

ON EVAPORATION OF DROPLETS IN ROCKET MOTOR CHAMBER

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(Communicated by R. S. Varma, F.N.I.)

(Received March 12; read August 3, 1956)

INTRODUCTION

An indication regarding the relative importance of two combustion mechanisms suggested by Penner (1953) may be obtained by determining whether liquid droplets are in a given motor chamber for a sufficiently long period of time to vaporise completely without the occurrence of exothermic liquid phase reactions. If the time required for the complete vaporisation of the liquid droplets is greater than the time spent by the liquid droplets in the motor chamber, then liquid phase reactions must occur, provided complete combustion takes place in the rocket chamber. It is, therefore, of interest to determine the minimum time required for the evaporation of liquid droplets in a rocket chamber.

A brief discussion and bibliography of the available literature on the evaporation of droplets has been given by Penner (1953).

In view of the obvious complexity of the problem Penner (1953) has employed an approximate treatment which is designed to estimate a reasonable upper limit for the rate of droplet vaporisation in a rocket chamber.

In his treatment although the droplet temperature changed with time it was assumed that the droplet remained isothermal during evaporation, i.e. the thermal conduction coefficients of evaporating liquids were taken as infinite.

Hartwig (1953) has considered a shell model of the evaporating liquid in which the droplet is composed of an inner core and an outer spherical shell of constant thickness ϵ . He assumed the inner sphere to be isothermal remaining at the original temperature T_0 of the evaporating liquid and the outer shell to be also isothermal, its temperature being determined by an appropriate heat balance equation.

Probert (1946) indicates that the rate of evaporation of small droplets is proportional to the diameter of the evaporating droplets rather than to the square of the diameter. Penner (1953) mentions this important conclusion but surprisingly enough both he and Hartwig (1953) used Knudsen (1915) equation which gives the rate of vaporisation proportional to the square of the diameter. They have not given any justification for the use of Knudsen equation in view of its non-validity for small droplets as has been pointed out by Probert (1946).

In this paper the authors have discussed the problem on the lines of Penner's (1953) treatment and on the basis of the well-known expression for the rate of evaporation due to Maxwell (1890) and Langmuir (1918) which gives the rate of evaporation proportional to the diameter of the droplet as suggested by Probert (1946). This relation has also been experimentally verified by Morse (1910), Namekawa and Takahashi (1937) and recently by Ranz and Marshall (1952).

The authors have also discussed the problem on the lines of Hartwig (1953) utilising the well-established expression for the rate of evaporation due to Maxwell (1890) and Langmuir (1918) in place of Knudsen (1915) equation. The results have been put in a much more convenient and neat form in place of the simultaneous

numerical solution of the two resulting differential equations of Penner (1953) and Hartwig (1953).

THEORY OF DROPLET VAPORISATION

The rate of droplet vaporisation depends upon the nature of the evaporating liquid and upon the heat transferred to the liquid droplet after injection into the combustion chamber. The minimum life of a liquid droplet in a rocket chamber may be calculated by using an upper limit for the heat transferred and also for the isothermal rate of decrease of droplet radius with time.

The order of magnitude for the upper limit for the isothermal rate of decrease of droplet radius with time can be calculated from the well-known equation due to Maxwell (1890) and Langmuir (1918) which may easily be put in the form

$$-\frac{dr}{dt} = \frac{DMp}{\rho RT} \cdot \frac{1}{r} \quad \dots \quad (1)$$

where

- r is the radius of the evaporating liquid,
- t is the time,
- D is the diffusion coefficient,
- M is the molecular weight of the liquid,
- p is the saturated vapour pressure of the liquid,
- ρ is the density of the liquid,
- T is the temperature,
- and R is the universal gas constant.

An upper limit for the heat transfer to the liquid droplets is obtained by maximizing the product $h(T_g - T)$ in the relation

$$Q = 4\pi r^2 h (T_g - T) \quad \dots \quad (2)$$

where

- Q is the rate of heat transfer to the droplet,
- h is overall coefficient of heat transfer,
- and T_g is the temperature of the hot gases.

Introducing certain approximations Penner (1953) suggests the reasonable relation for droplets of radius up to 10^{-2} cms.

$$h = \frac{k}{r} \quad \dots \quad (3)$$

where k is the thermal conductivity of hot gases.

ISOTHERMAL DROPLET

The rate of temperature rise with time of a spherical droplet receiving the energy Q is

$$\left(\frac{dT}{dt}\right)_1 = \frac{Q}{\left(\frac{4}{3}\pi r^3 \rho\right) c} = \frac{3h(T_g - T)}{\rho r c} = \frac{3k(T_g - T)}{\rho r^2 c} \quad \dots \quad (4)$$

where c is the specific heat of the evaporating liquid.

A liquid droplet is also cooled as a result of evaporation. The rate of decrease of temperature with time associated with the cooling effect of evaporation is given according to the relation

$$-\left(\frac{dT}{dt}\right)_2 = -\frac{L}{\frac{4}{3}\pi r^3 \rho c} \cdot \frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho\right) = -\frac{3L}{cr} \frac{dr}{dt} \quad \dots \quad (5)$$

where L is the latent heat of vaporisation of the liquid.

The net change of temperature with time is, therefore,

$$\begin{aligned} \frac{dT}{dt} &= \frac{3k(T_g - T)}{\rho r^2 c} - \frac{3L}{cr} \left(-\frac{dr}{dt}\right) \\ &= \frac{3k(T_g - T)}{\rho r^2 c} - \frac{3L}{cr} \cdot \frac{DMp}{\rho RT} \cdot \frac{1}{r} \\ &= \frac{3}{\rho r^2 c} \left\{ k(T_g - T) - \frac{LDMp}{RT} \right\} \quad \dots \quad (6) \end{aligned}$$

Combining equations (1) and (6)

$$-\frac{dr}{dT} = r \cdot \frac{(DMp/\rho RT)}{(3/\rho c) \{k(T_g - T) - (LDMp/RT)\}}$$

or

$$\int_{r_0}^r -\frac{dr}{r} = \int_{T_0}^T \frac{DMpc/3RT}{\{k(T_g - T) - (LDMp/RT)\}} dT$$

or

$$\log \frac{r_0}{r} = F(T). \quad \dots \quad (7)$$

Since D , p , C and L are functions of T .

From equation (7) we may put

$$r = \sqrt{f(T)} \quad \dots \quad (7a)$$

where

$$f(T) = r_0^2 \exp \{-2F(T)\} \quad \dots \quad (7b)$$

Hence from equation (6) we get

$$\frac{dT}{dt} = \frac{3}{\rho c} \cdot \frac{1}{f(T)} \{k(T_g - T) - (LDMp/RT)\}$$

or

$$\int_0^t dt = \frac{\rho c}{3} \int_{T_0}^T \frac{f(T) dT}{\{k(T_g - T) - (LDMp/RT)\}}$$

or

$$t = \phi(T). \quad \dots \quad (8)$$

Equations (7) and (8) give radius and time as functions of temperature. The two equations also express the relation between r and t in parametric form.

An Approximation: If we plot p against $\frac{1}{T}$ we find that for a small range of temperature the curve may be taken as nearly linear, i.e.

$$p = A - \frac{B}{T} \dots \dots \dots (9)$$

The constants A and B depend upon the range of temperature or the portion of the curve, selected, and may be obtained from the curve.

Putting

$$D = \sigma T^2, \dots \dots \dots (10)$$

assuming L and C to be constant and using equation (9) we get

$$\begin{aligned} F(T) &= \frac{\sigma Mc}{3R} \int_{T_0}^T \frac{AT - B}{k(T_g - T) - (ML\sigma/R)(AT - B)} dT \\ &= \frac{\sigma Mc}{3R} \int_{T_0}^T \frac{(AT - B)}{(C - \delta T)} dT \\ &= -\frac{a}{2}(T - T_0) - \frac{b}{2} \log \frac{C - \delta T}{C - \delta T_0} \dots \dots \dots (11) \end{aligned}$$

where

$$\begin{aligned} C &= kT_g + BLM\sigma/R \\ \delta &= k + ALM\sigma/R \\ a &= \frac{2c}{3} \cdot \frac{A\sigma M/R}{(k + ALM\sigma/R)} \\ b &= \frac{2kc}{3} \cdot \frac{(\sigma Mc/R)(AT_g - B)}{(k + ALM\sigma/R)^2} \end{aligned}$$

Thus we have

$$\sqrt{f(T)} = r = r_0 \left(\frac{C - \delta T}{C - \delta T_0} \right)^{b/2} \exp \left\{ \frac{a}{2} (T - T_0) \right\} \dots \dots (12)$$

When the drop has just completely evaporated $r = 0$, its temperature is given by

$$T_f = \frac{C}{\delta} \dots \dots \dots (13)$$

The life is given by equation (8) as

$$t = \frac{r_0^2 \rho c}{3} \int_{T_0}^{T_f} \frac{(C - \delta T)^{b-1}}{(C - \delta T_0)^b} \cdot \exp \{ a(T - T_0) \} dT \dots \dots (14)$$

which is readily numerically integrable.

If the range of temperature ($T_f - T_0$) is large it may be broken up into small ranges and the method can then be applied.

NONISOTHERMAL DROPLET

The mass of the isothermal shell of constant thickness is

$$m = \frac{4}{3} \pi \{r^3 - (r - \epsilon)^3\} \rho$$

If

$$\begin{aligned} \epsilon/r &<< 1, \\ m &= 4\pi r^2 \epsilon \rho. \quad \dots \dots \dots \dots \dots \dots (15) \end{aligned}$$

The mass of the inner core decreases as the radius decreases. This mass is transferred to the outer shell and since it is isothermal this mass has to be heated from a temperature T_0 to T . Hence the rate of absorption of energy to keep ϵ constant and the outer shell isothermal is

$$\begin{aligned} -c(T - T_0) \frac{d}{dT} \left\{ \frac{4}{3} \pi (r - \epsilon)^3 \right\} &= 4\pi \rho c (T - T_0) (r - \epsilon)^2 \left(-\frac{dr}{dt} \right) \\ &\approx 4\pi \rho c (T - T_0) r^2 \left(-\frac{dr}{dt} \right). \end{aligned}$$

In order to correct for this heat sink we may take effective rate of heat transfer as

$$\begin{aligned} Q' &= 4\pi r^2 h (T_g - T) - 4\pi \rho c (T - T_0) (r - \epsilon)^2 \left(-\frac{dr}{dt} \right) \\ &\approx 4\pi k r (T_g - T) - 4\pi \rho c r^2 (T - T_0) \left(-\frac{dr}{dt} \right) \end{aligned}$$

Thus the rate of increase of temperature due to heat transfer is given by

$$\left(\frac{dT}{dt} \right)_1 = \frac{Q'}{mc} = \frac{k(T_g - T)}{\rho c \epsilon r} - \frac{(T - T_0)}{\epsilon} \left(-\frac{dr}{dt} \right) \quad \dots \dots (16)$$

The rate of absorption of heat by evaporation is given by

$$-L \frac{d}{dt} \left(\frac{4}{3} \pi r^3 \rho \right) = 4\pi r^2 L \rho \left(-\frac{dr}{dt} \right)$$

Hence the rate of decrease of temperature of the outer shell due to evaporation is

$$-\left(\frac{dT}{dt} \right)_2 = \frac{4\pi r^2 \rho L (-dr/dt)}{c \cdot 4\pi r^2 \epsilon \rho} = \frac{L}{c \epsilon} \left(-\frac{dr}{dt} \right) \quad \dots \dots (17)$$

Neglecting the heat transfer between the two isothermal regions

$$\left(\frac{dT}{dt} \right) = \left(\frac{dT}{dt} \right)_1 + \left(\frac{dT}{dt} \right)_2$$

represents the rate of change of temperature with time. Thus

$$\frac{dT}{dt} = \frac{k(T_g - T)}{\rho c \epsilon r} - \left\{ \frac{L}{c \epsilon} + \frac{T - T_0}{\epsilon} \right\} \left(-\frac{dr}{dt} \right) \quad \dots \dots \dots (18)$$

Using equation (1), equation (18) may be put in the form

$$\begin{aligned} \frac{dT}{dt} &= \frac{k(T_g - T)}{\rho c \epsilon r} - \frac{DMp}{\rho RT} \cdot \frac{1}{r} \left[\frac{L}{c\epsilon} + \frac{T - T_0}{\epsilon} \right] \\ &= \frac{1}{\rho r \epsilon} \left[\frac{k}{c} (T_g - T) - \left(\frac{DMp}{RT} \right) \left\{ T - T_0 + \frac{L}{c} \right\} \right] \dots \dots (18a) \end{aligned}$$

From equations (18a) and (1) we get

$$\begin{aligned} - \int_{r_0}^r dr &= \int_{T_0}^T \frac{\epsilon(DMp/RT) dT}{\left[\frac{k}{c} (T_g - T) - \left(\frac{DMp}{RT} \right) \left(T - T_0 + \frac{L}{c} \right) \right]} \\ &= \psi(T) \end{aligned}$$

or

$$r = r_0 - \psi(T) \dots \dots \dots (19)$$

From equations (19) and (18a)

$$\begin{aligned} t &= \int_0^t dt \\ &= \int_{T_0}^T \frac{\rho \epsilon \left\{ r_0 - \psi(T) \right\}}{\left\{ \frac{k}{c} (T_g - T) - \frac{DMp}{RT} \left(T - T_0 + \frac{L}{c} \right) \right\}} \\ &= \chi(T) \dots \dots \dots (20) \end{aligned}$$

Equations (19) and (20) express radius and time as functions of temperature. The two equations also express the relation between r and t in parametric form.

For determining the life of the drop we have to put $T = T_f$ in equation (20) such that

$$r_0 = \psi(T)$$

An approximation: Using equations (9) and (10) we have

$$\begin{aligned} \psi(T) &= \int_{T_0}^T \frac{(\epsilon\sigma/R)(AT - B) dT}{\frac{k}{c} T_g + \frac{\sigma M \beta}{R} \left(\frac{L}{c} - T_0 \right) + T \left\{ \frac{\sigma MB}{R} - \frac{\sigma MA}{R} \left(\frac{L}{c} - T_0 \right) \right\} - \frac{\sigma MA}{R} T^2} \\ &= \frac{\epsilon\sigma}{R} \int_{T_0}^T (AT - B) / (a' + b'T - c'T^2) dT \\ &= \frac{\epsilon\sigma}{R} \left[- \frac{A}{2c'} \log (a' + b'T - c'T^2) + \frac{(b'A/2c') - B}{2\alpha} \log \frac{\alpha + T}{\alpha - T} \right]_{T_0}^T \dots \dots (21) \end{aligned}$$

where $\alpha^2 = a' + b'^2/4c'^2$.

AN EXAMPLE

We proceed to calculate the life of an aniline droplet of initial radius 10^{-2} cms and temperature $373^\circ K$ in hot air at atmospheric pressure and temperature $1598.4^\circ K$, on the isothermal model.

We have

$$M = 93.12$$

$$\sigma = 0.6095/273^2 \text{cm}^2/\text{sec}(\text{°K})^2$$

Knowing that the vapour pressure of aniline at 373°K and 393°K is 45.7 and 96.6 mms of mercury respectively we arrive at the following values:

$$AM\sigma/R = 1277.24 \times 10^{-8} \text{ gms/cm sec. °K}$$

$$BM\sigma/R = 455593.9 \times 10^{-8} \text{ gms/cm sec. °K}$$

Putting $k = 0.5 \times 10^{-4}$ cal./cm sec. °K.

$$C = 6721.91916 \times 10^{-4} \text{ cal./cm sec. °K}$$

$$\delta = 17.10412 \times 10^{-4} \text{ cal./cm sec. °K}$$

Taking $L = 130$ cal./gm and $c = .553$ cal./gm °K

$$b = 1.0000$$

$$a = 27.529 \times 10^{-4} (\text{°K})^{-1}$$

Hence

$$T_f = 393 \text{°K}$$

$$\frac{t}{r_0^2} = \frac{\rho c}{3} \cdot \frac{1}{C - \delta T_0} \int_{T_0}^{T_f} \exp \{a(T - T_0)\} dT = \frac{\rho c [\exp a \{T_f - T_0\} - 1]}{3a(C - \delta T_0)}$$

Putting $\rho = 0.94$ gms/cc.

$$t/r_0^2 = 1.0395 \times 10^2 \text{ secs/cm}^2$$

Taking $r_0 = 10^{-2}$ cms.

$$t = 1.0395 \times 10^{-2} \text{ seconds.}$$

ACKNOWLEDGEMENTS

The authors are highly grateful to Dr. D. S. Kothari and Dr. R. S. Varma for their kind encouragement.

ABSTRACT

Penner and Hartwig have proposed isothermal and nonisothermal models for the evaporation of droplets in rocket motor chamber. Their discussions are based on Knudsen's equation, which gives the rate of evaporation, proportional to the square of diameter of the drop. Probert and many others have found the rate of evaporation to be proportional to the diameter, in agreement with the expression given by Maxwell and Langmuir.

In this paper, the authors have discussed the problem on the lines of Penner and Hartwig and on the basis of the well-known expression for rate of evaporation, due to Maxwell and Langmuir. The results have been obtained in a much more neat and convenient form as compared to the simultaneous numerical solution of two resulting differential equations of Penner and Hartwig.

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