EQUATION OF STATE OF ELEMENTS FROM THE RELATIVISTIC THOMAS-FERMI THEORY

(Case of Extreme Degeneracy)

by V. Swarup Mathur, *Department of Physics, University of Delhi*

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The Thomas-Fermi statistical model of the atom (hereafter referred to as **TF**), devised to obtain a self-consistent potential and electron distribution around an atomic nucleus, was employed for the first time by Slater and Krutter (1935) for the study of metals. Later it was realised that the results of the Thomas-Fermi theory could be applicable to other elements also at such high pressures at which the detailed influence of the outer atomic structure would be obliterated on account of 'pressure ionisation'. Recently March (1955) has obtained the equation of state of elements at very high pressures from the Thomas-Fermi theory. He has made it clear that the pressure on the TF atom is simply that which would be exerted by a free electron gas of density equal to the TF density at the atomic boundary; so that it would, for our purpose, suffice only to determine the boundary value of the TF function $\phi(x)$, instead of obtaining a complete solution for $\phi(x)$ inside the atom. To this effect March has made use of an ingenious method, and his results suggest certain modifications to be made in the formulae of Gilvarry (1954*b*) who had, rather intuitively, fitted the available numerical data with the boundary functions having the correct limiting forms.

The TF equation used by March was, however, non-relativistic, whereas it is well known that the relativistic effects become significant at extremely high pressures which are of interest in astrophysics. An attempt to use relativistic Thomas-Fermi equation to determine the electron density at the atomic boundary has recently been made by Murty (1956) who based his equation on the 'density of states' derived from Schrödinger's equation. However, to obtain a correct relativistic TF equation it would, according to Gilvarry (1954*a*), be more appropriate to use the 'density of states' derived by Rudkjøbing (1952) from Dirac equation for a spherically symmetrical potential.

In this paper we have obtained the equation of state of elements from the relativistic Thomas-Fermi theory based on Rudkjøbing's expression for the 'density of states'. Both extreme relativistic and non-relativistic approximations have been considered. For the present we have confined our attention only to the case of 'extreme degeneracy', reserving the consideration of elevated temperatures for a subsequent paper. It may be added that so far we know of two forms of equations of state. One is that obtained by March (1955) from the non-relativistic Thomas-Fermi theory which, though takes account of the 'electron-nucleus' and 'electron-electron' interactions, ignores the relativistic corrections which are significant at very high densities. Another equation of state which is due to Chandrasekhar,* is that of a completely degenerate free electron gas which takes account of the relativistic effects but ignores the 'electron-nucleus' and 'electron-electron' interactions whose contributions are significant at comparatively low densities. The present

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* Chandrasekhar, S. (1939). Chap. XI, pp. 415-16. Eqs. (16), (17) and (18).

paper is an attempt to incorporate both 'relativistic' and 'electrostatic' corrections into the equation of state.

2. The Relativistic Thomas-Fermi Equation

According to Rudkjøbing (1952), the 'density of states' $n(r, E)\,dE$ of electrons with energies between E and $E+dE$ at a distance r from the origin of reference, where the potential (being spherically symmetrical) is $V(r)$, is given by—

$$n(r, E)\,dE = \sigma_1 \left[(E+eV)^2 - m^2c^4 - \left(re\frac{dV}{dr}\right)^2\right]^{\frac{1}{2}} (E+eV)\,dE, \quad .. \quad (1)$$

where $\sigma_1 = \dfrac{8\pi}{h^3c^3}$.

The term $\left(re\dfrac{dV}{dr}\right)^2$ in $n(r, E)\,dE$ is due to the 'spin orbit interaction' and appears as a result of using Dirac equation.

At temperature T the electron density at r would be given by

$$\rho(r) = \int_{E_l}^{\infty} \frac{n(r, E)\,dE}{\frac{1}{A}\exp\frac{E}{kT}+1} \quad .. \quad .. \quad .. \quad .. \quad (2)$$

where E_l is the lower limit to E, such that $\left\{(E+eV)^2 - m^2c^4 - \left(re\dfrac{dV}{dr}\right)^2\right\}$ vanishes for $E = E_l$.

Putting

$$\frac{E-mc^2+eV}{kT} = u,$$

$$A = \exp\frac{E_m}{kT},$$

$$re\frac{dV}{dr} = \lambda,$$

and denoting the values of u for $E = E_m$ and $E = E_l$ by u_0 and u_l respectively, we get

$$\rho(r) = \sigma_1(kT)^3 \int_{u_l}^{\infty} \frac{\frac{d\chi}{du}}{\exp(-u_0+u)+1}\,du, \quad .. \quad .. \quad (3)$$

where

$$\chi = \tfrac{1}{3}\left[u^2 + \frac{2mc^2}{kT}u - \frac{\lambda^2}{k^2T^2}\right]^{\frac{3}{2}}.$$

Since χ would vanish for $u = u_l$, the integral in (3) can be evaluated by using Sommerfeld's lemma,* viz.

$$\int_{u_l}^{\infty} \frac{\frac{d\chi}{du}}{\exp(-u_0+u)+1}\,du = \chi(u_0) + 2[c_2\chi''(u_0) + c_4\chi^{\text{iv}}(u_0) \ldots\ldots], \quad .. \quad (4)$$

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* Chandrasekhar, S. (1939), pp. 389.

where $c_2 = \frac{\pi^2}{12}$, $c_4 = \frac{7\pi^4}{720}$, and so on; provided $u_0 \gg 1$.

Proceeding in this way, and substituting for u_0 and λ, we get

$$\rho(r) = \frac{8\pi}{3h^3c^3}\left\{(E_m+eV)^2-m^2c^4-\left(re\frac{dV}{dr}\right)^2\right\}^{\frac{3}{2}}\times$$
$$\times\left[1+\frac{(\pi kT)^2}{2}\cdot\frac{2(E_m+eV)^2-m^2c^4-\left(re\frac{dV}{dr}\right)^2}{\left\{(E_m+eV)^2-m^2c^4-\left(re\frac{dV}{dr}\right)^2\right\}^2}\right]. \quad \ldots \quad (5)$$

This expression for $\rho(r)$ (the electron density at r) yields at once the 'temperature perturbed' relativistic Thomas-Fermi equation when substituted into the Poisson's equation, viz.

$$\nabla^2V(r) = 4\pi e\rho(r). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Taking the potential inside the atom to be spherically symmetrical and introducing the dimensionless variables $\phi(x)$ and x by

$$\left.\begin{aligned}\frac{Ze^2}{r}\phi(r) &= (E_m-mc^2+eV) = e(V-V_0),\\ \text{and} \quad ax &= r, \quad \text{where} \quad a = \frac{\hbar^2}{me^2},\end{aligned}\right\} \quad \ldots \quad \ldots \quad (7)$$

we get the desired equation as—

$$\frac{1}{x}\frac{d^2\phi}{dx^2} = \lambda\left\{\left(\frac{\phi}{x}\right)^2+\frac{\beta\phi}{x}-\left(\frac{d\phi}{dx}-\frac{\phi}{x}\right)^2\right\}^{\frac{3}{2}}\times$$
$$\times\left[1+\frac{(\pi kT)^2\left\{\left(\frac{\phi}{x}\right)^2+\beta\frac{\phi}{x}-\frac{1}{2}\left(\frac{d\phi}{dx}-\frac{\phi}{x}\right)^2+\frac{\beta^2}{8}\right\}}{\left(\frac{2mc^2}{\beta}\right)^2\left\{\left(\frac{\phi}{x}\right)^2+\beta\frac{\phi}{x}-\left(\frac{d\phi}{dx}-\frac{\phi}{x}\right)^2\right\}^2}\right]. \quad \ldots \quad (8)$$

where

$$\left.\begin{aligned}\lambda &= \frac{32\pi^2Z^2e^6}{3h^3c^3},\\ \text{and} \quad \beta &= \frac{2mc^2}{Ze^2}a.\end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

This equation, with the neglect of the 'spin orbit' term, viz. $\left(\frac{d\phi}{dx}-\frac{\phi}{x}\right)^2$, reduces to Murty's equation.

We shall consider here only the case of 'extreme degeneracy', i.e. when $u_0 \to \infty$. In this case the temperature dependent term in (8) can be neglected and we can write the equation as—

$$\frac{d^2\phi}{dx^2} = \lambda\frac{\phi^3}{x^2}\left\{1+\frac{\beta x}{\phi}-\left(\frac{x}{\phi}\frac{d\phi}{dx}-1\right)^2\right\}^{\frac{3}{2}}. \quad \ldots \quad \ldots \quad (10)$$

The solution of this equation should be found for the boundary conditions

$$\left.\begin{aligned} &\text{(i)}\ \left(\frac{d\phi}{dx}\right)_{x_0} = \frac{\phi(x_0)}{x_0}, \\ &\text{and (ii)}\ \ \phi(0) = 1, \end{aligned}\right\} \qquad (11)$$

where x_0 corresponds to the boundary of the atom.

Following March (1955), we assume, for $\phi(x)$, a Taylor's series solution about the boundary point, i.e.

$$\phi(x) = \phi(x_0)+\sum_{n=1}^{\infty} t_n(x_0-x)^n, \qquad (12)$$

where

$$t_n = \frac{(-1)^n}{n\,!}\left(\frac{d^n\phi}{dx^n}\right)_{x_0}. \qquad (13)$$

With the help of (13) we shall obtain terms up to t_5 in the expansion and then find $\phi(x_0)$ from the condition that for $x \to 0$, $\phi(x)$ goes over to unity. Although the Taylor's series expansion will not be strictly convergent for values of x in the neighbourhood of zero, yet it is useful from a practical point of view as a semi-convergent series.

The coefficients up to t_5 are

$$\left.\begin{aligned} t_1 &= -\frac{\phi(x_0)}{x_0}, \\ t_2 &= \frac{1}{2}\lambda\,\frac{\phi^3(x_0)}{x_0^2}\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{3}{2}}, \\ t_3 &= -\frac{\lambda}{6}\frac{\phi^3(x_0)}{x_0^3}\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{3}{2}}. \\ t_4 &= \frac{1}{8}\lambda^2\,\frac{\phi^5(x_0)}{x_0^4}\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^3\left(1-\frac{\sigma}{2}\right)-\frac{\lambda^3}{8}\,\frac{\phi^7(x_0)}{x_0^4}\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{7}{2}}, \\ t_5 &= -\frac{\lambda^2}{40}\frac{\phi^5(x_0)}{x_0^5}\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^3\left(1-\frac{\sigma}{2}\right)+\frac{3}{40}\lambda^3\,\frac{\phi^7(x_0)}{x_0^5}\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{7}{2}} \end{aligned}\right\} \qquad (14)$$

where $\sigma = \dfrac{\dfrac{\beta x_0}{\phi(x_0)}}{1+\dfrac{\beta x_0}{\phi(x_0)}}$.

We have to find $\phi(x_0)$ from the condition

$$\lim_{x\to 0}\left[\phi(x_0)+\sum_n t_n(x_0-x)^n\right] = 1. \qquad (15)$$

Considering first only the terms up to t_3 in (15) we get—

$$\phi^3(x_0)\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{3}{2}} = \frac{3}{\lambda}. \qquad (16)$$

The value of $\phi(x_0)$ as given by (16) will be denoted by $\phi^*(x_0)$. To obtain a more appropriate value of $\phi(x_0)$ we may next consider in (15) terms up to t_5 and in the last two terms put $\phi^*(x_0)$ in place of $\phi(x_0)$. Thus we get for $\phi(x_0)$—

$$\phi^3(x_0)\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{3}{2}} = \frac{3}{\lambda}\left[1-\frac{9}{10}\left\{\frac{1}{\phi^*(x_0)}-\frac{1}{2}\left(\frac{\lambda}{3}\right)^{\frac{2}{3}}\beta x_0\right\}+\frac{3^{\frac{7}{3}}\lambda^{\frac{2}{3}}}{20}\cdots\right] \quad .. \quad (17)$$

Though it appears difficult to use (17) as such to find the equation of state of elements, it seems convenient to do so in two different approximations which will be referred to as 'extreme-relativistic' and 'non-relativistic'. In the former case we shall have $\frac{\lambda^{\frac{1}{3}}\beta x_0}{2\,.\,3^{\frac{1}{3}}} \ll 1$, and in the latter $\frac{\lambda^{\frac{1}{3}}\beta x_0}{2\,.\,3^{\frac{1}{3}}} \gg 1$. (17) can be written in the ER approximation as—

$$\phi^3(x_0)\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{3}{2}} = \frac{3}{\lambda}\left[1-\frac{9}{10}\left(\frac{\lambda}{3}\right)^{\frac{1}{3}}\left\{1+\frac{\lambda^{\frac{2}{3}}\beta^2 x_0^2}{8\,.\,3^{\frac{1}{3}}}-\frac{3}{2}\left(\frac{\lambda}{3}\right)^{\frac{1}{3}}\right\}\cdots\right], \quad .. \quad (18)$$

and in the NR approximation as—

$$\phi^3(x_0)\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{3}{2}} = \frac{3}{\lambda}\left[1-\frac{9}{20}\left(\frac{\lambda}{3}\right)^{\frac{2}{3}}\beta x_0\left\{1+\frac{2\,.\,3^{\frac{2}{3}}}{\lambda^{\frac{2}{3}}\beta^2 x_0^2}\cdots\right\}+\frac{3^{\frac{7}{3}}\lambda^{\frac{2}{3}}}{20}\cdots\right], \quad (19)$$

where we have retained the lowest order terms in both cases.

3. The Pressure at the Boundary of a Relativistic Thomas-Fermi Atom

The pressure at the boundary of the atom will be given by

$$p = kT\int_{E_l}^{\infty} ln\left(1+A\exp-\frac{E}{kT}\right)n(r_0, E)\,dE, \quad .. \quad .. \quad (20)$$

where r_0 is the radius of the atom. Substituting for $n(r_0, E)$ and remembering that $\left(\frac{dV}{dr}\right)_{r_0} = 0$, we get, on integrating by parts,

$$p = \frac{8\pi}{3h^3c^3}\int_{E_l}^{\infty}\frac{\{(E+eV(r_0))^2-m^2c^4\}^{\frac{3}{2}}\,dE}{\exp\frac{E-E_m}{kT}+1}. \quad .. \quad .. \quad (21)$$

Introducing a variable u as before we get—

$$p = \frac{8\pi}{3h^3c^3}k^4T^4\int_{u_l}^{\infty}\frac{\frac{d\psi}{du}\,du}{\exp(u-u_0)+1}, \quad .. \quad .. \quad .. \quad (22)$$

where

$$\frac{d\psi}{du} = \left(u^2+\frac{2mc^2}{kT}u\right)^{\frac{3}{2}}. \quad .. \quad .. \quad .. \quad .. \quad (23)$$

Using Sommerfeld's lemma we get on substituting for u_0,

$$p = \frac{\pi m^4c^5}{3h^3}f(\eta)\left[1+\frac{4\pi^2k^2T^2}{m^2c^4}\frac{\eta(1+\eta^2)^{\frac{1}{2}}}{f(\eta)}\cdots\right] \quad .. \quad .. \quad (24)$$

where

$$f(\eta) = (2\eta^3-3\eta)(\eta^2+1)^{\frac{1}{2}}-3\sinh^{-1}\eta,$$

and

$$\eta = \frac{\{(E_m+eV(r_0))^2-m^2c^4\}^{\frac{1}{2}}}{mc^2}. \qquad \cdots \quad \cdots \quad (25)$$

Making use of (7) we have, for η, the expression

$$\eta = \frac{2}{\beta x_0}\,\phi(x_0)\left\{1+\frac{\beta x_0}{\phi(x_0)}\right\}^{\frac{1}{2}}. \qquad \cdots \quad \cdots \quad \cdots \quad (26)$$

As we are considering the case of extreme degeneracy, we shall neglect the temperature dependent terms in (24). We have thus for the pressure on the atomic boundary

$$p = \frac{\pi m^4c^5}{3h^3}\,f(\eta) \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \quad (24a)$$

where η is given by (26).

In the ER and NR approximations respectively, η can be expressed as

$$\eta = \frac{2\,.\,3^{\frac{1}{3}}}{\lambda^{\frac{1}{3}}\beta x_0}\left[1-\frac{3}{10}\left(\frac{\lambda}{3}\right)^{\frac{1}{3}}\left\{1+\frac{\lambda^{\frac{2}{3}}\beta^2x_0^2}{8\,.\,3^{\frac{2}{3}}}-\frac{6}{5}\left(\frac{\lambda}{3}\right)^{\frac{1}{3}}\right\}\cdots\right] \qquad \cdots \quad \cdots \quad (27)$$

for

$$\frac{\lambda^{\frac{1}{3}}\beta x_0}{2\,.\,3^{\frac{1}{3}}} \ll 1,$$

and

$$\eta = \frac{2\,.\,3^{\frac{1}{3}}}{\lambda^{\frac{1}{3}}\beta x_0}\left[1-\frac{3}{20}\left(\frac{\lambda}{3}\right)^{\frac{2}{3}}\beta x_0\left(1+\frac{2\,.\,3^{\frac{2}{3}}}{\lambda^{\frac{2}{3}}\beta^2x_0^2}\right)+\frac{3^{\frac{4}{3}}\lambda^{\frac{2}{3}}}{20}-\frac{9}{400}\left(\frac{\lambda}{3}\right)^{\frac{4}{3}}\beta^2x_0^2\cdots\right] \qquad (28)$$

for $\dfrac{10}{3^{\frac{5}{3}}\lambda^{\frac{1}{3}}} \gg \dfrac{\lambda^{\frac{1}{3}}\beta x_0}{2\,.\,3^{\frac{1}{3}}} \gg 1$.

Introducing the parameter $\eta_0 = \dfrac{2\,.\,3^{\frac{1}{3}}}{\lambda^{\frac{1}{3}}\beta x_0}$, we can rewrite (27) and (28) as

$$\eta = \eta_0\left[1-\frac{3}{10}\left(\frac{\lambda}{3}\right)^{\frac{1}{3}}\left\{1+\frac{1}{2\eta_0^2}-\frac{6}{5}\left(\frac{\lambda}{3}\right)^{\frac{1}{3}}\right\}\cdots\right] \qquad \cdots \quad \cdots \quad (27a)$$

for $\eta_0 \gg 1$ (ER approximation),
and

$$\eta = \eta_0\left[1-\frac{3^{\frac{2}{3}}}{10}\frac{\lambda^{\frac{1}{3}}}{\eta_0}\left(1+\frac{1}{2}\eta_0^2\right)+\frac{3^{\frac{4}{3}}}{20}\lambda^{\frac{2}{3}}-\frac{3^{\frac{4}{3}}}{100}\left(\frac{\lambda^{\frac{1}{3}}}{\eta_0}\right)^2+\cdots\right] \qquad \cdots \quad (28a)$$

for $\dfrac{3^{\frac{5}{3}}\lambda^{\frac{1}{3}}}{10} \ll \eta_0 \ll 1$ (NR approximation).

According to Chandrasekhar,* $f(\eta)$ can be expanded in the two approximations as

$$f(\eta) = 2\eta^4-2\eta^2-3\ln 2\eta-\frac{7}{4}+O\left(\frac{1}{\eta^2}\right) \quad \text{for} \quad \eta \gg 1,$$

and

$$f(\eta) = \frac{8}{5}\eta^5-\frac{4}{7}\eta^7+\frac{1}{3}\eta^9-\frac{5}{22}\eta^{11}\cdots \quad \text{for} \quad \eta \ll 1. \qquad \cdots \quad \cdots \quad (29)$$

* Chandrasekhar, S. (1939). Chap. X, p. 361.

In terms of η_0, the above expansions can be written as

$$f(\eta) = 2\eta_0^4\left[1-\frac{1}{\eta_0^2}-\frac{6}{5}\left(\frac{\lambda}{3}\right)^{\frac{1}{3}}+\frac{99}{50}\left(\frac{\lambda}{3}\right)^{\frac{2}{3}}\cdots\right] \qquad (30)$$

for $\eta_0 \gg 1$,
and

$$f(\eta) = \frac{8}{5}\eta_0^5\left[1-\frac{3^{\frac{2}{3}}}{2}\frac{\lambda^{\frac{1}{3}}}{\eta_0}-\frac{5}{14}\eta_0^2+\frac{3^{\frac{4}{3}}}{20}\left(\frac{\lambda^{\frac{1}{3}}}{\eta_0}\right)^2+\frac{5}{24}\eta_0^2+\frac{3^{\frac{7}{3}}}{10}\lambda^{\frac{2}{3}}\cdots\right] \qquad (31)$$

for $\frac{3^{\frac{5}{3}}\lambda^{\frac{1}{3}}}{10} \ll \eta_0 \ll 1$.

Defining ρ_m, the density of the material, by $\frac{Am_H}{\frac{4\pi}{3}(ax_0)^3}$ where A is the atomic weight of the element and m_H is the mass of the hydrogen atom, we can write the equations of state of elements in the two approximations as

$$p = \frac{hc}{8}\left(\frac{Z\rho_m}{Am_H}\right)^{\frac{4}{3}}\left(\frac{3}{\pi}\right)^{\frac{1}{3}}\left[1-\frac{1}{\eta_0^2}-\frac{6}{5}\left(\frac{4Z^2}{9\pi}\right)^{\frac{1}{3}}\alpha+\frac{99}{50}\left(\frac{4Z^2}{9\pi}\right)^{\frac{2}{3}}\alpha^2 + \text{higher terms in } \alpha \text{ and } \frac{1}{\eta_0}\right] \qquad (32)$$

for $\eta_0 \gg 1$,
and

$$p = \frac{h^2}{5m}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}}\left(\frac{Z\rho_m}{Am_H}\right)^{\frac{5}{3}}\left[1-\left(\frac{3Z^2}{2\pi}\right)^{\frac{1}{3}}\frac{\alpha}{\eta_0}-\frac{5}{14}\eta_0^2+\frac{1}{5}\left(\frac{3Z^2}{2\pi}\right)^{\frac{2}{3}}\frac{\alpha^2}{\eta_0^2} + \frac{5}{24}\eta_0^4+\frac{6}{5}\left(\frac{3Z^2}{2\pi}\right)^{\frac{2}{3}}\alpha^2 + \text{higher terms in } \frac{\alpha}{\eta_0}, \eta_0 \text{ and } \alpha\right] \qquad (33)$$

for
$$\left(\frac{324Z^2}{\pi}\right)^{\frac{1}{3}}\frac{\alpha}{10} \ll \eta_0 \ll 1,$$

where $\alpha = \frac{e^2}{\hbar c}$ = fine structure constant,

and

$$\eta_0 = \left(\frac{\rho_m}{B}\right)^{\frac{1}{3}} \text{ where } B = \frac{8\pi m^3c^3\frac{A}{Z}m_H}{3h^3} = 9{\cdot}82\times10^5\frac{A}{Z}\text{ gms./c.c.}$$

Eq. (32) holds in the extreme relativistic approximation when the densities are very high ($\gg 10^6$ gms./c.c.) so that $\eta_0 \gg 1$. The second term on the r.h.s. of (32) appears on account of the deviation from the extreme relativistic limit while the third and fourth terms appear as the 'electrostatic' and 'spin orbit' corrections. We see that in the extreme relativistic approximation the 'electrostatic' and 'spin orbit' terms are very small, being of the order of α and α^2 respectively. By writing the equation of state for the ER approximation in the parametric form as (24*a*) and (27*a*), we see that it differs from Chandrasekhar's equation of state of a completely degenerate electron gas only through terms of the order of the fine structure constant.

Eq. (33) holds, on the other hand, in the non-relativistic approximation when the densities are $\ll 10^6$ gms./c.c. However, for densities so small ($< {\cdot}04\,AZ$ gms./c.c.)

that $\eta_0 < \left(\frac{324Z^2}{\pi}\right)^{\frac{1}{3}} \frac{\alpha}{10}$, (33) loses its validity. Comparing (33) with the equation of state originally obtained by Kothari (1938) in a parametric form from his theory of 'pressure ionisation',* and later by March (1955) from the non-relativistic TF theory, we note that our equation contains some additional terms. Of these, the third term, which appears as a relativistic correction, is very significant particularly at high densities ($\approx 10^5$ gms./c.c.).

The following table gives, for various densities, the percentage correction on account of the relativistic terms to the pressure obtained from March's equation. (The calculations have been done for $Z \approx 10$).

ρ_m in gms./c.c.	η_0	Percentage correction
10^2	·03706	·17%
10^3	·07986	·34%
10^4	·1720	1·25%
10^5	·3707	4·86%
5×10^5	·6337	11·47%

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Abstract

In this paper, the author has obtained the equation of state of elements at very high pressures from the relativistic Thomas-Fermi equation based on the 'density of states' derived by Rudkjøbing (1952). Both extreme relativistic and non-relativistic approximations have been considered. Attention is, however, confined only to the case of 'extreme degeneracy'. The effect of 'exchange' is also neglected as it does not invalidate our results for high pressures and high atomic numbers.

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* If we take in Kothari's eq. the constants ν_1 and ν_2 to be $\frac{3}{4\pi}$ and $\frac{4\pi}{3}$ respectively, instead of taking them equal to unity, we find that it becomes identical with March's equation.

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