WAVE RESISTANCE OF A SHIP MOVING IN A CIRCULAR PATH

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Abstract

A formula for the wave resistance for a ship moving uniformly in a circle has been obtained, the width of the ship has been supposed to be small and the ship has been replaced by its central vertical section which is covered by a suitable distribution of sources and sinks. This distribution produces the necessary wave system which satisfies the boundary condition on the side of the ship.

1. Introduction

In this paper an attempt will be made to obtain a wave resistance formula for a slender ship moving with a uniform angular velocity in a circle on a deep sea. The problem was suggested by Lunde (1951). In §2 we have given a short account of the method followed by Havelock (1949) and Lunde (1951) for the solution of such problems. The ship is supposed to be symmetrical about a vertical central plane, and the elevation of the ship's surface on either side of this plane is regarded as small. The irrotational fluid motion due to the motion of the ship is supposed to be generated by a distribution of sources and sinks on this plane so that the ship may be replaced by a suitable distribution of sources and sinks on its central plane of symmetry. As this plane moves without rotation in a circular path, each source or sink traces out a circle of different radius. The resultant fluid motion will be due to this moving source-sink system.

The velocity potential due to each moving source is obtained by a method suggested by Havelock (1949) and employed by Lunde (1953) as follows: The path followed by a particular source is known. At the position it would occupy at any time τ, a source of strength $m(\tau)$ is supposed to be created suddenly without any impulsive disturbance of the free surface. The created source is maintained there for a short time $\delta\tau$ and then annihilated, the annihilation being supposed not to involve any impulsive disturbance. The non-transient effect of this transient source $m(\tau)$ is worked out. The effect of the source after it has been in continuous motion for a time t is obtained by summing up all such non-transient effects for varying τ up to the time t, and finally adding the effect of a source similarly created at time t in its appropriate position but no longer destroyed.

The distribution of source density on the central plane is kept open during the calculation of the resultant potential motion. Finally the potential motion is subjected to the condition that the discontinuity of its normal derivative on the plane should give the requisite source density. This is sufficient to determine the fluid motion from which the ship resistance may be calculated by the usual method.

2. General Analysis

Taking the origin on the undisturbed free surface and the z-axis vertically upwards, we imagine a source of strength m suddenly created at the point $(0, 0, -f)$. Since the impulse is zero at the free surface, the appropriate initial condition is $\phi = 0$ on $z = 0$. In order to satisfy this condition we must assume an image source of strength $-m$ at $(0, 0, f)$. Thus the velocity potential becomes in the usual notation

$$\frac{m}{r_1}-\frac{m}{r_2}=\frac{m}{2\pi}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}e^{k[i\bar{\omega}-(z+f)]}dk-\frac{m}{2\pi}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}e^{k[i\bar{\omega}+(z-f)]}dk, \quad .. \quad (1)$$

where

$$r_1^2=x^2+y^2+(z+f)^2, \quad r_2^2=x^2+y^2+(z-f)^2,$$

and

$$\bar{\omega}=x\cos\theta+y\sin\theta.$$

This velocity potential causes a velocity w at the free surface of the liquid. We shall now imagine that the source generated at $Q(0, 0, -f)$ at $t=0$ is maintained for a short interval of time $\delta\tau$, and then annihilated. The surface elevation due to this transient source of life time $\delta\tau$ is given by

$$\zeta_0=w\delta\tau=-\delta\tau\,.\,m\frac{\partial}{\partial z}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=\delta\tau\,.\,\frac{m}{\pi}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}e^{k(i\bar{\omega}-f)}\,k\,dk$$

$$=\delta\tau\,.\,\frac{m}{\pi}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}\cos k\bar{\omega}\,.\,e^{-kf}\,.\,k\,dk, \quad .. \quad .. \quad (2)$$

where only the real part is taken.

When the source is annihilated this surface elevation ζ_0 remains as the initial condition of the subsequent motion. The problem of the subsequent fluid motion is now reduced to that of an extension of the well-known two dimensional Poisson problem. It is easy to verify that all the conditions of the surface wave problem can be satisfied if we take the velocity potential,

$$\phi=\delta\tau\,.\,\frac{m}{\pi}g^{\frac{1}{2}}\int_{-\pi}^{\pi}d\theta\int_0^{t}\sin\sqrt{gk}t\,.\,e^{k[(z-f)+i\bar{\omega}]}\,.\,k^{\frac{1}{2}}\,dk. \quad .. \quad .. \quad (3)$$

In this problem the conditions to be satisfied are :

$$\nabla^2\phi=0,\text{ for } z<0\,;\ \phi\to 0,\text{ as } z\to-\infty\,;$$

$$\phi_t=g\zeta_0,\text{ on } z=0\text{ at } t=0\,;$$

which is equivalent to

$$\zeta_t=-\phi_z\text{ on } z=0$$

$$\phi_{tt}=-g\phi_z\text{ on } z=0. \qquad \text{(Lamb (1932))}.$$

Equation (3) may also be written as

$$\phi=\delta\tau.\ 2mg^{\frac{1}{2}}\int_0^{\infty}J_0(kr)\sin\sqrt{gk}t\,.\,e^{k(z-f)}\,.\,k^{\frac{1}{2}}\,dk. \quad .. \quad .. \quad (3a)$$

with $r=\sqrt{x^2+y^2}$, for

$$2J_0(kr)=\frac{1}{\pi}\int_{-\pi}^{\pi}e^{ik\bar{\omega}}\,d\theta.$$

We now proceed to consider the fluid motion due to a source moving in a horizontal circle of radius h at a depth f below the free surface, with constant angular velocity Ω. We shall use cylindrical co-ordinates taking the origin at the free surface vertically above the centre of the circle. If a source is created at time $t = \tau$ at the point whose cylindrical co-ordinates are $(h, \Omega\tau, -f)$, maintained for a period $\delta\tau$, and then annihilated, the velocity potential due to this transient source at any subsequent time t is given in accordance with (3a) by

$$\phi = 2mg^{\frac{1}{2}} . \delta\tau \int_0^\infty J_0(k\tilde{\omega}_1) \sin\left[\sqrt{gk}(t-\tau)\right] . e^{k(z-f)} k^{\frac{1}{2}}\, dk, \quad \ldots \quad \ldots \quad (4)$$

where $\tilde{\omega}_1 =$ perpendicular distance of the field point $P(\tilde{\omega}, \theta, z)$ from the vertical through the source. In the following figure P' is shown as the projection of P on the plane of the circle along which the source moves, and Q is the position of the source at time τ, and O' is the centre of the circle described by the source.

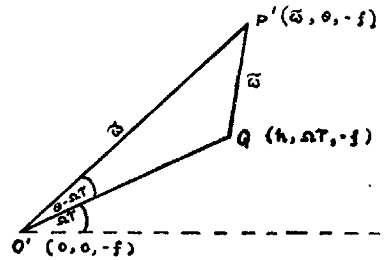

FIG. 1.

Hence

$$\tilde{\omega}_1^2 = \tilde{\omega}^2 + h^2 - 2\tilde{\omega}h \cos(\theta - \Omega\tau). \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

We may regard the fluid motion at time t as being made up of contributions of type (4), extending from $\tau = 0$ to $\tau = t$, together with that due to the sources m and $-m$ at the points $(h, \Omega t, -f)$ and $(h, \Omega t, f)$ respectively, these last two sources not being annihilated at time t. Hence

$$\phi(t) = \frac{m}{r_1} - \frac{m}{r_2} + 2g^{\frac{1}{2}} \int_0^t m(\tau)\, d\tau \int_0^\infty J_0(k\tilde{\omega}_1) . \sin\left[\sqrt{gk}(t-\tau)\right] . e^{k(z-f)} k^{\frac{1}{2}}\, dk, \quad \ldots \quad (6)$$

where r_1, r_2 represent the distances of $P(\tilde{\omega}, \theta, z)$ from $Q(h, \Omega t, -f)$, and $Q'(h, \Omega t, f)$ respectively, so that

$$r_1^2 = (\tilde{\omega} \cos\theta - h \cos\Omega t)^2 + (\tilde{\omega} \sin\theta - h \sin\Omega t)^2 + (z+f)^2$$

$$= \tilde{\omega}^2 + h^2 - 2\tilde{\omega}h \cos(\theta - \Omega t) + (z+f)^2$$

and similarly

$$r_2^2 = \tilde{\omega}^2 + h^2 - 2\tilde{\omega}h \cos(\theta - \Omega t) + (f-z)^2$$

and

$$\tilde{\omega}_1^2 = \tilde{\omega}^2 + h^2 - 2\tilde{\omega}h \cos(\theta - \Omega\tau). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

The results contained in equations (6) and (7) were obtained by Lunde (1951) in the paper already referred to.

3. Plane Distribution of Sources and Sinks

We shall use formula (6) to calculate the total disturbance due to the replacement of the ship by the source system. The ship, as we have stated before, is replaced by a rectangle forming its middle plane, which is supposed to be covered with sources of suitable density. The effect of the circular motion of the ship will be supposed to be equivalent to that of the motion of this rectangle without rotation such that its centre moves on the circle described by the centre of the ship. Every point of the rectangle will describe a circle. The rectangle in any position will in fact be contained within two concentric circular cylinders with the vertical axis through the centre as axis, and the midpoint of this rectangle will in any position touch the inner cylinder.

Let us now take a plane distribution of sources on the rectangle, the cylindrical co-ordinates of the centroid of the rectangle at time t being $(h, \Omega t, -f)$.

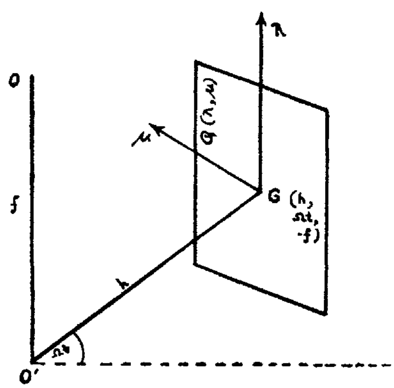

Fig. 2.

Let the horizontal and the vertical lines through the centroid be taken as a pair of axes (μ, λ) fixed in the moving plane. Let σ be the surface density of the distribution at any point $Q(\lambda, \mu)$. The cylindrical co-ordinates of this point Q with respect to the original fixed axes are

$$\sqrt{h^2+\mu^2},\ \Omega t+\tan^{-1}\frac{\mu}{h},\ -(f-\lambda).$$

Then the velocity potential due to this moving distribution of sources and sinks is obtained from (6) by writing $\sigma d\lambda d\mu$ for m and changing h, Ωt, and f into $\sqrt{h^2+\mu^2}$, $\Omega t+\tan^{-1}\frac{\mu}{h}$ and $f-\lambda$ respectively, and integrating over the entire distribution over the rectangle. Thus

$$\phi(t)=\iint \sigma\left(\frac{1}{r_1}-\frac{1}{r_2}\right)d\lambda\, d\mu$$

$$+2g^{\frac{1}{2}}\iint d\lambda\, d\mu\int_0^t \sigma d\tau\int_0^\infty J_0(k\tilde{\omega}_1)\sin\left[\sqrt{gk}(t-\tau)\right].e^{k(z-f+\lambda)}\ .k^{\frac{1}{2}}\,d_k. \quad (8)$$

where by r_1, r_2, $\tilde{\omega}_1$ we now mean

$$r_1^2 = \tilde{\omega}^2 + h^2 + \mu^2 - 2\tilde{\omega}\sqrt{h^2+\mu^2}\cos\left(\theta - \Omega t - \tan^{-1}\frac{\mu}{h}\right) + (z+f-\lambda)^2$$

$$r_2^2 = \tilde{\omega}^2 + h^2 + \mu^2 - 2\tilde{\omega}\sqrt{h^2+\mu^2}\cos\left(\theta - \Omega t - \tan^{-1}\frac{\mu}{h}\right) + (f-\lambda-z)^2$$

$$\tilde{\omega}_1^2 = \tilde{\omega}^2 + h^2 + \mu^2 - 2\tilde{\omega}\sqrt{h^2+\mu^2}\cos\left(\theta - \Omega \tau - \tan^{-1}\frac{\mu}{h}\right). \quad \ldots \quad \ldots \quad (9)$$

Geometrically $\tilde{\omega}_1$ represents the perpendicular distance of any point $P(\tilde{\omega}, \theta, z)$ in the fluid from an axis parallel to the z-axis through the position of the source at (λ, μ) at time τ.

4. Boundary Conditions

Let the hull of a slender ship-shaped body be given by

$$\nu = F(\lambda, \mu). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

The ship is considered so slender that ν may be regarded as small. Regarding the ship as represented by the plane distribution of sources referred to above we proceed to establish an approximate relation between the surface density of the distribution and the shape of the ship.

The approximate boundary condition at any point on the surface of the ship is as follows: (i) the normal fluid velocity at the point (λ, μ) on the rectangle = the normal component of the ship velocity at the corresponding point (λ, μ, ν) on the ship surface, (ii) the normal fluid velocity due to the plane distribution at any point (λ, μ) on the plane itself is $\pm 2\pi\sigma$, where σ is the surface density of sources at that point.

To consider the normal component of the ship velocity at the point (λ, μ, ν) on the surface of the ship, let us consider a section of the surface of the ship given by (10) by the plane $z = -f+\lambda$. Due to the motion of the ship this section rotates in a circle with its centre at its own level on the z-axis. This centre lies at a horizontal distance h from the z-axis.

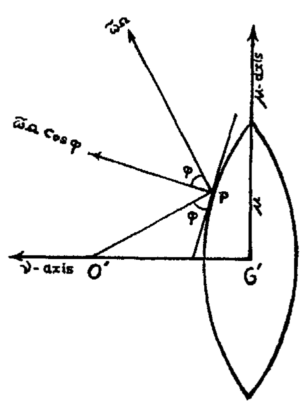

Fig. 3.

In figure 3, O', G' are points of intersection with the plane $z = -f+\lambda$ of the z-axis and the λ-axis respectively.

The ship velocity at the point (λ, μ, ν)

$$= \tilde{\omega}\Omega \text{ perpendicular to } O'P.$$

Its component along the normal to the side of the ship $= \tilde{\omega}\Omega \cos\phi$; $\tilde{\omega}$ and $\cos\phi$ may be obtained from the geometry (plane) of the section itself.

The tangent of the gradient (inclination to μ-axis) at P is $\dfrac{\partial\nu}{\partial\mu}$, and the tangent of the gradient of $O'P$ is $\dfrac{\nu-h}{\mu}$.

Hence

$$\tan\phi = \frac{\dfrac{\partial\nu}{\partial\mu}-\dfrac{\nu-h}{\mu}}{1+\dfrac{\partial\nu}{\partial\mu}\cdot\dfrac{\nu-h}{\mu}} = \frac{\dfrac{h}{\mu}-\dfrac{\nu}{\mu}+\dfrac{\partial\nu}{\partial\mu}}{1-\dfrac{h}{\mu}\dfrac{\partial\nu}{\partial\mu}}$$

$\left(\text{neglecting the second order term } \nu\dfrac{\partial\nu}{\partial\mu}\right)$; whence we obtain

$$\cos\phi = \frac{\mu-h\dfrac{\partial\nu}{\partial\mu}}{\sqrt{h^2+\mu^2}}\left(1+\frac{h}{h^2+\mu^2}\nu\right).$$

Further,

$$\tilde{\omega}^2 = (\nu-h)^2+\mu^2,$$

whence

$$\tilde{\omega} = \sqrt{h^2+\mu^2}\left(1-\frac{h\nu}{h^2+\mu^2}\right).$$

Hence

$$\tilde{\omega}\Omega\cos\phi = \Omega\left(\mu-h\frac{\partial\nu}{\partial\mu}\right). \qquad \ldots \quad \ldots \quad \ldots \quad (11)$$

This result is correct to the first order of approximation.

The connection between the density of the source distribution and the shape of the ship will be given by

$$2\pi\sigma = \Omega\left(\mu-h\frac{\partial\nu}{\partial\mu}\right)$$

$$\text{or} \quad \sigma = \frac{\Omega}{2\pi}\left(\mu-h\frac{\partial\nu}{\partial\mu}\right). \qquad \ldots \quad \ldots \quad \ldots \quad (12)$$

Here σ does not depend on time τ; substituting (12) in (8) we get

$$\phi(t) = \frac{\Omega}{2\pi}\iint\left(\mu-h\frac{\partial\nu}{\partial\mu}\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)d\lambda\,d\mu$$
$$+\frac{\Omega g^{\frac{1}{2}}}{\pi}\iint\left(\mu-h\frac{\partial\nu}{\partial\mu}\right)d\lambda\,d\mu\int_0^t d\tau\int_0^\infty J_0(k\tilde{\omega}_1)\sin\left[\sqrt{g}k(t-\tau)\right]e^{k(z-f+\lambda)}\,.\,k^{\frac{1}{2}}\,dk. \qquad \ldots \quad (13)$$

5. Wave Resistance

From Bernoulli's equation

$$\frac{p}{\rho}+\tfrac{1}{2}q^2+g\zeta-\phi_t = \text{constant},$$

the variable part of the pressure is given by

$$p = \rho\phi_t, \text{ neglecting } \tfrac{1}{2}q^2. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

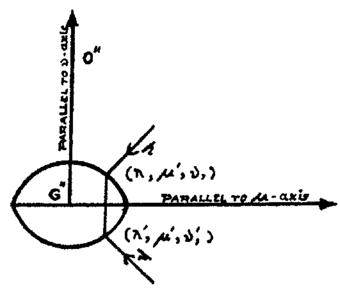

Fig. 4.

Let us consider the instantaneous position of the ship at any time. The figure 4 shows the section of this position by the horizontal plane $z = -f+\lambda'$, O'' and G'' being the points of intersection of the plane with the z-axis and the λ-axis respectively. If p be the pressure at the point (λ', μ', ν'), since the pressure at the corresponding point $(\lambda', \mu', -\nu')$ differs from it only by a small quantity owing to the thinness of the ship, the wave resistance may be defined as

$$R = -2\iint p\,\frac{\partial \nu'}{\partial \mu'}\,d\lambda'\,d\mu',$$

and to a first approximation the value of p may be taken at the point (λ', μ', O) on the central plane of the ship.

$$\therefore\quad R = -2\rho\iint \phi_t\,\frac{\partial \nu'}{\partial \mu'}\,d\lambda'\,d\mu', \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

the value of ϕ_t being taken at the point (λ', μ') of the distribution and the integration being taken over the entire distribution on the central plane of the ship. Equation (15) gives the wave resistance of the ship with ϕ given by (13), and r_1, r_2, $\tilde{\omega}_1$ involved in (13) given by (9). From (13) the first part of ϕ

$$= \frac{\Omega}{2\pi}\iint\left(\mu-h\frac{\partial \nu}{\partial \mu}\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)d\lambda\,d\mu$$

and its contribution to ϕ_t is

$$-\frac{\Omega}{2\pi}\iint\left(\mu-h\frac{\partial \nu}{\partial \mu}\right)\left(\frac{1}{r_1^2}\frac{\partial r_1}{\partial t}-\frac{1}{r_2^2}\frac{\partial r_2}{\partial t}\right)d\lambda\,d\mu.$$

Now from (9)

$$\frac{\partial r_1}{\partial t} = -\frac{\tilde{\omega}\Omega}{r_1}\sqrt{h^2+\mu^2}\,\sin\left(\theta-\Omega t-\tan^{-1}\frac{\mu}{h}\right)$$

and

$$\frac{\partial r_2}{\partial t} = -\frac{\tilde{\omega}\Omega}{r_2}\sqrt{h^2+\mu^2}\sin\left(\theta-\Omega t-\tan^{-1}\frac{\mu}{h}\right).$$

$\therefore$ the first part of ϕ_t

$$\frac{\tilde{\omega}\Omega^2}{2\pi}\iint\left(\mu-h\frac{\partial\nu}{\partial\mu}\right)\left(\frac{1}{r_1^3}-\frac{1}{r_2^3}\right)\sqrt{h^2+\mu^2}\;\sin\left(\theta-\Omega t-\tan^{-1}\frac{\mu}{h}\right)d\lambda\,d\mu$$

and its value is to be taken at the point (λ', μ') on the central plane, the cylindrical co-ordinates of the point being

$$\sqrt{h^2+\mu'^2},\quad \Omega t+\tan^{-1}\frac{\mu'}{h},\quad -f+\lambda'.$$

The values of r_1, r_2 in the above integral for ϕ_t may be obtained from (9) by substituting the above values for $\tilde{\omega}$, θ, and z. But these may be obtained more easily geometrically for r_1 and r_2 represent the distances of the point $P(\lambda', \mu')$ from the point $Q(\lambda, \mu)$ and its image point $Q'(2f-\lambda, \mu)$ in the free surface—all the three points lying in the central plane.

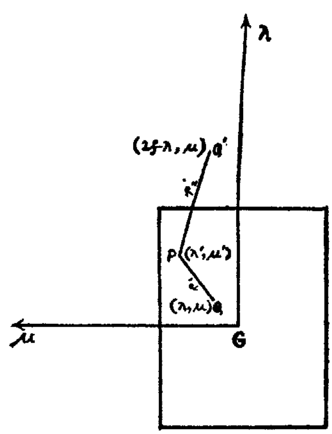

FIG. 5.

$$r_1^2 = (\lambda'-\lambda)^2+(\mu'-\mu)^2,$$
$$r_2^2 = (\lambda'+\lambda-2f)^2+(\mu'-\mu)^2.$$

Also

$$\sin\left(\theta-\Omega t-\tan^{-1}\frac{\mu}{h}\right) = \sin\left(\tan^{-1}\frac{\mu'}{h}-\tan^{-1}\frac{\mu}{h}\right) \text{ at } P(\lambda', \mu')$$

$$= \frac{h(\mu'-\mu)}{\sqrt{(h^2+\mu^2)(h^2+\mu'^2)}}.$$

Hence the value of the first part of ϕ_t at the point $P(\lambda', \mu')$ is

$$\frac{h\Omega^2}{2\pi}\iint\left(\mu-h\frac{\partial\nu}{\partial\mu}\right)(\mu'-\mu)\left[\left\{(\lambda'-\lambda)^2+(\mu'-\mu)^2\right\}^{-\frac{3}{2}}\right.$$
$$\left.-\left\{(\lambda'+\lambda-2f)^2+(\mu'-\mu)^2\right\}^{-\frac{3}{2}}\right]d\lambda\,d\mu$$

and hence the first part of R

$$= -\frac{\rho h\Omega^2}{\pi}\iint \frac{\partial \nu'}{\partial \mu'}\,d\lambda'\,d\mu' \iint \left(\mu - h\frac{\partial \nu}{\partial \mu}\right)(\mu'-\mu)\left[\{(\lambda'-\lambda)^2+(\mu'-\mu)^2\}^{-\frac{3}{2}} - \{(\lambda'+\lambda-2f)^2+(\mu'-\mu)^2\}^{-\frac{3}{2}}\right]d\lambda\,d\mu.$$

$$= -\frac{\rho h\Omega^2}{\pi}\iint \frac{\partial \nu'}{\partial \mu'}\,d\lambda'\,d\mu' \iint \mu(\mu'-\mu) \times \left[\{(\lambda'-\lambda)^2+(\mu'-\mu)^2\}^{-\frac{3}{2}} - \{(\lambda'+\lambda-2f)^2+(\mu'-\mu)^2\}^{-\frac{3}{2}}\right]d\lambda\,d\mu. \quad .. \quad (16)$$

for the other part of the integral will cancel out in pairs since the integral is to be taken twice over the distribution.
From (13) the second part of ϕ

$$= \frac{\Omega g^{\frac{1}{2}}}{\pi}\iint\left(\mu - h\frac{\partial \nu}{\partial \mu}\right)d\lambda\,d\mu\int_0^t d\tau\int_0^\infty J_0(k\tilde{\omega}_1)\sin\left[\sqrt{gk}(t-\tau)\right]e^{k(z-f+\lambda)}\,.\,k^{\frac{1}{2}}\,dk,$$

and remembering that

$$\frac{d}{d\alpha}\int_0^\alpha f(\beta,\alpha)d\beta = \int_0^\alpha \frac{d}{d\alpha}f(\beta,\alpha)d\beta + f(\alpha,\alpha)\frac{d\alpha}{d\alpha}, \quad .. \quad .. \quad (17)$$

we have the second part of ϕ_t

$$= \frac{g\Omega}{\pi}\iint\left(\mu - h\frac{\partial \nu}{\partial \mu}\right)d\lambda\,d\mu\int_0^t d\tau\int_0^\infty J_0(k\tilde{\omega}_1)\cos\left[\sqrt{gk}(t-\tau)\right]e^{k(z-f+\lambda)}\,.\,k\,dk,$$

and the value of this integral also is to be taken at the point (λ', μ') on the central plane, the cylindrical co-ordinates of the point being

$$\sqrt{h^2+\mu'^2},\ \Omega t + \tan^{-1}\frac{\mu'}{h},\ -f+\lambda'.$$

From (9) the value of $\tilde{\omega}_1$ at the above point is given by

$$\tilde{\omega}_1^2 = 2h^2+\mu^2+\mu'^2-2[(h^2+\mu\mu')\cos\Omega(t-\tau) - h(\mu'-\mu)\sin\Omega(t-\tau)]. \quad .. \quad (18)$$

Then the second part of $\phi_t(\lambda', \mu')$ is given by

$$\frac{g\Omega}{\pi}\iint\left(\mu - h\frac{\partial \nu}{\partial \mu}\right)d\lambda\,d\mu\int_0^t d\tau\int_0^\infty J_0(k\tilde{\omega}_1)\cos\left[\sqrt{gk}(t-\tau)\right]e^{k(\lambda'+\lambda-2f)}\,.\,k\,dk,$$

where $\tilde{\omega}_1$ is given by (18).
The second part of R

$$= -\frac{2g\rho\Omega}{\pi}\iint\frac{\partial \nu'}{\partial \mu'}\,d\lambda'\,d\mu'\iint\left(\mu - h\frac{\partial \nu}{\partial \mu}\right)d\lambda\,d\mu\int_0^t d\tau\int_0^\infty J_0(k\tilde{\omega}_1) \times \cos\left[\sqrt{gk}(t-\tau)\right].\,e^{k(\lambda'+\lambda-2f)}.\,k\,dk. \quad .. \quad (19)$$

Combining (16) and (19) the complete wave resistance is obtained as

$$R = -\frac{\rho h \Omega^2}{\pi} \iint \frac{\partial \nu'}{\partial \mu'} d\lambda' d\mu' \iint \mu(\mu'-\mu)$$

$$\times \left[\{(\lambda'-\lambda)^2+(\mu'-\mu)^2\}^{-\frac{3}{2}} - \{(\lambda'+\lambda-2f)^2+(\mu'-\mu)^2\}^{-\frac{3}{2}} \right] d\lambda\, d\mu$$

$$-\frac{2g\rho\Omega}{\pi} \iint \frac{\partial \nu'}{\partial \mu'} d\lambda'\, d\mu' \iint \left(\mu - h\frac{\partial \nu}{\partial \mu}\right) d\lambda\, d\mu \int_0^t d\tau \int_0^\infty J_0(k\tilde{\omega}_1)$$

$$\times \cos\left[\sqrt{gk}(t-\tau)\right] . e^{k(\lambda'+\lambda-2f)} k\, dk, \qquad \ldots \qquad (20)$$

where $\tilde{\omega}_1$ is given by (18).

The first term in the resistance is of the form const. × density × acceleration of the ship; the constant depends entirely on the dimensions of the ship. The second term is contributed by the source distribution equivalent to the ship and the waves generated.

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