ON CONSIDERATIONS OF BORE RESISTANCE AND HEAT TRANSFER ON THE INTERNAL BALLISTICS OF A GUN USING COMPOSITE CHARGES

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Introduction

Hunt and Goldie (1951) have discussed the validity of the method of allowing for bore resistance by increasing the equivalent mass moved, which implies the assumption that bore resistance is proportional to pressure. They have pointed out that a certain amount of evidence exists for the approximate constancy of bore resistance after band engraving. The assumption of increasing the equivalent mass moved to allow for bore resistance is, therefore, not a good approximation. A step by step numerical solution of the Internal Ballistics equations of a gun taking bore resistance into account has been suggested by Hunt and Goldie (1951). In an earlier paper (under communication) Jain and Sodha have discussed the effect of constant bore resistance on the Internal Ballistics of a gun. In Section I of this paper the author has discussed the effect of constant bore resistance on the Internal Ballistics of a gun using composite charges.

The phenomenon of heat transfer to gun barrel has been discussed by Hicks and Thornhill (1951) and Corner (1950). At present heat transfer is allowed for in Internal Ballistics of a gun by using a modified value of γ for the propellant. This method involves the assumption that heat loss to the gun barrel at any instant is proportional to the kinetic energy of the shot. In an earlier paper (under communication) Jain and Sodha have discussed the Internal Ballistics of a gun, taking heat loss to the gun barrel as proportional to shot travel, which is the most useful approximation obtained from numerical results as pointed out by Hicks and Thornhill (1951). In Section II of this paper the author has discussed the effect of heat transfer on the Internal Ballistics of a gun using composite charges when heat transfer has been taken to be proportional to shot travel. The total heat loss to a gun may be estimated by means of empirical formulae given by Hicks and Thornhill (1951). Thus we may obtain the heat loss per unit length of shot travel.

Section I

Basic Equations

Assuming constant bore resistance, linear rate of burning, $b_1 = \frac{1}{\delta_1}$ and $b_2 = \frac{1}{\delta_2}$, the fundamental equations of Internal Ballistics of a gun using composite charges are:—

$$\frac{F_1C_1Z_1+F_2C_2Z_2}{\gamma-1} = \frac{PA\,(x+l)}{\gamma-1}+\frac{1}{2}W'V^2+AKx \quad \ldots \quad (1)$$

$$W'V\frac{dV}{dx} = A(P-k) \quad \ldots \quad (2)$$

$$D_1 \frac{df_1}{dt} = -\beta_1 P \qquad \ldots \qquad (3a)$$

$$D_2 \frac{df_2}{dt} = -\beta_2 P \qquad \ldots \qquad (3b)$$

$$Z_1 = (1-f_1)(1+\theta_1 f_1) \qquad \ldots \qquad (4a)$$

$$Z_2 = (1-f_2)(1+\theta_2 f_2) \qquad \ldots \qquad (4b)$$

where

suffix 1 represents the first charge,
suffix 2 represents the second charge,
F is the force constant of the propellant,
C is the charge weight,
Z is the fraction of the charge burnt,
P is the mean pressure of propellant gases,
A is the bore area,
l is the equivalent chamber length,
γ is the ratio of the specific heats of propellant gases at constant pressure and at constant volume,
$W' = 1{\cdot}01\,W + \frac{C_1}{3} + \frac{C_2}{3}$, C_1, C_2 being the charge weights and W being the shot weight,
x is the shot travel,
V is the velocity of the shot at any instant,
K is the bore resistance per unit area,
b is the covolume of propellant gases,
δ is the density of the propellant,
D is the web size of the propellant,
θ is the form factor of the propellant,
f is the fraction of D remaining unburnt at any instant, and β is the rate of burning coefficient.

Putting

$$\xi = 1+\frac{x}{l},$$

$$\eta_1 = \frac{VAD_1}{F_1 C_1 \beta_1},$$

$$\eta_2 = \frac{VAD_2}{F_2 C_2 \beta_2},$$

$$\zeta_1 = \frac{PAl}{F_1 C_1},$$

$$\zeta_2 = \frac{PAl}{F_2 C_2},$$

$$M_1 = \frac{A^2 D_1^2}{F_1 C_1 \beta_1^2 W'},$$

$$M_2 = \frac{A^2 D_2^2}{F_2 C_2 \beta_2^2 W'},$$

$$\rho_1 = \frac{KAl}{F_1 C_1},$$

and

$$\rho_2 = \frac{KAl}{F_2C_2},$$

in equations (1), (2), (3*a*) and (3*b*) we have

$$\frac{F_1C_1}{F_2C_2}Z_1+Z_2 = \zeta_2\xi+\frac{\eta_2^2}{2M_2}(\gamma-1)+\rho_2(\gamma-1)(\xi-1) \quad \ldots \quad (5)$$

$$\eta_1\frac{d\eta_1}{d\xi} = M_1(\zeta_1-\rho_1) \quad \ldots \quad (6a)$$

$$\eta_2\frac{d\eta_2}{d\xi} = M_2(\zeta_2-\rho_2) \quad \ldots \quad (6b)$$

$$\eta_1\frac{df_1}{d\xi} = -\zeta_1 \quad \ldots \quad (7a)$$

$$\eta_2\frac{df_2}{d\xi} = -\zeta_2 \quad \ldots \quad (7b)$$

Solution

Combining (7*a*) and (7*b*), integrating and using the boundary condition that initially $f_1 = f_{10}$ and $f_2 = f_{20}$ we have

$$f_1 = \frac{\beta_1D_2}{\beta_2D_1}f_2+f_{10}-\frac{\beta_1D_2}{\beta_2D_1}f_{20} \quad \ldots \quad (8)$$

It is easy to see (Venkatesan and Patni, 1953) that if

(i) $\beta_1D_2f_{20} > \beta_2D_1f_{10}$ then f_2 cannot become zero before f_1, hence charge c_1 must burn out earlier.

(ii) $\beta_1D_2f_{20} < \beta_2D_1f_{10}$ then f_1 cannot become zero before f_2, hence charge c_2 must burn out earlier.

(iii) $\beta_1D_2f_{20} = \beta_2D_1f_{10}$ both the charges will have to burn out simultaneously.

For the sake of definiteness we shall assume that the charge C_1 burns out first. From (6*b*) and (7*b*) we have

$$\frac{d\eta_2}{df_2} = -M_2\left(1-\frac{\rho_2}{\zeta_2}\right)$$

$$\approx -M_2\left(1-\frac{\rho_2}{\bar{\zeta}_2}\right)$$

$$= -\bar{M}_2$$

since ρ_2 is small as compared to ζ_2,

where

$$\bar{\zeta}_2 = \frac{1}{M_2}\frac{\int_0^{\eta_{2,b}}\eta_2\frac{d\eta_2}{d\xi}d\xi}{\int_1^{\xi_b}d\xi} = \frac{\eta_{2,b}^2}{2M_2(\xi_b-1)}$$

Integrating the above equation and using the boundary condition that initially $f_2 = f_{20}$ and $\eta_2 = 0$, we have

$$\eta_2 = \overline{M}_2(f_{20}-f_2) \quad \ldots \quad (9)$$

Combining (9) and (4b) we have

$$Z_2 = \left(1-f_{20}+\frac{\eta_2}{\overline{M}_2}\right)\left[1+\theta_2 f_{20}-\theta_2\frac{\eta_2}{\overline{M}_2}\right] \quad \ldots \quad (10)$$

with the help of (9), (4a) and (8) we get

$$Z_1 = \left[1-f_{10}+\frac{\beta_1 D_2}{\beta_2 D_1}\frac{\eta_2}{\overline{M}_2}\right]\left[1+\theta_1 f_{10}-\theta_1\frac{\beta_1 D_2}{\beta_2 D_1}\frac{\eta_2}{\overline{M}_2}\right] \quad \ldots \quad (11)$$

From equations (10), (11), (6b) and (5) we have

$$\frac{F_1C_1}{F_2C_2}\left[1-f_{10}+\frac{\beta_1 D_2}{\beta_2 D_1}\frac{\eta_2}{\overline{M}_2}\right]\left[1+\theta_1 f_{10}-\theta_1\frac{\beta_1 D_2}{\beta_2 D_1}\frac{\eta_2}{\overline{M}_2}\right]$$
$$+\left[1-f_{20}+\frac{\eta_2}{\overline{M}_2}\right]\left[1+\theta_2 f_{20}-\theta_2\frac{\eta_2}{\overline{M}_2}\right]$$
$$=\frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}\xi+\frac{\eta_2^2}{2M_2}(\gamma-1)+\rho_2[\gamma(\xi-1)+1]$$

or

$$A+B\eta_2-C\eta_2^2=\frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}\xi+\rho_2[\gamma(\xi-1)+1] \quad \ldots \quad (12)$$

where

$$A=\frac{F_1C_1}{F_2C_2}(1-f_{10})(1+\theta_1 f_{10})+(1-f_{20})(1+\theta_2 f_{20})$$

$$B=\frac{F_1C_1}{F_2C_2}\left[\frac{\beta_1 D_2}{\beta_2 D_1}(1+2\theta_1 f_{10}-\theta_1)\right]\frac{1}{\overline{M}_2}+[1+2\theta_2 f_{20}-\theta_2]\frac{1}{\overline{M}_2}$$

$$C=\frac{F_1C_1}{F_2C_2}\frac{\beta_1^2 D_2^2}{\beta_2^2 D_1^2}\frac{\theta_1}{\overline{M}_2^2}+\frac{\theta_2}{\overline{M}_2^2}+\frac{\gamma-1}{2M_2}$$

For the approximate solution of (12) we first proceed to solve

$$A+B\eta_2-C\eta_2^2=\frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi'}\xi'$$

or

$$\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}=M_2\frac{d\xi'}{\xi'} \quad \ldots \quad (12a)$$

Integrating (12a) and using the boundary condition that $\eta_2 = 0$ when $\xi' = 1$ we have

$$\xi'=\exp\frac{1}{M_2}\int_0^{\eta_2}\frac{\eta_2 d\eta_2}{(A+B\eta_2-C\eta_2^2)} \quad \ldots \quad (13)$$

putting $\xi = \xi'+y'$ in equation (12) we have

$$(A+B\eta_2-C\eta_2^2)\frac{d(\xi'+y')}{d\eta_2} = \frac{\eta_2}{M_2}(\xi'+y')+\rho_2[\gamma(\xi'+y'-1)+1]\frac{d(\xi'+y')}{d\eta_2}$$

Using equation (12*a*) and the fact that the disturbance in shot travel due to bore resistance is small, i.e. $y' << \xi'$, we have

$$\frac{dy'}{d\eta_2}-\frac{1}{M_2}\frac{\eta_2 y'}{(A+B\eta_2-C\eta_2^2)} = \frac{\rho_2[\gamma(\xi'-1)'+1]}{(A+B\eta_2-C\eta_2^2)^2}\eta_2\frac{\xi'}{M_2}$$

or

$$y' = \frac{\rho_2}{M_2}\exp\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}\times\int_0^{\eta_2}\left\{\frac{[\gamma(\xi'-1)+1]\eta_2\xi'}{(A+B\eta_2-C\eta_2^2)^2}\times\exp\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}\right\}d\eta_2 \quad \text{.. (14)}$$

ξ' as a function of η_2 being given by (13)

or
$$y' = \phi_1(\eta_2)$$

$$\therefore \quad \xi = \xi'+\phi_1(\eta_2) \quad \text{.. .. .. .. (15)}$$

The pressure at any instant when both the charges are burning is given by

$$\zeta_2 = \left(\frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}+\rho_2\right) \quad \text{.. .. .. (16)}$$

WHEN THE FIRST CHARGE HAS BURNT OUT

From (6*b*) and (7*b*) we have

$$d\eta_2 = -M_2 df_2+M_2\frac{\rho_2}{\zeta_2}df_2$$
$$= -M_2 df_2-M_2\rho_2\frac{d\xi}{\eta_2}$$

or

$$\eta_2 \approx M_2(f_{20}-f_2)-\rho_2\int_0^{\eta_2}\frac{\xi'}{A+B\eta_2-C\eta_2^2}d\eta_2 \quad \text{.. .. (17)}$$

ξ' as a function of η_2 being given by (13).

Putting $f_1 = 0$ or $f_2 = f_{20}-\frac{\beta_2 D_1}{\beta_1 D_2}f_{10}$, the shot velocity when the first charge has burnt out is given by

$$\eta_{2b'} = M_2\frac{\beta_2 D_1}{\beta_1 D_2}f_{10}-\rho_2\int_0^{M_2\frac{\beta_2 D_1}{\beta_1 D_2}f_{10}}\frac{\xi'}{A+B\eta_2-C\eta_2^2}d\eta_2 \quad \text{.. .. (18)}$$

From equation (15) the shot travel when the first charge has burnt out is given by

$$\xi_{b'} = \exp\frac{1}{M_2}\int_0^{\eta_{2b'}} \frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2} + \frac{\rho_2}{M_2}\exp\frac{1}{M_2}\int \frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2} \times$$

$$\int_0^{M_2\frac{\beta_2 D_1}{\beta_1 D_2}f_{10}} \left\{ \frac{\gamma(\xi'-1)\eta_2\xi'}{(A+B\eta_2 C\eta_2^2)^2} \exp - \frac{1}{M_2}\int \frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2} \right\} d\eta_2 \quad (19)$$

ξ' as a function of η_2 being given by (13).

The first charge has burnt out and the second charge is burning

Putting $Z_1 = 1$ in equation (5) and using (6*b*) and (10) we have

$$\frac{F_1 C_1}{F_2 C_2} + \left[1-f_{20}+\frac{\eta_2}{M_2}\right]\left[1+\theta_2 f_{20}-\theta_2\frac{\eta_2}{M_2}\right] = \frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}\xi + \frac{\eta_2(\gamma-1)}{2M_2} + \rho_2[\gamma(\xi-1)+1]$$

or

$$A'+B'\eta_2-C'\eta_2^2 = \frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}\xi+\rho_2[\gamma(\xi-1)+1] \quad \ldots \quad \ldots \quad (20)$$

where

$$A' = \frac{F_1 C_1}{F_2 C_2} + (1-f_{20})(1+\theta_2 f_{20})$$

$$B' = \frac{1}{M_2}[1+\theta_2 f_{20}] - \frac{\theta_2}{M_2}(1-f_{20})$$

$$C' = \frac{\theta_2}{M_2^2} + \frac{\gamma-1}{2M_2}$$

For the approximate solution of (20) we first proceed to solve

$$A'+B'\eta_2-C'\eta_2^2 = \frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi''}\xi''$$

or

$$\frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2} = M_2\frac{d\xi''}{\xi''} \quad \ldots \quad \ldots \quad \ldots \quad (20a)$$

Integrating the above and using the boundary condition that $\eta_2 = \eta_{2b'}$ when $\xi'' = \xi_{b'}$ we have

$$\xi'' = \xi_{b'}\exp\frac{1}{M_2}\int_{\eta_{2b'}}^{\eta_2} \frac{\eta_2 d\eta_2}{(A'+B'\eta_2-C'\eta_2^2)} \quad \ldots \quad \ldots \quad (21)$$

Putting $\xi = \xi''+y''$ in equation (20) we have

$$\frac{d(\xi''+y'')}{d\eta_2}(A'+B'\eta_2-C'\eta_2^2) = \frac{\eta_2}{M_2}(\xi''+y'')+\rho_2[\gamma(\xi''+y''-1)+1]\frac{d(\xi''+y'')}{d\eta_2}$$

Using equation (20a) and the fact that the disturbance in shot travel due to bore resistance is small, i.e. $y'' < < \xi''$, we have

$$\frac{dy''}{d\eta_2} - \frac{1}{M_2}\frac{\eta_2}{(A'+B'\eta_2-C'\eta_2^2)}y'' = \frac{\rho_2[\gamma(\xi''-1)+1]\eta_2\xi''}{(A'+B'\eta_2-C'\eta_2^2)^2 M_2}$$

or

$$y'' = \frac{\rho_2}{M_2}\exp\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{(A'+B'\eta_2-C'\eta_2^2)^2}\times\int_{\eta_{2b'}}^{\eta_2}\left\{\frac{\eta_2\xi''[\gamma(\xi''-1)+1]}{(A'+B'\eta_2-C'\eta_2^2)^2}\right.$$

$$\left.\times\exp -\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2}\right\}d\eta_2 \quad .. \quad (22)$$

ξ'' as a function of η_2 being given by (21)

or $$y'' = \phi_2(\eta_2)$$

$$\xi = \xi''+y'' = \xi''+\phi_2(\eta_2) \quad .. \quad .. \quad .. \quad .. \quad (23)$$

The pressure at any instant when the first charge has burnt out and the second charge is burning is given by

$$\zeta_2 = \left(\frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi}+\rho_2\right) \quad .. \quad .. \quad .. \quad .. \quad .. \quad (24)$$

Both the charges have burnt

From (6b) and (7b) we have

$$d\eta_2 = -M_2 df_2 + M_2\frac{\rho_2}{\zeta_2}df_2$$

$$= -M_2 df_2 - M_2\rho_2\frac{d\xi}{\eta_2}$$

or

$$(\eta_2-\eta_{2b'}) \approx M_2(f_{20}-\frac{\beta_2 D_1}{\beta_1 D_2}f_{10}-f_2)$$

$$-\rho_2\int_{\eta_{2b'}}^{\eta_2}\frac{\xi''}{(A'+B'\eta_2-C'\eta_2^2)}d\eta_2$$

ξ'' as a function of η_2 being given by (21).

Putting $f_2 = 0$ at all burnt the shot velocity at all burnt is given by

$$\eta_{2b''} = \eta_{2b'}+M_2 f_{20}-M_2\frac{\beta_2 D_1}{\beta_1 D_2}f_{10}-\rho_2\int_{\eta_{2b'}}^{M_2 f_{20}}\frac{\xi''}{(A'+B'\eta_2-C'\eta_2^2)}d\eta_2 \quad .. \quad (25)$$

From equation (23) the shot travel at all burnt is given by

$$\xi_{b''} = \xi_{b'} \exp \frac{1}{M_2} \int_{\eta_{2b'}}^{\eta_{2b}} \frac{\eta_2 d\eta_2}{(A'+B'\eta_2-C'\eta_2^2)} + \frac{\rho_2}{M_2} \exp \frac{1}{M_2} \int \frac{\eta_2 d\eta_2}{(A'+B'\eta_2-C'\eta_2^2)}$$

$$\times \int_{\eta_{2b'}}^{\eta_{2b''}} \left\{ \frac{\eta_2 \xi''[\gamma(\xi''-1)+1]}{(A'+B'\eta_2-C'\eta_2^2)^2} \exp - \frac{1}{M} \int \frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2} \right\} d\eta_2 \quad .. \quad (26)$$

ξ'' as a function of η_2 being given by (21).

After all burnt

Putting $Z_1 = Z_2 = 1$ in equation (5) and using (6*b*) we have

$$\left(1+\frac{F_1C_1}{F_2C_2}\right) = \frac{\eta_2}{M_2} \frac{d\eta_2}{d\xi} \xi + \frac{\gamma-1}{2M_2} \eta_2^2 + \rho_2[\gamma(\xi-1)+1]$$

or

$$\frac{\eta_2}{M_2} \frac{d\eta_2}{d\xi} \xi^{\gamma-1} + \frac{\gamma-1}{2M_2} \eta_2^2 \xi^{\gamma-2} = \left(1+\frac{F_1C_1}{F_2C_2}\right) \xi^{\gamma-2} - \rho_2[\gamma(\xi-1)+1]\xi^{\gamma-2}$$

Integrating the above and using the usual boundary condition we have

$$\left[\frac{\eta_2^2 \xi^{\gamma-1}}{2M_2}\right]_{\eta_{2b''},\ \xi_{b''}}^{\eta_2,\ \xi} = \left(1+\frac{F_1C_1}{F_2C_2}\right) \frac{\xi^{\gamma-1}-\xi_{b''}^{\gamma-1}}{(\gamma-1)} - \rho_2(\xi^{\gamma}-\xi_{b''}^{\gamma}) + \rho_2(\xi^{\gamma-1}-\xi_{b''}^{\gamma-1})$$

or

$$\eta_2^2 = \frac{2M_2}{\xi^{\gamma-1}} \left[\frac{\eta_{2b''}^2}{2M_2} \xi_{b''}^{\gamma-1} + \left(1+\frac{F_1C_1}{F_2C_2}\right) \frac{\xi^{\gamma-1}-\xi_{b''}^{\gamma-1}}{(\gamma-1)} - \rho_2(\xi^{\gamma}-\xi_{b''}^{\gamma}) + \rho_2(\xi^{\gamma-1}-\xi_{b''}^{\gamma-1}) \right] \quad .. \quad (27)$$

Section II

Basic Equations

Assuming a linear rate of burning, $b_1 = \frac{1}{\delta_1}$, $b_2 = \frac{1}{\delta_2}$ and the heat transferred to the gun barrel as proportional to shot travel, the bore equations of Internal Ballistics of a gun using composite charges are:

$$\frac{F_1C_1Z_1+F_2C_2Z_2}{(\gamma-1)} = \frac{PA(x+l)}{(\gamma-1)} + \tfrac{1}{2}W^1V^2 + Gx \quad .. \quad .. \quad .. \quad (28)$$

$$W'V\frac{dV}{dx} = AP \quad .. \quad .. \quad .. \quad .. \quad .. \quad (29)$$

and equations (3*a*), (3*b*), (4*a*) and (4*b*) of Section I, where G is the heat transferred to the barrel by hot gases per unit of shot travel.

Solution

With the usual transformations employed in Section I of this paper and

$$\rho = \frac{Gl(\gamma-1)}{F_2C_2}$$

in equations (28), (29) and (3a) and (3b) we have

$$\frac{F_1C_1}{F_2C_2}Z_1+Z_2 = \zeta_2\xi+\frac{\gamma-1}{2M_2}\eta_2^2+\rho(\xi-1) \quad \ldots \quad (30)$$

$$\eta_1\frac{d\eta_1}{d\xi} = M_1\zeta_1 \quad \ldots \quad (31a)$$

$$\eta_2\frac{d\eta_2}{d\xi} = M_2\zeta_2 \quad \ldots \quad (31b)$$

and equations (7a) and (7b) of Section I.

Both the charges are burning

It is easy to see with the help of Section I that when both the charges are burning

$$\eta_2 = M_2(f_{20}-f_2) \quad \ldots \quad (32)$$

$$\xi = \exp\frac{1}{M_2}\int_0^{\eta_2}\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}+\frac{\rho}{M_2}\exp\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}$$

$$\times\int_0^{\eta_2}\left\{\frac{\eta_2(\xi'-1)\xi'}{(A+B\eta_2-C\eta_2^2)^2}\exp-\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}\right\}d\eta_2 \quad \ldots \quad (33)$$

ξ' as a function of η_2 being given by (13) and where the constant A remains unchanged and the constants B and C are given by

$$B = \frac{F_1C_1}{F_2C_2}\left[\frac{\beta_1D_2}{\beta_2D_1}(1+2\theta_1f_{10}-\theta_1)\right]\frac{1}{M_2}+[1+2\theta_2f_{20}-\theta_2]\frac{1}{M_2}$$

$$C = \frac{F_1C_1}{F_2C_2}\frac{\beta_1^2D_2^2}{\beta_2^2D_1^2}\frac{\theta_1}{M_2^2}+\frac{\theta_2}{M_2^2}+\frac{\gamma-1}{2M_2}$$

The pressure at any instant when both the charges are burning is given by

$$\zeta_2 = \frac{\eta_2}{M_2}\frac{d\eta_2}{d\xi} \quad \ldots \quad (34)$$

When the first charge has burnt out

Putting $f_1 = 0$ or $f_2 = f_{20}-\frac{\beta_2D_1}{\beta_1D_2}f_{10}$ in equation (32) the shot velocity when the first charge has burnt out is given by

$$\eta_{2b'} = M_2\frac{\beta_2D_1}{\beta_1D_2}f_{10} \quad \ldots \quad (35)$$

From equation (33) the shot travel when the first charge has burnt out is given by

$$\xi_{b'} = \exp\frac{1}{M_2}\int_0^{\eta_{2b'}}\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}+\frac{\rho}{M_2}\exp\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}$$

$$\times\int_0^{\eta_{2b'}}\left\{\frac{\eta_2\xi'(\xi'-1)}{(A+B\eta_2-C\eta_2^2)^2}\times\exp-\frac{1}{M_2}\int\frac{\eta_2 d\eta_2}{A+B\eta_2-C\eta_2^2}\right\}d\eta_2 \quad \ldots \quad (36)$$

ξ' as a function of η_2 being given by (13) bearing in mind that the constants B and C have changed.

The first charge has burnt out and the second charge is burning

It follows from Section I of the paper that the shot travel at any instant, when the first charge has burnt out and the second charge is burning, is given by

$$\xi = \xi_{b'} \exp \frac{1}{M_2} \int_{\eta_{2b'}}^{\eta_2} \frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2} + \frac{\rho}{M_2} \exp \frac{1}{M_2} \int \frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2}$$

$$\times \int_{\eta_{2b'}}^{\eta_2} \left\{ \frac{\eta_2(\xi''-1)\xi''}{(A'+B'\eta_2-C''\eta_2^2)^2} \times \exp - \frac{1}{M_2} \int \frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2} \right\} d\eta_2 \quad \ldots \quad (37)$$

ξ'' as a function of η_2 being given by (21) and where the constant A' remains unchanged and B' and C' are given by

$$B' = \frac{1}{M_2}(1+\theta_2 f_{20}) - \frac{\theta_2}{M_2}(1-f_{20})$$

$$C' = \frac{\theta_2}{M_2^2} + \frac{\gamma-1}{2M_2}$$

The pressure at any instant when the first charge has burnt out and the second charge is burning is given by

$$\zeta_2 = \frac{\eta_2}{M_2} \frac{d\eta_2}{d\xi} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

Both the charges have burnt

Putting $f_2 = 0$ in equation (32) the shot velocity at all burnt is given by

$$\eta_{2b''} = M_2 f_{20} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

From equation (36) we see that the shot travel at all burnt is given by

$$\xi_{b''} = \xi_{b'} \exp \frac{1}{M_2} \int_{\eta_{2b'}}^{\eta_{2b''}} \frac{\eta_2 d\eta_2}{(A'+B'\eta_2-C'\eta_2^2)} + \frac{\rho}{M_2} \exp \frac{1}{M_2} \int \frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2} \times$$

$$\int_{\eta_{2b'}}^{\eta_{2b''}} \left\{ \frac{\eta_2 \xi''(\xi''-1)}{(A'+B'\eta_2^2-C'\eta_2^2)^2} \exp - \frac{1}{M_2} \int \frac{\eta_2 d\eta_2}{A'+B'\eta_2-C'\eta_2^2} \right\} d\eta_2 \quad \ldots \quad \ldots \quad (40)$$

ξ'' as a function of η_2 being given by (21) and bearing in mind that the constants B' and C' have changed.

After all burnt

Putting $Z_1 = Z_2 = 1$ in equation (30) and using equation (31b) we have

$$\left(1+\frac{F_1C_1}{F_2C_2}\right) = \frac{\eta_2}{M_2} \frac{d\eta_2}{d\xi} \xi + \frac{\gamma-1}{2M_2} \eta_2^2 + \rho(\xi-1)$$

or

$$\frac{\eta_2}{M_2} \frac{d\eta_2}{d\xi} \xi^{\gamma-1} + \frac{\gamma-1}{2M_2} \eta_2^2 \xi^{\gamma-2} = \left(1+\frac{F_1C_1}{F_2C_2}\right) \xi^{\gamma-1} - \rho(\xi-1)\gamma^{-2}$$

Integrating the above and using the boundary condition that $\eta_2 = \eta_{2b''}$ when $\xi = \xi_{b''}$ we have

$$\left[\frac{\eta_2^2 \xi^{\gamma-1}}{2M_2}\right]_{\eta_{2b''},\, \xi_{b''}}^{\eta_2,\, \xi} = \left(1+\frac{F_1C_1}{F_2C_2}\right)\left(\frac{\xi^{\gamma-1}-\xi_{b''}^{\gamma-1}}{\gamma-1}\right)$$

$$-\rho\frac{(\xi^{\gamma}-\xi_{b''}^{\gamma})}{\gamma}+\rho\left(\frac{\xi^{\gamma-1}-\xi_{b''}^{\gamma-1}}{\gamma-1}\right)$$

or

$$\eta_2^2 = \frac{2M_2}{\xi^{\gamma-1}}\left[\frac{\eta_{2b''}^2}{2M_2}\xi_{b''}^{\gamma-1}+\left(1+\frac{F_1C_1}{F_2C_2}\right)\left(\frac{\xi^{\gamma-1}-\xi_{b''}^{\gamma-1}}{\gamma-1}\right)\right.$$

$$\left.-\rho\frac{(\xi^{\gamma}-\xi_{b''}^{\gamma})}{\gamma}+\rho\left(\frac{\xi^{\gamma-1}-\xi_{b''}^{\gamma-1}}{\gamma-1}\right)\right] \quad \ldots \quad \ldots \quad (41)$$

An example

Consider a gun, shot and propellants combination such that

$\theta_1 = \theta_2 = 0$

$\gamma = 1{\cdot}25$

$\beta_1 D_2 = \beta_2 D_1$ (case of simultaneous burning taken for the sake of simplicity in calculations)

$F_1 = F_2$

$C_1 = \frac{C_2}{10}$

$M_1 = M_2 = 1$

$f_{10} = f_{20} = 1$

$\rho = {\cdot}01$

Thus

$A = 0$

$B = 1{\cdot}1$

$C = {\cdot}125$

Let the barrel length of the gun be six times the chamber length, so that $\xi = 7$ at the muzzle. We proceed to calculate the muzzle velocity of the shot when

(a) Heat transfer is neglected ($\rho = 0$).
(b) Heat transfer is allowed for in the usual manner.
(c) Heat transfer is allowed for by author's method.

(a) Neglecting heat transfer we have

$$\xi_{b''} = 2{\cdot}625$$

$$\eta_{b''} = 1$$

Putting $\xi_{b''} = 2{\cdot}625$, $\eta_{b''} = 1$, $\xi = 7$ at the muzzle and $\rho = 0$ in equation (41) we have

$$\eta_2^2 = 2{\cdot}697 \qquad (a)$$

or

$$\eta_2 = 1{\cdot}643$$

(*b*) Heat transfer is allowed for in the usual manner by using a modified value $\bar{\gamma}$ given by

$$\bar{\gamma}-1 = (1+\chi)(\gamma-1)$$

where χ is the ratio of the total heat loss to the muzzle energy. In this example

$$\chi = \frac{\rho(\xi-1)}{(\gamma-1)\left(\dfrac{\eta_2^2}{2M_2}\right)} = \frac{6\times{\cdot}01\times 2}{{\cdot}25\times 2{\cdot}697} = {\cdot}1779$$

$$\bar{\gamma} = 1+(1+\chi)(\gamma-1) = 1{\cdot}2945$$

$$A = 0$$

$$B = 1{\cdot}1$$

$$C = {\cdot}14725$$

Hence

$$\xi_{b''} = 2{\cdot}656$$

Putting $\xi_{b''} = 2{\cdot}656$, $\eta_{b''} = 1$, $\xi = 7$ at the muzzle $\gamma = \bar{\gamma} = 1{\cdot}2945$ and $\rho = 0$ in equation (41) we have

$$\eta_2^2 = 2{\cdot}60571 \qquad (b)$$

or

$$\eta_2 = 1{\cdot}614$$

(*c*) Heat transfer is allowed for by the author's method
Referring to the notation in this paper we have

$$\eta_{b''} = 1$$

$$\xi_{b''} = 2{\cdot}625+{\cdot}03023$$

$$= 2{\cdot}65523$$

Putting $\xi_{b''} = 2{\cdot}65523$, $\eta_{b''} = 1$, $\xi = 7$ at muzzle and $\rho = {\cdot}01$ in equation (41) we have

$$\eta_2^2 = 2{\cdot}6212 \qquad (c)$$

or

$$\eta_2 = 1{\cdot}619$$

This value is in reasonable agreement with that obtained by the usual method (*b*).

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Abstract

In this communication the author has discussed (i) the effect of bore resistance and (ii) the effect of heat transfer to gun barrel by hot gases, on the Internal Ballistics of a gun using composite charges. The bore resistance is taken to be constant and the heat transferred to the gun barrel at any instant is taken to be proportional to the shot travel. Both these facts have considerable evidence to support them.

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