ON RELATION BETWEEN SPECTRAL NUMBER AND ULTRASONIC WATTAGE FROM RAMAN AND NATH'S THEORY

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ABSTRACT

Raman and Nath's relationship for the distribution of intensity in the ultrasonic spectra has been critically studied, with a view to finding out the relation between the observed spectral number and ultrasonic wattage. It is shown that no explicit relationship is obtainable and a graphical representation based on any particular minimum intensity for observation is possible. The graphical relation is non-linear for smaller order numbers and is linear for higher order numbers.

INTRODUCTION

According to Raman and Nath's theory (1936) of diffraction of light by sound waves in liquids the intensity of the nth spectral order is proportional to $J_n^2(v)$ where $J_n(v)$ is the Bessel function of nth order and v is given by $\frac{2\pi\Delta\mu l}{\lambda}$, where $\Delta\mu$ is the maximum variation of refractive index and is proportional to the square root of ultrasonic intensity, λ is the wave-length of the incident light, and l is the length of the light path in the liquid through ultrasonic beam. The theoretical relation has been found to be well substantiated by the experiment of Sanders (1936).

Raman's relation, as it stands, is, however, not well suited to give us any direct relationship between the spectra observed and the values of 'v' or the ultrasonic wattage applied. Samal *et al.* (1955) suggested empirically that the ultrasonic wattage can be related with the visible spectral number in the form

$$w = bn^2 + cn^3$$

where w is the input electrical wattage, b and c are constants. This n-w graph came in coincidence with n-v^2 graph obtained from Raman and Nath's diagrammatic representation of ultrasonic spectral intensity. But Sanders had suggested from his experiment that n-v graph is linear. It is to be discussed here whether a relationship is obtainable between n and v from Raman and Nath's theory.

DISCUSSION OF THE THEORY

Any decision regarding the observed spectral number, however, has associated with it a minimum intensity or the threshold intensity for the visibility of a spectrum. Sanders took the threshold intensities amounting to 0·86 per cent and 0·57 per cent of the unscattered beam. On expansion of the function $J_n^2(v)$ according to Neumann (Watson, 1944) we have

$$J_n^2(v) = \frac{(\frac{1}{2}v)^{2n}}{(n!)^2}\left[1 - \frac{T_2 v^2}{1\,.\,(2n+1)} + \frac{T_4 v^4}{1\,.\,2\,.\,(2n+1)(2n+2)} - \cdots\right]$$

where

$$T_2 = \frac{2n+1}{2n+2},\; T_4 = \frac{(2n+1)(2n+3)}{(2n+2)(2n+4)},\; T_6 = \frac{(2n+1)(2n+3)(2n+5)}{(2n+2)(2n+4)(2n+6)}.$$

For a particular threshold intensity of visibility the relation may be expressed in the form

$$J_n^2(v) = I_{\min} = \text{const} = \frac{(\frac{1}{2}v)^{2n}}{(n\,!)^2}\left[1-\frac{v^2}{(2n+2)}+\frac{(2n+3)v^4}{2(2n+2)^2(2n+4)}-\ldots\right]$$

where $I_{\min}$ is the fixed minimum intensity in terms of the unscattered beam intensity. It is clear from the above equation that n and v cannot be put into a linear relationship. It is also not possible to separate them such that n can be expressed explicitly in terms of v. One is thus forced to have only an indirect approach to the solution of the problem and to see if any graphical representation can be obtained from the theoretical relationship.

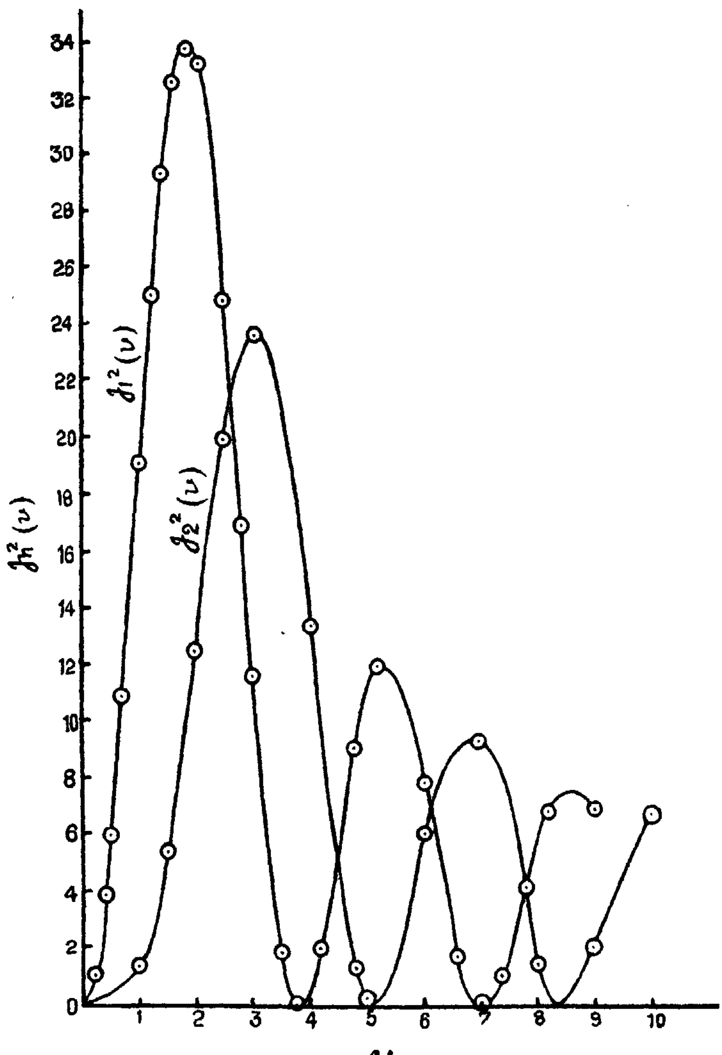

FIG. 1. Plot of $J_n^2(v)-v$ when $n = 1$ and 2.

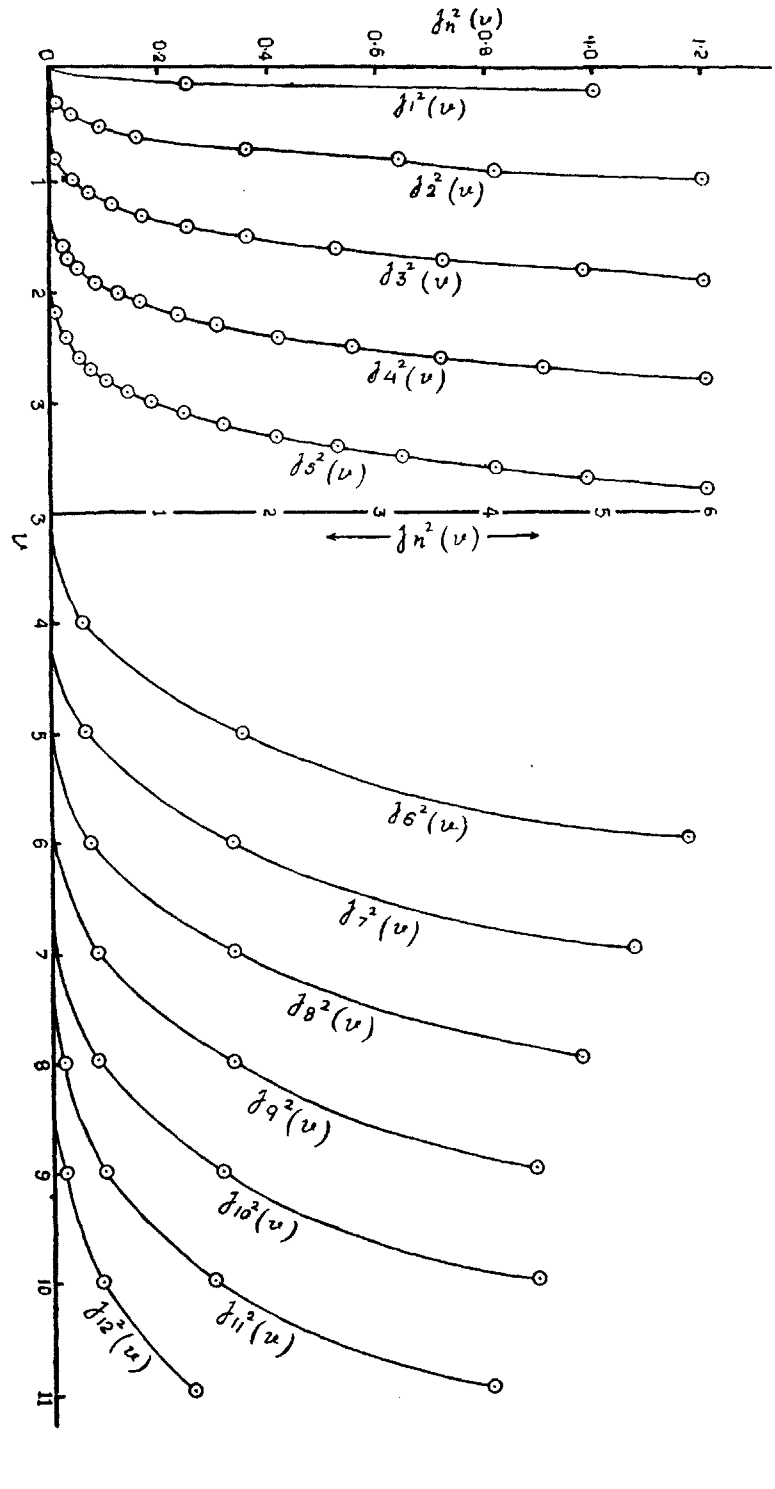

FIG. 2. Plot of $J_n^2(v)$ in per cent of intensity of unscattered beam against v in two different scales. n varies from 1 to 5 in one scale and from 6 to 12 in the other.

The graphical representation of $J_n^2(v)$ against v for any particular value of n is of a fluctuating nature as shown in Fig. 1. So for any particular value of n a minimum or threshold intensity, i.e. constant value of $J_n^2(v)$, will be satisfied for a number of v-values. We are, however, concerned not with all the values of v to give us a particular minimum intensity but the minimum v-value which would be required to give us the limiting intensity I_{min}, for a particular order number n. This will determine the first appearance of spectrum. It has to be noted that this minimum v-value increases with the order number n as will be clear from Fig. 2 where Fig. 1 has been redrawn with only the initial stages of $J_n^2(v)$ against v for greater range of n-values.

Fixing up any I_{min}-value as the threshold intensity for observation, a graph can be drawn from Fig. 2 between the spectral number and the minimum v-values required for their first appearance. With threshold intensities of 0·4 per cent, 0·57 per cent and 0·86 per cent v-values are calculated from Fig. 2 and are given in Table I. The graphs have been drawn in Fig. 3.

TABLE I

n-values	Minimum v-values		
	$J_n^2(v) = 0{\cdot}4\%$	$J_n^2(v) = 0{\cdot}57\%$	$J_n^2(v) = 0{\cdot}86\%$
1	0·14	0·15	0·18
2	0·73	0·80	0·93
3	1·54	1·61	1·78
4	2·40	2·50	2·70
5	3·30	3·42	3·64
6	4·25	4·30	4·56
7	5·23	5·30	5·66
8	6·10	6·28	6·56
9	7·03	7·22	7·50
10	8·00	8·22	8·50
11	9·00	9·18	9·50
12	10·20	10·25	10·66

The courses of the curves are similar in nature and show throughout that the graph is non-linear for smaller spectral orders and tends to be linear for larger values of n. They show also that for the threshold intensities, utilized by Sanders, a linear relationship between the spectral number and ultrasonic wattage is not at all possible from the point of view of Raman and Nath's theory. On the other hand the diagrammatic representation of Raman and Nath's tabulated intensities gives rise to a graph which is in agreement with the n-v graph obtained here with 0·4 per cent as the threshold intensity.

To verify the graphical relationship, so obtained, the visible spectral number was noted at different distances from the ultrasonic source for a number of liquids. The ultrasonic wattages at different distances were calculated by Samal (1957) on utilization of this n-v graph with I_{min}-value as 0·4 per cent of the unscattered beam. The calculated values of ultrasonic absorption coefficients for the liquids are in good agreement with that of other methods.

It has, however, to be understood that there must be a limitation of the total spectral number observed with higher and higher v-values, i.e. with higher and higher ultrasonic wattages. As the necessary limit of the threshold intensity is fixed at any particular value, say 0·4 per cent of the unscattered beam, and also as the intensities of many of the spectral orders will be larger than the threshold

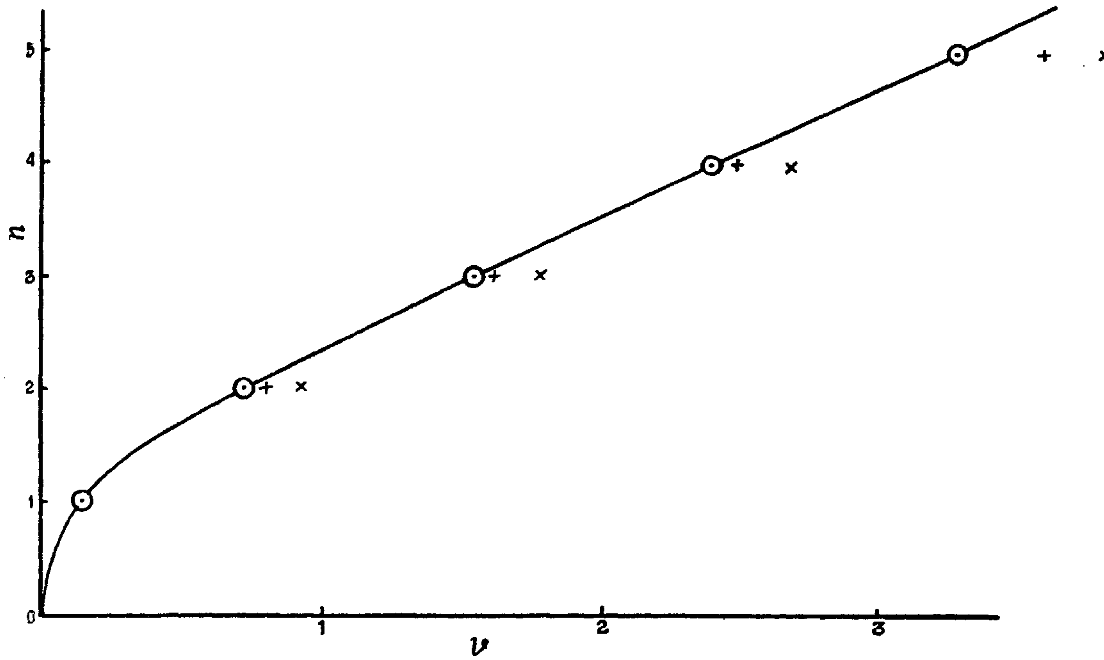

FIG. 3. A graph between n and minimum v when $I_{min} = 0{\cdot}4$ per cent. Points indicated by + and × refer to $I_{min} = 0{\cdot}57$ per cent and $0{\cdot}86$ per cent respectively.

intensity, one cannot go beyond a certain limiting number of spectra, however high the ultrasonic wattage may be. This will evidently be limited by the strength of the light source. It may be noted that the light sources with which we are working are generally very intense and the ultrasonic energy is sufficiently weak, so that the region of the limitation of the spectral number is not reached in ordinary experimental conditions.

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