A DYNAMIC TREATMENT OF THE PROBLEM OF PRESSURE ESTIMATION IN IMPACT SENSITIVITY EXPERIMENTS ON EXPLOSIVES

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Abstract

The solution of the spherical wave equation for an infinite elastic medium with an exponentially decaying pulse as a boundary condition at the cavity inside it, as provided by Goldsmith and Allen, has been applied to the estimation of peak pressures in impact sensitivity experiments making use of our own data and are compared with the values obtained experimentally. This is a 'dynamic' approach to the problem and is distinctly different from the static one based on the Hertz theory of impact.

1. Introduction

In a recent publication one of us (Murgai, 1956) has given a theoretical treatment of the problem of pressure estimation in impact sensitivity experiments on explosives. It is mainly based on the assumptions embodied in the Hertz theory of impact. This line of approach is a static treatment of the problem, the displacements of the colliding bodies, produced by the impulsive forces generated during impact, being calculated from the deformations under equivalent static loads. This assumption is justifiable for cases where the times of impact are much larger than those required for the waves to travel through the system, and has yielded reasonably correct results, under conditions satisfying this requirement. A more rigorous solution of the problem would come under the scope of the theory of propagation of stress waves in solids. This subject has received, comparatively speaking, less attention in recent years. The rôle of internal friction in metals, the dependence of the elastic constants on the rate of loading, etc., are not yet clearly understood. There exist, however, solutions for a few problems under some well defined assumptions. One of such cases is that of the problem of the propagation of pulses in infinite elastic media due to the detonation of an explosive in a small cavity in the medium. This has been solved by Goldsmith and Allen (1955). It consists, essentially, in the solution of the spherical wave equation for elastic media in terms of the scalar displacement potential, with an exponentially decaying pulse at the cavity as the boundary condition. These authors have provided expressions for the radial stress (in addition to tangential and shear stresses, particle and wave velocities, etc.), for various distances and times from the instant of explosion. We have adapted these results to the investigation of the problem of stress wave propagation in impact sensitivity experiments.

We propose to describe in the present paper an adaptation of the above theoretical results by Goldsmith and Allen (1955) to the investigation of the problem of stress wave propagation in impact sensitivity experiments, taking our own data (details reported elsewhere), and their comparison with our own experimental values as well as those by other workers (Taylor and Weale, 1938 ; Hollies *et al.*, 1953) on pressure estimations for mercury fulminate.

2. Theoretical

We describe in this section the basis of the approximation on which our dynamic treatment of the problem of pressure estimation is based.

Fig. 1(a) shows the positions that a falling sphere takes after striking an infinite plane. The arcs B, C and D correspond to the successive surfaces of deformation starting from the point A, when the impact first takes place. At this instant a wave of compression travels down the medium with a velocity equal to that of sound in steel. This wave is subsequently followed by waves of greater and greater amplitude which have come from their respective surfaces B, C and D. At the instant the wave from B is just started that from A is at A' (say). At a later instant when the wave from C is starting those from A and B are at A'' and B' respectively. If the arc D be taken to be that corresponding to the maximum deformation, the instant when the wave of maximum amplitude is at D, others will be at A''', B'' and C' respectively. After some time the waves will reach the point O. The region round O will therefore be subjected to a gradually rising pressure pulse. From the point of view of calculating the maximum pressure at the point O, therefore, one has to find the amplitude at this place of the pulse which originated from the surface D. Strictly speaking the radii of these surfaces of deformation will go on changing. But for small deformations—which will be the case while working within the framework of the elastic theory on which the analysis is based—and for larger radii* ($R \gg a$ the radius of the circle of contact—AP in Fig. 1(a)), the change in R may be neglected and the radius of the surface of deformation D considered equal to that of the ball itself with centre now at O_1.

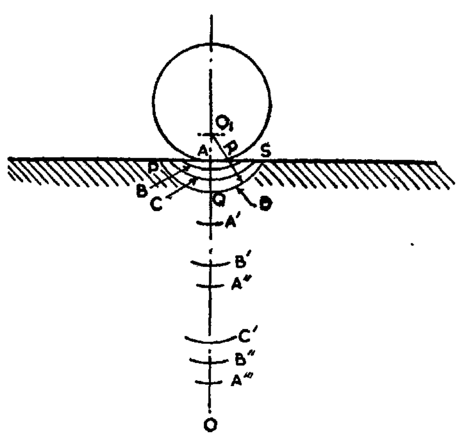

FIG. 1(a). Wave propagation in an infinite medium.

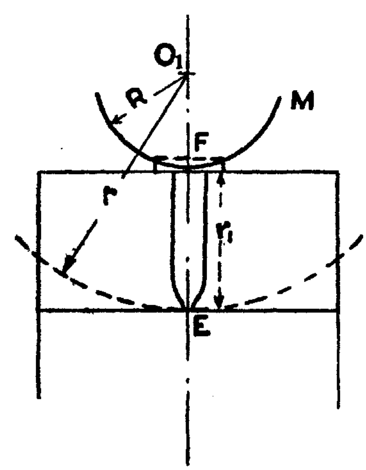

FIG. 1(b). Outline of the essentials of the impact machine.

Fig. 1(b) shows an outline of the essentials of our impact machine, E being the position of the explosive between the anvils under the striking pin at which we are interested in calculating the maximum pressure due to the falling ball M. This we do by making use of the results of Goldsmith and Allen (1955) for spherical wave propagation in an infinite medium due to a pulse applied at the cavity, the radius of the cavity being equal to that of the striking ball. This would imply disregarding the members of the assembly in Fig. 1(b) and assuming it to be a single piece. To what extent such an approximation would hold under actual conditions of experiments would depend upon the relative magnitude of the impacting spheres and the dimensions, etc., of the striking pins. We expect that for balls having radii small compared to the area of cross-sections of the pins, this may not

* This forms the basis of the analysis of deformations under distributed loads and its adaptation to the problem of impact of spheres. For details see Timoshenko and Goodier (1951).

be far from the truth. The calculated results seem to support this view. (It may be of interest to mention here that the value of the stress in the wave front in a conical bar at a certain distance due to an exponentially rising pulse, applied to the spherical surface of a certain radius at its apex, is given by an expression (London and Quinney, 1923) similar to that in equation (1), namely $P_{rd} = P_m \cdot \frac{R}{r}$). In order to make use of this solution we further assume that the pressure at the region of contact rises to the maximum value exponentially in half the time of impact. Since both the maximum pressure and the time of impact be determined from the impact parameters, the exponent of the exponential (corresponding to the decay constant in Goldsmith's problem) can thus be found out. With this, therefore, one can map out the network of stresses for various distances and times from the solution provided by the above authors. One thing however remains to be pointed out. The solution given by them is for a decaying pulse, while in the experiments on impact the pulse first rises and then falls subsequently. But, as would appear, the above results are based on the solution of a second order homogeneous differential equation which remains invariant under the transformation $t = -t$. The pressure and velocity profiles, therefore, will be symmetrical about their maxima. For the purpose of our present discussion, however, only the peak values of the radial stresses are relevant*, and the expressions for these do not involve the decay constant (this, in fact, is a confirmation of the above statement, namely the invariance of the original differential equation under the transformation $t = -t$). All the same this point deserves mention.

The value of the maximum radial stress under the dynamic condition is given by

$$P_{rd} = P_m \frac{R}{r} \quad \ldots \quad (1)$$

where†

$$P_m = 0{\cdot}135 \frac{a}{R(\phi_1+\phi_2)}$$

$$a = 1{\cdot}241\,[(\phi_1+\phi_2)mv^2R^2]^{\frac{1}{5}}$$

$$\phi_1+\phi_2 = 0{\cdot}318\left[\frac{1-\sigma_1}{(1-2\sigma_1)(K_1+1{\cdot}33G_1)}+\frac{1-\sigma_2}{(1-2\sigma_2)(K_2+1{\cdot}33G_2)}\right]$$

in which R is the radius of the ball, m its mass, v the velocity of ball, r the distance from the centre of the ball to the seat of explosive pellet (distance O_1E in Fig. 1(*b*)), E, K, G, and σ being the Young's modulus, bulk modulus, modulus of Shear and Poisson's ratio respectively. The subscripts 1 and 2 refer to the material of the ball and that of the impact machine.

The numerical values of the elastic constants used in the present calculation are given below :—

$$E = 21{\cdot}39\times10^{11} \text{ dynes per sq. cm.}$$

$$\sigma = 0{\cdot}31$$

$$K = \frac{E}{3(1-2\sigma)} = 18{\cdot}7\times10^{11} \text{ dynes per sq. cm.}$$

$$G = \frac{E}{2(1+\sigma)} = 8{\cdot}2\times10^{11} \text{ dynes per sq. cm.}$$

* The experimental pressures, with which these calculated stresses have been compared subsequently, correspond obviously to radial compressive stress.

† For details see Timoshenko and Goodier (1951, p. 383).

3. Results and Discussions

Table I below gives the values of P_{rd} as calculated* by equation (1) and are compared to the values obtained experimentally by the present authors as

Table I

Explosive	Weight of steel balls in gm.	Fifty per cent explosion efficiency heights in cm.	Maximum radial stress P_{rd} in dynes/cm.^{2} : 10^9		Experimental values dynes/cm.^{2} : 10^9
			1	2	
Mercury fulminate	101·90	39·60	13·4	3·16	1·56–1·95 (Hollies *et al.*, 1953).
	60·80	44·44	12·2	2·90	
	44·45	52·49	11·7	2·78	1·06 (Taylor and Weale, 1938).
	28·00	55·26	10·6	2·52	0·92–2·36 (present authors)
	19·00	67·20	8·9	2·04	

* The exact values of the heights of the pins r_1 in the experiments of Taylor and Weale (1932) and Hollies *et al.* (1953) are not available. We give below the calculated values of P_{rd} for some likely values of r_1 (for their experiments). These could not be very much different from the actual values. P_{rd} thus calculated is of the correct order.

Explosive	Weight of the ball in gm.	Height for 50 per cent explosives efficiency	r_1	Approximation A (maximum stress) in dynes/cm.^{2} : 10^9		Experimental values in dynes/cm.^{2} : 10^9
				1	2	
Mercury fulminate	113·4	4·32	1·0	14·13	3·35	1·56–1·95
			2·0	10·10	2·40	(Hollies *et al.*, 1953)
(Taylor and Weale, 1932)	14·2	35·6	1·0	15·36	3·65	
			2·0	9·78	2·33	1·06
(Hollies *et al.*, 1953)	44·65	65·50	1·0	21·4	4·9	(Taylor and Weale, 1938)
			2·0	14·5	3·3	
Trinitroresorcinate mixture	85·06	7·87	1·0	15·26	3·63	
			2·0	10·74	2·56	
(Taylor and Weale, 1932)	28·3	22·86	1·0	15·94	3·79	
			2·0	10·54	2·51	
Lead azide	66·68	105·0	1·0	24·8	5·75	2·8–3·5
			2·0	17·2	4·0	
(Hollies *et al.*, 1953)			3·0	13·2	3·05	(Hollies *et al.*, 1953)

well as by the other workers (Taylor and Weale, 1938; Hollies *et al.*, 1953). The dependence of the elastic constants on the rate of loading has been taken into account in a general way (Murgai, 1956) by finding two values of P_m and a—one for steel to steel impact and the other for steel to a solid having elastic constants one-tenth that of steel—expecting that the experimental pressures would lie in between the two values of P_{rd}, calculated corresponding to the two sets of P_m and a. These two values of P_{rd} (values of P_{rd} based on the dynamic treatment) are shown as 1 and 2 in the above table. One could obtain the theoretical mass vs. height relation, by substituting for P_{rd}, the experimentally measured value of pressure. This, in fact, would be just a reinterpretation of the above result.

Acknowledgement

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References

Goldsmith, W., and Allen, W. A. (1955). Spherical propagation of explosive pulses in elastic medium. *J. Acoust. Soc. Am.*, **27**, 47.
Hollies, N. R. S., Legge, N. R., and Morrison, J. L. (1953). The sensitivity of initiator explosives to mechanical impact. *Can. J. Chem.*, **31**, 746.
London, J. W., and Quinney, H. (1923). Experiments with the Hopkinson pressure bar. *Proc. Roy. Soc.* (London), **A103**, 622.
Murgai, M. P. (1956). On the pressure estimation in impact sensitivity experiments on explosives and the problem of initiation. *J. Chem. Phys.*, **25**, 762.
Taylor, W., and Weale, A. (1932). The mechanism of the initiation and propagation of detonations in solid explosives. *Proc. Roy. Soc.* (London), **A138**, 92.
———(1938). Conditions for the initiation and propagation of detonation in solid explosives. *Trans. Faraday Soc.*, **34**, 995.
Timoshenko, S., and Goodier, J. N. (1951). Theory of Elasticity, pp. 372–384. McGraw-Hill Book Company Inc.

ERRATA

Paper by M. P. Murgai and A. K. Ray entitled 'A dynamic treatment of the problem of pressure estimation in the impact sensitivity experiments on explosives' (published in the Proceedings, Vol. 24, A, No. 2, March 1958).

Page 104. Table in foot-note, column 4

read $\dfrac{r_1}{\text{cm.}}$ *in place of* r_1

Page 104. Table in foot-note, column 5

read Maximum stress in dynes/cm.^{2} : 10^9 *in place of* Approximation A (maximum stress) in dynes/cm.^{2} : 10^9