_1 = B_2 = 0$

(*iii*) Neglecting heat transfer and bore resistance. Equations (5*a*), (5*b*), (6*a*), (6*b*) and (7) become

$$\eta_1 \frac{df_1}{d\xi} = -\zeta_1 \qquad \text{(13a)}$$

$$\eta_2 \frac{df_2}{d\xi} = -\zeta_2 \qquad \text{(13b)}$$

$$\eta_1 \frac{d\eta_1}{d\xi} = M_1\zeta_1[1-a(\xi-1)]^2 \qquad \text{(14a)}$$

$$\eta_2 \frac{d\eta_2}{d\xi} = M_2\zeta_2[1-a(\xi-1)]^2 \qquad \text{(14b)}$$

$$\frac{F_1C_1}{F_2C_2} Z_1 + Z_2 = \zeta_2\left[\xi - a(\xi-1)^2 + \frac{a^2}{3}(\xi-1)^3\right] + \frac{\gamma-1}{2M_2}\eta_2^2 \qquad \text{(15)}$$

From (14*b*) and (13*b*) we have

$$\frac{d\eta_2}{df_2} = -M_2[1-a(\xi-1)]^2$$

Since, for a small taper, $[1-a(\xi-1)]^2 \approx 1$ we may replace it by its average value before all burnt.

Hence

$$\frac{d\eta_2}{df_2} = -\overline{M}_2 \qquad \text{(16)}$$

where

$$\overline{M}_2 = M_2 \frac{\int_1^{\xi_b} [1-a(\xi-1)]^2 d\xi}{\int_1^{\xi_b} d\xi} = M_2\left[1 - a(\xi_b - 1) + \frac{(\xi_b-1)^2}{3} a^2\right]$$

where ξ_b is value of the shot velocity at all burnt taking $a = 0$.

Integrating (15) and using the boundary condition that $\eta_2 = 0$ when $f_2 = 1$ we have

$$\eta_2 = \overline{M}_2(1-f_2) \qquad \text{(17)}$$

From (17) (and 1*b*)

$$Z_2 = \frac{\eta_2}{\overline{M}_2}\left[1+\theta_2-\theta_2\frac{\eta_2}{\overline{M}_2}\right] \qquad \text{(18)}$$

With the help of (17), (1*a*) and (8) and the boundary condition that $f_1 = 1$ when $\eta_2 = 0$ we have

$$Z_1 = \frac{\beta_1 D_2}{\beta_2 D_1}\frac{\eta_2}{\overline{M}_2}\left[1+\theta_1-\theta_1\frac{\beta_1 D_2}{\beta_2 D_1}\frac{\eta_2}{\overline{M}_2}\right] \qquad \text{(19)}$$

Combining (19), (18), (15) and (13b) we have

$$\frac{\eta_2}{\overline{M}_2}\left[1+\theta_2-\theta_2\frac{\eta_2}{\overline{M}_2}\right]+\frac{F_1C_1}{F_2C_2}\frac{\beta_1D_2}{\beta_2D_1}\frac{\eta_2}{\overline{M}_2}\left[1+\theta_1-\theta_1\frac{\beta_1D_2}{\beta_2D_1}\frac{\eta_2}{\overline{M}_2}\right]$$
$$=\frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}\left[1-2a(\xi-1)+a^2(\xi-1)^2\right]^{-1}\left[\xi-a(\xi-1)^2+\frac{a^2}{3}(\xi-1)^3\right]$$
$$+\frac{\gamma-1}{2}\frac{\eta_2^2}{M_2} \quad \ldots \quad (20)$$

Neglecting higher powers of a than a^2 we have

$$\frac{d\eta_2}{A-\mu\eta_2}=M_2d\xi\left[\frac{1}{\xi}+\frac{a^2}{\xi^3}+\frac{1}{\xi^2}(a-5a^2/_3)+a(a-1)-\frac{a^2}{3}\xi\right]$$

where

$$A=\frac{1+\theta_2}{\overline{M}_2}+\frac{F_1C_1}{F_2C_2}\frac{\beta_1D_2}{\beta_2D_1}\frac{(1+\theta_1)}{\overline{M}_2}$$

and

$$\mu=\frac{\theta_2}{\overline{M}_2^2}+\frac{F_1C_1}{F_2C_2}\frac{\beta_1^2D_2^2}{\beta_2^2D_1^2}\frac{\theta_1}{\overline{M}_2^2}+\frac{\gamma-1}{2M_2}$$

Integrating the above and using the boundary condition that $\eta_2=0$ when $\xi=1$ we have

$$\log\left(\frac{A-\mu\eta_2}{A}\right)=-\mu M_2\left[\log\xi+\frac{a^2}{2\xi^2}(\xi^2-1)+\left(a-\frac{5a^2}{3}\right)\left(\frac{\xi-1}{\xi}\right)\right.$$
$$\left.+a(a-1)(\xi-1)-\frac{a^2}{6}(\xi^2-1)\right]$$

or

$$\eta_2=\frac{A}{\mu}\left[1-\xi^{-\mu M_2}F(\xi)\right] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

where

$$F(\xi)=\mathrm{Exp}-\mu M_2\left[\frac{a^2}{2\xi^2}(\xi^2-1)+a(1-5a/_3)\frac{\xi-1}{\xi}-a(1-a)(\xi-1)-\frac{a^2}{6}(\xi^2-1)\right]$$

Also

$$\zeta_2=\frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}\frac{1}{[1-a(\xi-1)]^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

(21) and (22) hold so long as both the charges are burning.

THE STAGE WHEN THE FIRST CHARGE HAS JUST BURNT OUT

Combining (13b) and (14b) we have

$$M_2\frac{d\eta_2}{df_2}=-\frac{1}{[1-a(\xi-1)]^2}$$

or

$$M_2(1-f_2)=\int_0^{\eta_2}\frac{d\eta_2}{[1-a(\xi-1)]^2}$$
$$\approx\int_0^{\eta_2}[1+2a(\xi-1)+3a^2(\xi-1)^2]d\eta_2$$

Since

$$\eta_2 \approx \frac{A}{\mu}\{1-\xi^{-\mu}\} \text{ we have}$$

$$M_2(1-f_2) = \int_0^{\eta_2}(1-2a+3a^2)d\eta_2+2(a-3a^2)A\int_1^{\xi}\xi^{-\mu}\,d\xi+3a^2A\int_1^{\xi}\xi^{-\mu+1}d\xi$$

when the first charge has burnt out $f_1 = 0$ so that

$$f_2 = 1-\frac{\beta_2 D_1}{\beta_1 D_2}$$

Substituting this value of f_2 the value of η_2 when the first charge has burnt out is given by

$$\eta_{2b'} = \frac{M_2\beta_2 D_1}{(1-2a+3a^2)\beta_1 D_2}-\frac{A}{(1-2a+3a^2)}\left[2(a-3a^2)\frac{\xi_{b'}^{1-\mu}-1}{1-\mu}+3a^2\frac{\xi_{b'}^{2-\mu}-1}{2-\mu}\right] \quad (23)$$

From (21) we have

$$\xi_{b'} = \left[\frac{A-\mu\eta_{2b'}}{AF(\xi_{b'})}\right]^{-\frac{1}{\mu M_2}} \quad \ldots \quad (24)$$

We may evaluate $\xi_{b'}$ for a given value of $\eta_{2b'}$ by solving equation (24) by the method of successive approximations. If $\xi_{b',n}$ is the value of shot travel when the first charge has burnt out to the nth order of accuracy we have

$$\xi_{b',n+1} = \left[\frac{A-\mu\eta_{2b'}}{AF(\xi_{b',n})}\right]^{-\frac{1}{\mu M_2}} \quad \ldots \quad (25)$$

This value of $\xi_{b'}$ may be substituted in equation (23) to get a more accurate value of $\eta_{2b'}$.

Thus we may evaluate $\eta_{2b'}$ and $\xi_{b'}$ to any desired degree of accuracy.

The first charge has burnt out and the second charge is still burning

Putting $Z_1 = 1$ in equation (15) and using (14b) and (18) we have

$$\frac{F_1C_1}{F_2C_2}+\frac{\eta_2}{M_2}\left[1+\theta_2-\theta_2\frac{\eta_2}{M_2}\right]$$

$$= \frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}\left[1-2a(\xi-1)+a^2(\xi-1)^2\right]^{-1}\left[\xi-a(\xi-1)^2+\frac{a^2}{3}(\xi-1)^3\right]+\frac{\gamma-1}{2M_2}\eta_2^2 \quad (26)$$

or

$$\frac{\eta_2 d\eta_2}{c+d\eta_2-e\eta_2^2} = M_2\left[\frac{1}{\xi}+\frac{a^2}{\xi^3}+\frac{1}{\xi^2}\left(a-\frac{5a^2}{3}\right)-a(1-a)-\frac{a^2}{3}\xi\right]d\xi$$

where

$$c = \frac{F_1C_1}{F_2C_2}$$

$$d = \frac{1+\theta^2}{M^2}$$

$$e = \frac{\theta_2}{M_2{}^2}+\frac{\gamma-1}{2M_2}$$

or

$$\int_{\eta_{2b'}}^{\eta_2} \frac{\eta_2 d\eta_2}{c+d\eta_2-e\eta_2^2} = M_2\left[\log \xi - \frac{a^2}{2\xi^2} - \frac{a(1-5a/3)}{\xi} - a(1-a)\xi - \frac{a^2}{6}\xi^2\right]_{\xi_{b'}}^{\xi}$$

$$\therefore \xi = \operatorname{Exp} \frac{1}{M_2}\left[\int_{\eta_{2b'}}^{\eta_2} \frac{\eta_2 d\eta_2}{c+d\eta_2-e\eta_2^2} + \left(\frac{a^2}{2\xi^2} - \frac{a^2}{2\xi_{b'}^2}\right) + a(1-5a/3)\left(\frac{1}{\xi} - \frac{1}{\xi_{b'}}\right)\right.$$
$$\left. + a(1-a)(\xi-\xi_{b'}) + \frac{a^2}{6}(\xi^2-\xi_{b'}^2)\right] \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

Also

$$\zeta_2 = \frac{\eta_2}{M_2} \frac{1}{[1-a(\xi-1)]^2} \frac{d\eta_2}{d\xi}$$
$$= (c+d\eta_2-e\eta_2^2)\left[\xi - a(\xi-1)^2 + \frac{a^2}{3}(\xi-1)^3\right] \quad \ldots \quad \ldots \quad (28)$$

ξ as a function of η_2 being given by (27).

Both the charges have burnt

Combining (13*b*) and (14*b*) we have

$$M_2 \frac{d\eta_2}{df_2} = -\frac{1}{[1-a(\xi-1)]^2}$$

Integrating the above and using the boundary condition that $f_2 = 1 - \frac{\beta_2 D_1}{\beta_1 D_2} f_{2b'}$ when $\eta_2 = \eta_{2b'}$ we have

$$M_2(f_{2b'}-f_2) = \int_{\eta_{2b'}}^{\eta_2} [1+2a(\xi-1)+3a^2(\xi-1)^2]d\eta_2.$$

Since

$$\xi \approx \operatorname{Exp} \frac{1}{M_2}\int_{\eta_{2b'}}^{\eta_2} \frac{\eta_2 d\eta_2}{c+d\eta_2-e\eta_2^2} = \phi(\eta_2)$$

we have

$$M_2(f_{2b'}-f_2) = \int_{\eta_{2b'}}^{\eta_2} (1-2a+3a^2)d\eta_2$$
$$+2A(a-3a^2)\int_{\eta_{2b'}}^{\eta_2} \phi(\eta_2)d\eta_2$$
$$+3a^2 A\int_{\eta_{2b'}}^{\eta_2} \{\phi(\eta_2)\}^2 d\eta_2$$

Putting $f_2 = 0$, velocity at all burnt is given by

$$\eta_{2b''} = \frac{M_2 f_{2b'}}{(1-2a+3a^2)} + \eta_{2b'}$$

$$- \frac{A}{(1-2a+3a^2)}\left[2(a-3a^2)\int_{\eta_{2b'}}^{\eta_2} \phi(\eta_2)d\eta_2 + 3a^2\int_{\eta_{2b'}}^{\eta_2} \{\phi(\eta_2)\}^2 d\eta_2\right] \quad .. \quad (29)$$

From equation (27)

$$\xi_{b''} = \text{Exp}\,\frac{1}{M_2}\left[\int_{\eta_{2b'}}^{\eta_{2b''}} \frac{\eta_2 d\eta_2}{c+d\eta_2-e\eta_2^2} + \left(\frac{a^2}{2\xi^2} - \frac{a^2}{2\xi_{b'}^2}\right) + a(1-5a/3)\left(\frac{1}{\xi} - \frac{1}{\xi_{b'}}\right)\right.$$

$$\left. + a(1-a)(\xi-\xi_{b'}) + \frac{a^2}{6}(\xi^2-\xi_{b'}^2)\right] \quad .. \quad .. \quad .. \quad (27a)$$

From equations (27*a*) and (29) we can get the values of $\xi_{b''}$ and $\eta_{2b''}$ to any desired degree of accuracy by the method of successive approximation.

After all burnt

Putting $Z_1 = Z_2 = 1$ in equation (15) and using (14*b*) we have

$$\frac{F_1C_1}{F_2C_2} + 1 = \frac{1}{M_2[1-a(\xi-1)]^2}\left[\xi - a(\xi-1)^2 + \frac{a^2}{3}(\xi-1)^3\right]\eta_2\frac{d\eta_2}{d\xi} + \frac{\gamma-1}{2M_2}\eta_2^2 \quad (30)$$

or

$$\frac{dX}{d\xi} + \frac{\gamma-1}{\xi}\left[1-a(\xi-1)\right]^2\left[1 - \frac{a}{\xi}(\xi-1)^2 + \frac{a^2}{3\xi}(\xi-1)^3\right]^{-1} X$$

$$- \frac{\left(\frac{F_1C_1}{F_2C_2}+1\right)\left[1-a(\xi-1)\right]^2\left[1-\frac{a}{\xi}(\xi-1)^2+\frac{a^2}{3\xi}(\xi-1)^3\right]^{-1}}{\xi} = 0$$

where

$$X = \frac{\eta_2^2}{2M_2}$$

The solution of the above equation is

$$\left[X\Psi(\xi)\right]_{\xi_{b''},\,\eta_{2b''}}^{\xi,\,\eta} = \left(\frac{F_1C_1}{F_2C_2}+1\right)\left[\frac{\Psi(\xi)-\Psi(\xi_{b''})}{(\gamma-1)}\right] \quad .. \quad .. \quad (31)$$

where

$$\Psi(\xi) = \text{Exp}\left[(\gamma-1)\int\left\{1-a(\xi-1)\right\}^2\left\{1-\frac{a}{\xi}(\xi-1)^2+\frac{a^2}{3\xi}(\xi-1)^3\right\}^{-1}\xi^{-1}d\xi\right]$$

$$= \text{Exp}(\gamma-1)\int\left[\frac{1}{\xi}+\frac{a^2}{\xi^3}+\frac{1}{\xi^2}\left(a-\frac{5a^2}{3}\right)-a(1-a)-\frac{a^2}{3}\xi\right]d\xi$$

when powers of a higher than a^2 are neglected.

Thus

$$\Psi(\xi) = \xi^{\gamma-1}\text{Exp}\left[(\gamma-1)\left\{-\frac{a^2}{2\xi^2} - \left(a-\frac{5a^2}{3}\right)\frac{1}{\xi} - a(1-a)\xi - \frac{a^2}{6}\xi^2\right\}\right]$$

the muzzle velocity is given by

$$\eta_2^2 = \frac{2M_2}{\Psi(\xi)}\left[\frac{\eta_{2b''}^2\Psi(\xi_{b''})}{2M_2} + \frac{\left(\frac{F_1C_1}{F_2C_2}+1\right)\left\{\Psi(\xi)-\Psi(\xi_{b''})\right\}}{\gamma-1}\right] \quad \ldots \quad \ldots \quad (32)$$

AN EXAMPLE

Consider a gun, propellants and shot combination such that

$$\theta_1 = \theta_2 = 0$$
$$F_1 = F_2$$
$$C_1 = C_2/10$$

$\beta_1 D_2 = \beta_2 D_1$—Case of simultaneous burning of the two charges to facilitate calculations.

$$r = 1{\cdot}25$$
$$M_1 = M_2 = 1$$

Let the barrel length of the gun be six times the chamber length so that $\xi = 7$ at the muzzle velocity.

(a) when the gun is an orthodox one

(b) when the gun is tapered bore gun corresponding to $a = {\cdot}01$.

(a) *Orthodox Gun*

The shot velocity and shot travel at all burnt are given by

$$\eta_b = 1$$
$$\xi_b = 2{\cdot}625$$

Putting $\xi_b = 2{\cdot}625$, $\xi = 7$, $\eta_b = 1$ and $a = 0$ in equation (32) we have

$$\eta_2^2 = 2{\cdot}69711 \quad \text{or} \quad \eta_2 = 1{\cdot}643$$

(b) *Tapered Bore Gun* with $a = {\cdot}01$:—

Putting $a = {\cdot}01$ and $\xi_b = 2{\cdot}625$ as a first approximation in equation (23) we get

$$\eta_{b,1} = {\cdot}9865$$

To a first approximation the shot travel is given by equation (24) as

$$\xi'_{b,1} = \left[\frac{A-\mu\times{\cdot}9865}{AF(2{\cdot}625)}\right]^{-8}$$
$$= \left[\frac{1{\cdot}118-{\cdot}125\times{\cdot}9865}{1{\cdot}118\times1{\cdot}001318}\right]^{-8} = 2{\cdot}577$$
$$\xi''_{b,1} = \left[\frac{1{\cdot}118-{\cdot}125\times{\cdot}9865}{1{\cdot}118F(2{\cdot}577)}\right]^{-8}$$
$$= \left[\frac{1{\cdot}118-{\cdot}125\times{\cdot}9865}{1{\cdot}118\times1{\cdot}0012065}\right]^{-8} = 2{\cdot}572$$
$$\xi'''_{b,1} = \left[\frac{1{\cdot}118-{\cdot}125\times{\cdot}9865}{1{\cdot}118\times1{\cdot}001201}\right]^{-8}$$
$$= 2{\cdot}572$$
$$= \xi''_{b,1}$$

Hence $\xi_{b,1} = 2{\cdot}572$

Putting $\xi_{b,1} = 2{\cdot}572$ in equation (23) we have

$$\eta_{b,2} = {\cdot}9866$$

The shot travel is given by equation (24) as

$$\xi^{1}_{b,2} = \left[\frac{1{\cdot}118 - {\cdot}125 \times {\cdot}9866}{1{\cdot}118 \times 1{\cdot}001201}\right]^{-8} = 2{\cdot}572 = \xi_{b,1}$$

Hence we need not proceed further

$$\therefore \eta_b = {\cdot}9866$$
$$\xi_b = 2{\cdot}572$$

Putting $\eta_b = {\cdot}9866$, $\xi_b = 2{\cdot}572$, $a = {\cdot}01$ and $\xi = 7$ in equation (32) the muzzle velocity is given by

$$\eta_2^2 = 2{\cdot}641$$
$$\text{or } \eta_2 = 1{\cdot}625$$

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Abstract

In this communication the author has suggested a step by step numerical solution of the basic equations of Internal Ballistics of a tapered bore gun using composite charges, considering the same power law of burning. An analytic solution has also been presented assuming a linear rate of burning and neglecting covolume bore resistance and heat transfer.

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