EFFECT OF BORE RESISTANCE ON INTERNAL BALLISTICS OF GUNS TAKING INTO ACCOUNT THE CO-VOLUME TERMS

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1. Introduction

The main problem of internal ballistics is the calculation of maximum pressure and muzzle velocity for given loading conditions. In most of the systems of internal ballistics, the co-volume term is neglected, the effects produced by the resistance during the engraving of the band is taken into account by the use of 'shot-start' pressure and the resistance which persists after the engraving of the band is generally neglected.

Corner (1949) has given a method by which the ballistic effects of bore resistance of any desired form can be obtained. His method is based upon the choice of a standard bore resistance, the determination of its ballistic effects and the construction of more realistic laws by superimposition of standard forms suitably spaced along the travel. He assumes that $\eta = \frac{1}{\delta}$, i.e. the co-volume of the propellant gases is equivalent to the reciprocal of the density of the solid propellant. Recently the author (1956) extended Corner's theory to the case of composite charge. In this paper the author has found the effects of bore resistance on the internal ballistics of guns by taking $\eta \neq \frac{1}{\delta}$.

If A be the area of cross-section of the bore, then as standard bore resistance we take a force AP_0, constant over a travel Δx near the point x_0, and zero at all other shot positions, it is assumed that Δx is infinitesimal and $P_0 \Delta x$ is finite. The effects on ballistics to the first order in $P_0 \Delta x$ is calculated by subtracting an energy $AP_0 \Delta x$ from the kinetic energy of the projectile as it passes through the point x_0. For travels less than x_0 the solution is the normal solution of the ballistic equations and for travels greater than x_0, the ballistic equations are exactly the same as before except for a perturbing term linear in $P_0 \Delta x$. By taking a specific example, Corner has shown that first order theory is adequate to represent the effects of a concentrated bore resistance except when this is sufficient to stop the shot for a period of time, so that the method cannot be employed near the start of the motion. Another limitation of his method is that, if the kinetic energy of the shot when it meets the resistance is less than $AP_0 \Delta x$, the method breaks down, for it gives the projectile an imaginary velocity after crossing the resistance. In practice, the projectile would come to rest and would restart after the pressure had built up behind the shot, which would take a finite time. This is again a matter of $\Delta x / V_0$ (time of action of resistance) not being infinitesimal.

2. The Basic Equations

With the usual notations the basic equations of the internal ballistics before the occurrence of resistance are as follows:

The energy equation is

$$z(T_0-T) = \frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C}{3}\right)\frac{V^2}{CR}; \quad \dots \quad (1)$$

where $\bar{\gamma}$ is the value of γ suitably increased to allow for heat loss to the barrel.

The equation of state of the gas is

$$P\left[U+Ax-\frac{C(1-z)}{\delta}-Cz\eta\right] = \frac{CzRT\left(1+\frac{C}{2W_1}\right)}{\left(1+\frac{C}{3W_1}\right)}; \quad \dots \quad (2)$$

the form function is

$$z = (1-f)(1+\theta f); \quad \dots \quad (3)$$

the rate of burning equation is

$$D\frac{df}{dt} = -\beta P; \quad \dots \quad (4)$$

the dynamical equation of the motion of the shot is

$$\left(W_1+\frac{C}{2}\right)\frac{dV}{dt} = AP; \quad \dots \quad (5)$$

where W_1 is the effective mass of the projectile accounting for its rotational inertia.

3. Solution of the Equations

The basic equations (2) to (5) remain unaltered after the projectile passes the point x_0, while the energy equation (1) is replaced by

$$z(T_0-T) = \frac{\bar{\gamma}-1}{CR}\left[\left(W_1+\frac{C}{3}\right)\frac{V^2}{2}+AP_0\Delta x\right]. \quad \dots \quad (6)$$

Also just before reaching the resistance the kinetic energy of the charge and projectile is $\frac{1}{2}\left(W_1+\frac{C}{3}\right)V_0^2$ and immediately afterwards it is $\frac{1}{2}\left(W_1+\frac{C}{3}\right)V_0^2-AP_0\Delta x$. If V_1 is the velocity of the shot immediately after the resistance then assuming that the total kinetic energy is at all times $\frac{1}{2}\left(W_1+\frac{C}{3}\right)V^2$, where V is the velocity, we have

$$\frac{1}{2}\left(W_1+\frac{C}{3}\right)V_1^2 = \frac{1}{2}\left(W_1+\frac{C}{3}\right)V_0^2-AP_0\Delta x. \quad \dots \quad (7)$$

Eliminating P from (4) and (5) and integrating, we have

$$V = V_1+\frac{AD(f_0-f)}{\beta\left(W_1+\frac{C}{2}\right)}. \quad \dots \quad (8)$$

Therefore equation (6) becomes

$$\frac{CRz(T_0-T)}{\bar{\gamma}-1} = \frac{1}{2}\left(W_1+\frac{C}{3}\right)\left[V_1^2+\left\{\frac{AD(f_0-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right\}^2+\frac{2ADV_1(f_0-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right]+AP_0\Delta x.$$

Making use of equation (7) this can be written as

$$\frac{CRz(T_0-T)}{\bar{\gamma}-1} = \frac{1}{2}\left(W_1+\frac{C}{3}\right)\left[V_0^2+\left\{\frac{AD(f_0-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right\}^2+\frac{2ADV_1(f_0-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right]. \quad \cdots \quad (9)$$

Combining equations (2) and (9), we obtain

$$CRT_0z = \frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C}{3}\right)\left[V_0^2+\left\{\frac{AD(f_0-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right\}^2+\frac{2ADV_1(f_0-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right] + \frac{AP[x+l(1-bz)]\left(W_1+\frac{C}{3}\right)}{\left(W_1+\frac{C}{2}\right)}. \quad (10)$$

Where

$$Al = \left(U_1-\frac{C}{\delta}\right) \text{ (Initial free space)}$$

and

$$b = \frac{C\left(\eta-\frac{1}{\delta}\right)}{Al}.$$

Up to this point the equations have been exact. Now keeping only first order terms in equations (7), (8) and (10) we have

$$V_1 = V_0 - \frac{AP_0\Delta x}{\left(W_1+\frac{C}{3}\right)V_0}, \quad \cdots \quad (11)$$

$$V = \frac{AD(1-f)}{\beta\left(W_1+\frac{C}{2}\right)} - \frac{AP_0\Delta x}{\left(W_1+\frac{C}{3}\right)V_0}, \quad \cdots \quad (12)$$

$$CRT_0z = \frac{\bar{\gamma}-1}{2}\left(W_1+\frac{C}{3}\right)\left[\left\{\frac{AD(1-f)}{\beta\left(W_1+\frac{C}{2}\right)}\right\}^2 - \frac{2AP_0\Delta x(f_0-f)}{\left(W_1+\frac{C}{3}\right)(1-f_0)}\right] + \frac{AP[x+l(1-bz)]\left(W_1+\frac{C}{3}\right)}{\left(W_1+\frac{C}{2}\right)}. \quad (13)$$

Equation (13) can be written as

$$(1-f)(1+\theta f) = \frac{\bar{\gamma}-1}{2}M(1-f)^2 - \frac{(\bar{\gamma}-1)AP_0\Delta x(f_0-f)}{CRT_0(1-f_0)} - \frac{df}{dx}\left[M(1-f)-\frac{AP_0\Delta x}{CRT_0(1-f_0)}\right][x+l(1-bz)]\,; \quad \cdots \quad (14)$$

where

$$M = \frac{A^2D^2\left(1+\frac{C}{3W_1}\right)}{\beta^2 CRT_0 W_1\left(1+\frac{C}{2W_1}\right)^2}.$$

Equation (14) written in a different form is

$$Z+M[x+l(1-bz)]\frac{df}{dx} = \frac{AP_0\Delta x}{CRT_0(1-f_0)(1-f)}\left[\{x+l(1-bz\}\frac{df}{dx}-(\bar{\gamma}-1)(f_0-f)\right]; \quad (15)$$

where

$$Z = 1-\frac{\bar{\gamma}-1}{2}M+\theta_1 f, \quad \ldots \quad (16)$$

$$\theta_1 = \theta+\frac{\bar{\gamma}-1}{2}M. \quad \ldots \quad (17)$$

But if there were no bore resistance, then

$$Z+M[x+l(1-bz)]\frac{df}{dx} = 0. \quad \ldots \quad (18)$$

Thus we can obtain a solution correct to terms of order $P_0\Delta x$ by using equation (18) on the right of equation (15).
This gives

$$Z+M[x+l(1-bz)]\frac{df}{dx} = -\frac{AP_0\Delta x}{CRT_0(1-f_0)(1-f)}\left[\frac{Z}{M}+(\bar{\gamma}-1)(f_0-f)\right] \quad \ldots \quad (19)$$

or

$$\frac{dx}{df}+\left[\frac{M}{Z}-\frac{AP_0\Delta x}{CRT_0(1-f_0)(1-f)}\left\{\frac{1}{Z}+\frac{M(\bar{\gamma}-1)(f_0-f)}{Z^2}\right\}\right](x+l)$$
$$= blz\left[\frac{M}{Z}-\frac{AP_0\Delta x}{CRT_0(1-f_0)(1-f)}\left\{\frac{1}{Z}+\frac{M(\bar{\gamma}-1)(f_0-f)}{Z^2}\right\}\right]. \quad \ldots \quad (20)$$

Integrating this equation and taking the lower limit the point, suffix zero, at which resistance occurred, we get the relation between the shot-travel and the parameter f as

$$[x+l(1-bz)] = l\left(\frac{1+\theta}{Z}\right)^{\frac{M}{\theta_1}}\left[1-\frac{b\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\right]+\frac{blZ[\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f)]}{(M+\theta_1)(M+2\theta_1)}$$

$$+\frac{AP_0\Delta x}{CRT_0(1-f_0)}\left[l\left(\frac{1+\theta}{Z}\right)^{\frac{M}{\theta_1}}\left[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\log\frac{Z(1-f_0)}{Z_0(1-f)}\right.\right.$$

$$\left.\left.+\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left(\frac{1}{Z_0}-\frac{1}{Z}\right)\right]\right.$$

$$+bl\left[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2 Z^{M/\theta_1}}\left\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z_0}{1-f_0}\right.\right.$$

$$-Z^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z}{1-f}$$

$$+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}-Z^{\frac{M}{\theta_1}+1}\right)+\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\int_{f_0}^{f}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df\Bigg\}$$

$$-\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)Z^{M/\theta_1}}\Bigg\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)}$$

$$-Z^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f)}{M(M+\theta_1)}\Bigg\}\Bigg]\Bigg]. \quad \ldots \quad (21)$$

If we neglect the bore resistance terms in this relation, we get the same equation as obtained by Corner [(1950), equation (16), page 179]. Also if we neglect the co-volume terms but retain the bore resistance terms, then we obtain Corner's equation (109), page 218.

If suffix 'B' refers to the all-burnt position then the change in $[x_B+l(1-b)]$ due to the bore resistance is

$$\Delta[x_B+l(1-b)]=\frac{AP_0\Delta x}{CRT_0(1-f_0)}\Bigg[l\left(\frac{1+\theta}{Z_B}\right)^{\frac{M}{\theta_1}}\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\log\frac{Z_B(1-f_0)}{Z_0}$$

$$+\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)\Bigg]$$

$$+bl\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2Z_B^{M/\theta_1}}\Bigg\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z_0}{1-f_0}$$

$$-Z_B^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\log Z_B-\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\int_0^{f_0}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df$$

$$+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}-Z_B^{\frac{M}{\theta_1}+1}\right)\Bigg\}-\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)Z_B^{M/\theta_1}}\times$$

$$\times\Bigg\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)}-Z_B^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M}{M(M+\theta_1)}\Bigg\}\Bigg]\Bigg] \quad (22)$$

Maximum Pressure: From equation (13), we have

$$P[x+l(1-bz)]=\frac{CRT_0\left(1+\frac{C}{2W_1}\right)}{A\left(1+\frac{C}{3W_1}\right)}\left[(1+f)Z+\frac{(\bar{\gamma}-1)AP_0\Delta x(f_0-f)}{CRT_0(1-f_0)}\right]. \quad (23)$$

Also from equations (13) and (20), we obtain

$$\frac{dx}{df} = -\frac{CRT_0\left(1+\frac{C}{2W_1}\right)}{AP\left(1+\frac{C}{3W_1}\right)}\left[M(1-f)-\frac{AP_0\Delta x}{CRT_0(1-f_0)}\right]. \quad \ldots \quad (24)$$

Now we introduce the dimensionless quantity

$$k = \frac{\left(\eta-\frac{1}{\delta}\right)P_m\left(1+\frac{C}{3W_1}\right)}{RT_0\left(1+\frac{C}{2W_1}\right)}, \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

so that the equation (23) can be written as

$$P(x+l) = \frac{CRT_0\left(1+\frac{C}{2W_1}\right)(1-f)}{A\left(1+\frac{C}{3W_1}\right)}\left[Z+k(1+\theta f)+\frac{(\bar{\gamma}-1)AP_0\Delta x(f_0-f)}{CRT_0(1-f_0)(1-f)}\right]. \quad (26)$$

Now for maximum pressure $\frac{dP}{df} = 0$. Therefore differentiating equation (26) with respect to f and putting $\frac{dP}{df} = 0$, we obtain the value of f at maximum pressure as

$$f_m = \frac{\gamma' M+\theta-1}{\gamma' M+2\theta} + \text{a term proportional to } AP_0\Delta x, \quad \ldots \quad \ldots \quad (27)$$

where $\gamma' = \bar{\gamma}/(1+k)$.

Since we have to find only the first order effects of bore resistance on maximum pressure, we will substitute $f_m = \frac{\gamma' M+\theta-1}{\gamma' M+2\theta}$ in equations (21) and (23).

Now for pressures up to 25 tons/sq. in., RT_0 greater than 60 (tons/sq. in.) × (c.c./g), η less than 1·02 c.c./g, and δ less than 1·67 g/c.c., the maximum value of k is 0·175. It is convenient to introduce also a quantity

$$\xi = \gamma' - (\bar{\gamma}+1). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

Then

$$1-f_m = \frac{1+\theta}{\xi M+2\theta_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

and

$$Z_m = \frac{(1+\theta)(\xi M+\theta_1)}{(\xi M+2\theta_1)}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

Also as deduced by Corner we can write to a sufficient degree of accuracy,

$$\left(\frac{1+\theta}{Z_m}\right)^{\frac{M}{\theta_1}} = \frac{(\xi M+\theta_1)(eM+4\theta_1)}{(\xi M+2\theta_1)^2}. \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

Substituting from (29), (30) and (31) in equation (21), we obtain

$$\frac{[x_m+l(1-bz_m)](\xi M+2\theta_1)^2}{(\xi M+\theta_1)} = l(eM+4\theta_1)\left[1-\frac{b\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\right]$$

$$+\frac{bl(1+\theta)^2[\bar{\gamma}M(\xi M+2\theta_1)-2\theta(M+\theta_1)]}{(M+\theta_1)(M+2\theta_1)}$$

$$+\frac{AP_0\Delta x}{CRT_0(1-f_0)}\left[l(eM+4\theta_1)\left[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\log\frac{(\xi M+\theta_1)(1-f_0)}{Z_0}\right.\right.$$

$$\left.-\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left\{\frac{\xi M+2\theta_1}{(1+\theta)(\xi M+\theta_1)}-\frac{1}{Z_0}\right\}\right]$$

$$+bl\left[\frac{\{1+\theta-M(\bar{\gamma}-1)(1+f_0)\}(eM+4\theta_1)}{(1+\theta)^{\frac{M}{\theta_1}+2}}\left\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\times\right.\right.$$

$$\times\log\frac{Z_0}{1-f_0}$$

$$-\frac{(1+\theta)^{\frac{M}{\theta_1}+2}}{(eM+4\theta_1)}\frac{\bar{\gamma}M(\xi M+2\theta_1)-2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\log(\xi M+\theta_1)$$

$$+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}-\frac{(1+\theta)^{\frac{M}{\theta_1}+1}(\xi M+2\theta_1)}{(eM+4\theta_1)}\right)+$$

$$\left.+\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\int_{f_0}^{f_m}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df\right\}$$

$$-\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}(eM+4\theta_1)}{\theta_1(1+\theta)^{\frac{M}{\theta_1}+1}}\left\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)}\right.$$

$$\left.\left.\left.-\frac{(\xi M+2\theta_1)(1+\theta)^{\frac{M}{\theta_1}+1}}{(\xi M+\theta_1)(eM+4\theta_1)}\frac{(M-2\theta+\theta_1)(\xi M+2\theta_1)-2\theta M}{M(M+\theta_1)}\right\}\right]\right]. \quad \ldots \quad \ldots \quad (32)$$

Also equation (23) at the position of maximum pressure gives

$$\frac{(\xi M+2\theta_1)^2[x_m+l(1-bz_m)]}{(\xi M+\theta_1)} = \frac{CRT_0\left(1+\frac{C}{2W_1}\right)(1+\theta)^2}{AP_m\left(1+\frac{C}{3W_1}\right)}\times$$

$$\times\left[1+\frac{(\bar{\gamma}-1)AP_0\Delta x\left(f_0-1+\frac{1+\theta}{\xi M+2\theta_1}\right)(\xi M+2\theta_1)^2}{CRT_0(1-f_0)(1+\theta)^2(\xi M+\theta_1)}\right]. \quad \ldots \quad (33)$$

Dividing equation (32) by equation (33) and expanding in terms of $\left(\eta - \frac{1}{\delta}\right)$ and retaining only first power of $\left(\eta - \frac{1}{\delta}\right)$ we obtain

$$
\begin{aligned}
\frac{(eM+4\theta_1)}{(1+\theta)^2} = \\
& \frac{C}{Al}\left[\frac{RT_0\left(1+\frac{C}{2W_1}\right)}{P_m\left(1+\frac{C}{3W_1}\right)} + \left(\eta-\frac{1}{\delta}\right)\left\{\frac{\bar{\gamma}M(eM+4\theta_1)-\bar{\gamma}M(M+2\theta_1)+2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\right\}\right] \\
& + \frac{AP_0\Delta x}{CRT_0(1-f_0)}\left[\frac{(eM+4\theta_1)}{(1+\theta)^2}\left[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\log\frac{Z_0}{(M+\theta_1)(1-f_0)}\right.\right. \\
& \left.\left. + \frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left(\frac{M+2\theta_1}{(M+\theta_1)(1+\theta)} - \frac{1}{Z_0}\right)\right]\right. \\
& + \frac{(\bar{\gamma}-1)CRT_0\left(1+\frac{C}{2W_1}\right)(M+2\theta_1)\{\bar{\gamma}M+2\theta)f_0+1-\theta-\bar{\gamma}M\}}{AlP_m\left(1+\frac{C}{3W_1}\right)(M+\theta_1)(1+\theta)^2} \\
& + \frac{C\left(\eta-\frac{1}{\delta}\right)}{Al}\left[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}(eM+4\theta_1)}{(1+\theta)^{M/\theta_1+4}}\times\right. \\
& \qquad \times\left\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{1-f_0}{Z_0}\right. \\
& + \frac{(1+\theta)^{\frac{M}{\theta_1}+2}}{(eM+4\theta_1)}\frac{\bar{\gamma}M(M+2\theta_1)-2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\log(M+\theta_1) \\
& + \frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(\frac{(1+\theta)^{\frac{M}{\theta_1}+1}(M+2\theta_1)}{(eM+4\theta_1)} - Z_0^{\frac{M}{\theta_1}+1}\right) - \frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\times \\
& \qquad \left.\times\int_{f_0}^{\frac{\bar{\gamma}M+\theta-1}{\bar{\gamma}M+2\theta}}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df\right\} \\
& + \frac{M(\bar{\gamma}-1)(eM+4\theta_1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)^{\frac{M}{\theta_1}+3}}\left\{Z_0^{\frac{M}{\theta_1}}\frac{(M-2\theta+\theta_1)(1+\theta)-2\theta M(1-f_0)}{M(M+\theta_1)}\right.
\end{aligned}
$$

$$-\frac{(M+2\theta_1)(1+\theta)^{\frac{M}{\theta_1}+1}}{(M+\theta_1)(eM+4\theta_1)}\frac{(M-2\theta+\theta_1)(M+2\theta_1)-2\theta M}{M(M+\theta_1)}\Bigg\}$$

$$+\frac{M\bar{\gamma}(eM+4\theta_1)AlP_m\left(1+\frac{C}{3W_1}\right)}{CRT_0(M+\theta_1)^2(1+\theta)^4\left(1+\frac{C}{2W_1}\right)}\Bigg\{(1+\theta)\left(\theta-\frac{\bar{\gamma}+1}{2}M\right)-$$

$$-M(\bar{\gamma}-1)(1-f_0)(M+2\theta_1)\Bigg\}$$

$$+\frac{M\bar{\gamma}(\bar{\gamma}-1)\{\theta_1(1+\theta)+M(1-f_0)(M+2\theta_1)\}}{(1+\theta)^2(M+\theta_1)^2}\Bigg]\Bigg] \quad \ldots \quad (34)$$

Hence the change in maximum pressure due to a bore resistance $AP_0\Delta x$ occurring at a certain point during the period of burning, in terms of the characteristics of the solution without resistance, is obtained from

$$\frac{CRT_0\left(1+\frac{C}{2W_1}\right)}{Al\left(1+\frac{C}{3W_1}\right)}\Delta\left(\frac{1}{P_m}\right)=\frac{AP_0\Delta x}{CRT_0(1-f_0)}\Bigg[\frac{(eM+4\theta_1)}{(1+\theta)^2}\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}$$

$$\log\frac{(M+\theta_1)(1-f_0)}{Z_0}$$

$$-\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left\{\frac{M+2\theta_1}{(M+\theta_1)(1+\theta)}-\frac{1}{Z_0}\right\}\Bigg]\Bigg]$$

$$-\frac{(\bar{\gamma}-1)CRT_0\left(1+\frac{C}{2W_1}\right)(M+2\theta_1)\{(\bar{\gamma}M+2\theta)f_0+1-\theta-\bar{\gamma}M\}}{AlP_m\left(1+\frac{C}{3W_1}\right)(M+\theta_1)(1+\theta)^2}$$

$$+\frac{C\left(\eta-\frac{1}{\delta}\right)}{Al}\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}(eM+4\theta_1)}{(1+\theta)^{\frac{M}{\theta_1}+4}}\times$$

$$\times\left\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z_0}{1-f_0}\right.$$

$$-\frac{(1+\theta)^{\frac{M}{\theta_1}+2}}{(eM+4\theta_1)}\frac{\bar{\gamma}M(M+2\theta_1)-2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\log(M+\theta_1)+\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\times$$

$$\times\int_{f_0}^{\frac{\bar{\gamma}M+\theta-1}{\bar{\gamma}M+2\theta}}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df$$

$$+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}+\frac{(1+\theta)^{\frac{M}{\theta_1}+1}(M+2\theta_1)}{(eM+4\theta_1)}\right)\Bigg\}$$

$$-\frac{M(\bar{\gamma}-1)(eM+4\theta_1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)^{\frac{M}{\theta_1}+3}}\Bigg\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)}$$

$$-\frac{(1+\theta)^{\frac{M}{\theta_1}+1}(M+2\theta_1)}{(M+\theta_1)(eM+4\theta_1)}\frac{(M-2\theta+\theta_1)(M+2\theta_1)-2\theta M}{M(M+\theta_1)}\Bigg\}$$

$$+\frac{M\bar{\gamma}(eM+4\theta_1)AlP_m\left(1+\frac{C}{3W_1}\right)}{CRT_0(1+\theta)^4(M+\theta_1)^2\left(1+\frac{C}{2W_1}\right)}\Bigg\{M(\bar{\gamma}-1)(1-f_0)(M+2\theta_1)-$$

$$-(1+\theta)\left(\theta-\frac{\bar{\gamma}+1}{2}M\right)\Bigg\}$$

$$-\frac{M\bar{\gamma}(\bar{\gamma}-1)\{\theta_1(1+\theta)+M(1-f_0)(M+2\theta_1)\}}{(1+\theta)^2(M+\theta_1)^2}\Bigg]\Bigg] \quad \ldots \quad (35)$$

Here also we observe that if we neglect the co-value terms, i.e. put $\eta = \frac{1}{\delta}$, we get the same expression as obtained by Corner.

The above holds only if

$$f_0 \geqslant \frac{\gamma' M+\theta-1}{\gamma' M+2\theta} > 0.$$

If

$$f_0 < \frac{\gamma' M+\theta-1}{\gamma' M+2\theta} \geqslant 0$$

then there is no change in the maximum pressure. On the other hand if

$$f_0 \geqslant 0 > \frac{\gamma' M+2\theta-1}{\gamma' M+2\theta}$$

then the maximum pressure occurs at all-burnt. In that case we have from (22) and (23),

$$\frac{CRT_0\left(1+\frac{C}{2W_1}\right)Z_B}{AP_B\left(1+\frac{C}{3W_1}\right)}=l\left(\frac{1+\theta}{Z_B}\right)^{\frac{M}{\theta_1}}\left[1-\frac{b\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\right]+$$

$$+\frac{blZ_B[\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)]}{(M+\theta_1)(M+2\theta_1)}$$

$$
+\frac{AP_0\Delta x}{CRT_0(1-f_0)}\Bigg[l\left(\frac{1+\theta}{Z_B}\right)^{\frac{M}{\theta_1}}\Bigg(\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\log\frac{Z_B(1-f_0)}{Z_0}
$$

$$
+\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)-\frac{(\bar{\gamma}-1)f_0CRT_0\left(1+\frac{C}{2W_1}\right)}{AP_B\left(1+\frac{C}{3W_1}\right)}\Bigg)
$$

$$
+bl\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2Z_B^{M/\theta_1}}\Bigg\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z_0}{1-f_0}
$$

$$
-Z_B^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\log Z_B
$$

$$
+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}-Z_B^{\frac{M}{\theta_1}+1}\right)-\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\int_0^{f_0}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df\Bigg\}
$$

$$
-\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)Z_B^{M/\theta_1}}\Bigg\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)}
$$

$$
-Z_B^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M}{M(M+\theta_1)}\Bigg\}\Bigg]\Bigg] \quad \ldots \quad (36)
$$

Hence the change in maximum pressure in this case is given by

$$
\frac{CRT_0\left(1+\frac{C}{2W_1}\right)Z_B}{A\left(1+\frac{C}{3W_1}\right)}\Delta\left(\frac{1}{P_B}\right)=\frac{AP_0\Delta x}{CRT_0(1-f_0)}\Bigg[l\left(\frac{1+\theta}{Z_B}\right)^{\frac{M}{\theta_1}}\Bigg(\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\times
$$

$$
\times\log\frac{Z_B(1-f_0)}{Z_0}
$$

$$
+\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)-\frac{(\bar{\gamma}-1)f_0CRT_0\left(1+\frac{C}{2W_1}\right)}{AP_B\left(1+\frac{C}{3W_1}\right)}\Bigg)
$$

$$
+bl\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2Z_B^{M/\theta_1}}\Bigg\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z_0}{1-f_0}
$$

$$
-Z_B^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\log Z_B-\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\int_0^{f_0}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df
$$

$$+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}-Z_B^{\frac{M}{\theta_1}+1}\right)\Bigg\}-\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)Z_B^{M/\theta_1}}\times$$

$$\times\left\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)}-Z_B^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M}{M(M+\theta_1)}\right\}\Bigg]\Bigg] \quad (37)$$

Muzzle velocity: Since after all-burnt position the expansion of gases is adiabatic, the pressure at any travel x greater than x_B is

$$P = P_B r^{-\bar{\gamma}}\,; \quad \ldots \quad (38)$$

where

$$r = \frac{x+l(1-b)}{x_B+l(1-b)}\,. \quad \ldots \quad (39)$$

Also the equation of the motion of the shot is

$$\left(W_1+\frac{C}{2}\right)\frac{dV}{dt} = AP\,. \quad \ldots \quad (40)$$

Therefore from (38), (39) and (40), we obtain the velocity as

$$V^2 = V_B^2+\frac{AP_B[x_B+l(1-b)]\Phi}{\left(W_1+\frac{C}{2}\right)}\,; \quad \ldots \quad (41)$$

where

$$\Phi = \frac{2}{\bar{\gamma}-1}\left(1-r^{1-\bar{\gamma}}\right)\ldots \quad (42)$$

Hence if suffix E refers to the values at shot ejection, we have

$$V_E^2 = V_B^2+\frac{AP_B[x_B+l(1-b)]\Phi_E}{\left(W_1+\frac{C}{2}\right)}\,; \quad \ldots \quad (43)$$

where

$$\Phi_E = \frac{2}{\bar{\gamma}-1}\left[1-\left\{\frac{x_E+l(1-b)}{x_B+l(1-b)}\right\}^{1-\bar{\gamma}}\right]. \quad \ldots \quad (44)$$

In equation (43), V_B, $P_B[x_B+l(1-b)]$ and Φ_E are all altered by the resistance. From equation (23),

$$\frac{AP_B[x_B+l(1-b)]}{\left(W_1+\frac{C}{2}\right)} = \frac{CRT_0Z_B}{\left(W_1+\frac{C}{3}\right)}+\frac{(\bar{\gamma}-1)f_0AP_0\Delta x}{(1-f_0)\left(W_1+\frac{C}{3}\right)}\,, \quad \ldots \quad (45)$$

also

$$\Delta\Phi_E = 2r_E^{-\bar{\gamma}}\Delta r_E = -2r_E^{1-\bar{\gamma}}\frac{\Delta[x_B+l(1-b)]}{x_B+l(1-b)}\,; \quad \ldots \quad (46)$$

and from equation (12),

$$\Delta(V_B^2) = -\frac{2AP_0\Delta x}{\left(W_1+\frac{C}{3}\right)(1-f_0)}. \quad \ldots \quad (47)$$

Therefore we have

$$\Delta(V_E^2) = -\frac{2AP_0\Delta x\left(1-f_0+f_0 r_E^{1-\bar{\gamma}}\right)}{\left(W_1+\frac{C}{3}\right)(1-f_0)} - \frac{2Z_B r_E^{1-\bar{\gamma}} AP_0\Delta x}{\left(W_1+\frac{C}{3}\right)(1-f_0)[x_B+l(1-b)]}\times$$

$$\times\Bigg[l\left(\frac{1+\theta}{Z_B}\right)^{\frac{M}{\theta_1}}\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\log\frac{Z_B(1-f_0)}{Z_0}+$$

$$+\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)\Bigg]$$

$$+bl\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2 Z_B^{M/\theta_1}}\Bigg\{Z_0^{\frac{M}{\theta_1}}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z_0}{1-f_0}$$

$$-Z_B^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)}{(M+\theta_1)(M+\theta_1)}\log Z_B+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}-Z_B^{\frac{M}{\theta_1}+1}\right)$$

$$-\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\int_0^{f_0}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df\Bigg\} - \frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)Z_B^{M/\theta_1}}\times$$

$$\times\left\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)} - Z_B^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M}{M(M+\theta_1)}\right\}\Bigg]\Bigg] \quad (48)$$

and finally

$$\frac{\left(W_1+\frac{C}{3}\right)V_E(1-f_0)\Delta V_E}{AP_0\Delta x} = -1+f_0-f_0 r_E^{1-\bar{\gamma}} - \frac{Z_B r_E^{1-\bar{\gamma}}}{[x_B+l(1-b)]}\Bigg[l\left(\frac{1+\theta}{Z_B}\right)^{\frac{M}{\theta_1}}\times$$

$$\times\left[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2}\log\frac{Z_B(1-f_0)}{Z_0}+\frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)}\left(\frac{1}{Z_0}-\frac{1}{Z_B}\right)\right]$$

$$+bl\Bigg[\frac{\{1+\theta-M(\bar{\gamma}-1)(1-f_0)\}}{(1+\theta)^2 Z_B^{M/\theta_1}}\Bigg\{Z_0^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)(1-f_0)}{(M+\theta_1)(M+2\theta_1)}\log\frac{Z_0}{1-f_0}$$

$$-Z_B^{\frac{M}{\theta_1}+1}\frac{\bar{\gamma}M(1+\theta)-2\theta(M+\theta_1)}{(M+\theta_1)(M+2\theta_1)}\log Z_B+\frac{2\theta(1+\theta)}{(M+\theta_1)(M+2\theta_1)}\left(Z_0^{\frac{M}{\theta_1}+1}-Z_B^{\frac{M}{\theta_1}+1}\right)$$

$$-\frac{\bar{\gamma}M(1+\theta)^2}{(M+\theta_1)(M+2\theta_1)}\int_0^{f_0}\frac{Z^{\frac{M}{\theta_1}}}{1-f}df\Bigg\} - \frac{M(\bar{\gamma}-1)\{1+\theta-\theta_1(1-f_0)\}}{\theta_1(1+\theta)Z_B^{M/\theta_1}}\times$$

$$\times\left\{Z_0^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M(1-f_0)}{M(M+\theta_1)} - Z_B^{\frac{M}{\theta_1}}\frac{(1+\theta)(M-2\theta+\theta_1)-2\theta M}{M(M+\theta_1)}\right\}\Bigg]\Bigg]. \quad (49)$$

This gives the change of muzzle velocity due to a bore resistance $AP_0\Delta x$ occurring at the point (f_0, Z_0), in terms of the characteristics of the solution without resistance. If the resistance occurs at all-burnt position, $f_0 = 0$, then

$$\frac{\left(W_1+\frac{C}{3}\right)V_E\Delta V_E}{AP_0\Delta x} = -1.$$

When the resistance occurs during the adiabatic expansion the kinetic energy of the shot and the gases at the muzzle are reduced by the work done on the resistance, without other effects on the ballistics (except on the time to travel the bore). Hence

$$\frac{\left(W_1+\frac{C}{3}\right)V_E\Delta V_E}{AP_0\Delta x} = -1$$

if the resistance occurs after all-burnt.

4. Calculations of the Effect of Long Stretches of Bore Resistance

If we wish to find the effects of bore resistance extending over a large part of the travel and varying in any manner then such a resistance can be represented as an integral over suitably chosen resistance of our standard type. Thus the total change in maximum pressure due to the long stretch of bore resistance is given by

$$\Delta P_m = \int\frac{\Delta P_m}{\Delta x}\frac{dx}{df}df$$

$$= AP_m^2\int\frac{MP_0[x+l(1-bz)]}{Z(1-f)}\,\frac{\Delta P_m(1-f)}{P_m^2AP_0\Delta x}df. \quad (50)$$

The weighting factor, $\frac{(1-f)\Delta P_m}{P_m^2AP_0\Delta x}$ in terms of f, is obtained from (35). Since $[x+l(1-bz)]$ is a function of f and P_0 is a function of x, i.e. of f, so the integral (50) can be easily evaluated by a numerical integration. Similarly, total change in muzzle velocity due to the long stretch of bore resistance is given by

$$\Delta V_E = \int\frac{\Delta V_E}{\Delta x}\frac{dx}{df}df$$

$$= -\frac{A}{\left(W_1+\frac{C}{3}\right)V_E}\int\frac{MP_0[x+l(1-bz)]}{Z(1-f)}\,\frac{\left(W_1+\frac{C}{3}\right)V_E(1-f)\Delta V_E}{AP_0\Delta x}df \quad (51)$$

where the weighting factor

$$\left[\frac{\left(W_1+\frac{C}{3}\right) V_E(1-f)\,\Delta V_E}{AP_0\Delta x}\right]$$

is given by (49).

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Abstract

By neglecting the co-volume terms $\left(\eta = \frac{1}{\delta}\right)$, Corner has found the ballistic effects of bore resistance on the internal ballistics of a gun. In this communication the author has extended Corner's theory by taking account of the co-volume terms. The weighting factors are calculated for each point along the path of the motion and thus the effect of bore resistance for any given dependence on travel can be found rapidly by integration of the product of resistance and the weighting factor.

References

Corner, J. (1949). The ballistic effects of bore resistance. *Quart. J. Mech. and Applied Maths.*, **2**, 232.

————(1950). Theory of the Interior Ballistics of Guns, New York.

Tawakley, V. B. (1956). Effect of bore resistance on maximum pressure and muzzle velocity for composite charge. *PNISIPS.*, **22**, 105–128.

Issued October 21. 1957.