SCATTERING OF A LONGITUDINALLY POLARISED ELECTRON BEAM BY A UNIFORM MAGNETIC FIELD

by HARIDAS BANERJEE, *Research Scholar, Department of Physics, Indian Institute of Technology, Kharagpur*

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ABSTRACT

Polarisation effects of the scattering of longitudinally polarised electron beam by a uniform magnetic field has been considered. In evaluating the differential cross-section for the scattering process the S-matrix formalism of Feynman and Dyson has been followed. Detailed analysis of the results has been given for two special cases, *viz.* (1) when the magnetic field is longitudinal and (2) when it is transverse with respect to the momentum of the incident beam. Finally, it has been indicated how by detecting the polarisation of the scattered beam by an analyser the magnetic moment of a free electron could be determined.

1. INTRODUCTION

In their paper on double-scattering of electrons with intervening magnetic field Mendlowitz and Case (1955) have considered in detail the spin kinematics of an electron with anomalous magnetic moment in the presence of a magnetic field. From the Heisenberg equations of motion of the spin-operator in the Foldy-Wouthuysen representation they have obtained how the spin orientation of an electron changes with time in the presence of a magnetic field. Tolhoek and De Groot (1951) have also investigated how a uniform magnetic field changes the spin-direction of an electron with the help of solutions of Dirac's equation up to the first order in the magnetic field. More recently, the same phenomenon has been studied by Carrassi (1957) using exact solutions of Dirac's equation in the presence of a uniform transverse magnetic field, given by Huff (1931).

The present work is a digression from the above both in the nature of the phenomenon studied and in its treatment. Here we consider the scattering of a longitudinally polarised electron beam by a static and uniform magnetic field which is non-vanishing only in a bounded region. The polarisation-dependent differential cross-section for the scattering has been obtained with the help of the S-matrix formalism of Feynman and Dyson. The fact that a magnetic field cannot polarise an unpolarised beam of electrons requires that in any experiment designed for studying the effects of magnetic field on electron polarisation, the incident beam should be polarised. Previously the possible sources of polarised electrons, known to us, were the electrons scattered by the Coulomb field of the nucleus and the recoil electrons in Compton scattering. Recently, the suggestion of Lee and Yang (1956), that electrons emitted in β-decay are longitudinally polarised with degree of polarisation $-v/c$, and its later experimental verification by Wu *et al.* (1957) with electrons from Co^{60} nuclei has opened up before us a potential source of polarised electron beam which can be directly scattered by a magnetic field and the effects of the field on the spin orientations of the electrons can be studied. Therefore, the incident beam has been taken to be longitudinally polarised.

Explicit expressions for the orientation of the polarisation vector of the scattered electrons have been obtained in two cases : (1) when the magnetic field is longitudinal and (2) when it is transverse to the beam. It has been found that the degree of polarisation remains unaltered by the scattering, whereas the relative orientation between the directions of polarisation and momentum changes. It is thus possible in both the cases to obtain transversely polarised electron beams for particular values of the scattering angle. The polarisation direction, in the first case, does not depend on the magnetic field. In the second case, however, the polarisation direction of the scattered beam depends on the magnetic field and the magnetic moment of the electron. It has also been indicated how the detection of this polarisation by an analyser which consists of Coulomb scattering at heavy nuclei may yield values of the magnetic moment of free electrons.

2. S-MATRIX ELEMENT AND DIFFERENTIAL CROSS-SECTION

In this paper the natural system of units has been adopted with $\hbar = c = 1$. We choose the fundamental metric tensor $g_{\mu\nu}$, defined by

$$g_{11} = g_{22} = g_{33} = -g_{00} = 1$$

and

$$g_{\mu\nu} = 0, \text{ when } \mu \neq \nu.$$

The iteration solution of the S-operator can be written as

$$S = 1 - i \int \mathscr{H}(t)\, dt \qquad (1)$$

where $\mathscr{H}(t)$, the interaction operator between the electron field ψ and the external electromagnetic field ϕ_μ, is given by

$$\mathscr{H}(t) = ie\,. \int \bar{\psi}\gamma_\mu \psi \phi^\mu\, d^3x \qquad (2)$$

The static magnetic field $\boldsymbol{H}$ is uniform in the region

$$-l_1 \leqslant x_1 \leqslant l_1\,;\; -l_2 \leqslant x_2 \leqslant l_2\,;\; -l_3 \leqslant x_3 \leqslant l_3$$

and vanishes elsewhere. The dependence of ϕ_μ on $\boldsymbol{H}$ can be expressed as

with
$$\left.\begin{aligned} \boldsymbol{H} &= (\phi_{23}, \phi_{31}, \phi_{12}), \\ \phi_{\mu\nu} &= \partial_\mu \phi_\nu - \partial_\nu \phi_\mu. \end{aligned}\right\} \qquad (3)$$

We therefore take $\phi_\mu(\phi_0, \boldsymbol{\phi})$ such that

$$\phi_0 = 0, \quad \boldsymbol{\phi} = \tfrac{1}{2}\boldsymbol{H} \times \boldsymbol{r} \qquad (4)$$

We now decompose the current operator, *i.e.*, $-ie\,\bar{\psi}\gamma_\mu\psi$ by a well-known method into convection and polarisation current

$$-ie\,\bar{\psi}\gamma_\mu\psi = \frac{ie}{2m}\,.\left\{\bar{\psi}\partial_\mu\psi - \partial_\mu\bar{\psi}\psi\right\} - \frac{e}{2m}\,.\,\partial^\nu\left\{\bar{\psi}\sigma_{\mu\nu}\psi\right\}, \qquad (5)$$

where

$$\sigma_{\mu\nu} = \frac{1}{2i}\left(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right).$$

From (1) and (5) we have

$$S = 1 - \frac{e}{2m}\,.\int\left(\bar{\psi}\partial_\mu\psi - \partial_\mu\bar{\psi}\psi\right)\phi^\mu\, d^4x - \frac{ie}{2m}\,.\int \partial^\nu\left(\bar{\psi}\,\sigma_{\mu\nu}\psi\right)\phi^\mu\, d^4x + \ldots.$$

Partial integration then gives

$$S = 1 - \frac{e}{2m}\int (\bar{\psi}\partial_\mu\psi - \partial_\mu\bar{\psi}\psi)\, d^4x - \frac{ie}{2m}\cdot\frac{1}{2}\cdot\int \bar{\psi}\sigma_{\mu\nu}\psi\phi^{\mu\nu}\, d^4x + \ldots.$$

The S-matrix element in the momentum space for the scattering of an electron with momentum $\boldsymbol{p}_1$ into a state with momentum $\boldsymbol{p}_2$, up to the first order in the external field, is

$$(p_2 \,|\, S \,|\, p_1) = -\left(p_2 \left| \left\{ \frac{e}{2m}\int (\bar{\psi}\partial_\mu\psi - \partial_\mu\bar{\psi}\psi)\, \phi^\mu\, d^4x + \frac{i}{2}\mu_0 \int \bar{\psi}\,\sigma\,\psi\,.\,\boldsymbol{H}\, d^4x \right\} \right| p_1\right) \qquad (6)$$

where $\mu_0 = \dfrac{e}{2m}$. From analogy with the non-relativistic theory we identify μ_0 with the magnetic moment of the electron without radiative corrections.

We take the plane-wave decomposition of the spinor-field given by Jauch (1955, p. 61),

$$\psi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int d^3p \sqrt{\frac{m}{\epsilon}}\, \{a_r(\boldsymbol{p})\, u_r(\boldsymbol{p})\, e^{ip\cdot x} + b_r^*(\boldsymbol{p})\, v_r(\boldsymbol{p})\, e^{-ip\cdot x}\}$$

where, as usual, ϵ is the energy and m is the mass of the particle; a_r, b_r are the annihilation operators of the electron and its antiparticle respectively and u, v are the scalar spinor-amplitudes. With this decomposition of the field we, therefore, get

$$\left(p_2 \left| \frac{e}{2m}\int (\bar{\psi}\partial_\mu\psi - \partial_\mu\bar{\psi}\psi)\, d^4x \right| p_1\right)$$
$$= \frac{ie}{2m}\cdot\frac{1}{(2\pi)^3}\sqrt{\frac{m^2}{\epsilon_2\epsilon_1}}\, \bar{u}(\boldsymbol{p}_2)\, u(\boldsymbol{p}_1)\cdot(p_1+p_2)_\mu \int \phi^\mu(x)\, e^{-i(p_2-p_1)\cdot x}\,.\, d^4x.$$

But

$$\left.\begin{aligned} \int \phi^\mu(x)\, e^{-i(p_2-p_1)\cdot x}\, d^4x &= 2\pi i\delta(\epsilon_2-\epsilon_1)\,\eta(\boldsymbol{H}+\boldsymbol{Q})^\mu, &&\text{if } \mu \neq 0 \\ &= 0, &&\text{if } \mu = 0 \end{aligned}\right\} \quad \ldots \qquad (7)$$

where

$$\left.\begin{aligned} \eta &= 4\prod_{i=1}^{3} \frac{\sin\,(p_2-p_1)_i\, l_i}{(p_2-p_1)_i} \\ Q_i &= l_i \cot\,(p_2-p_1)_i\, l_i - \frac{1}{(p_2-p_1)_i}, \quad (i = 1, 2, 3) \end{aligned}\right\} \quad \ldots \quad \ldots \qquad (8)$$

and

so that

$$\left(p_2 \left| \frac{e}{2m}\cdot\int (\bar{\psi}\partial_\mu\psi - \partial_\mu\bar{\psi}\psi)\, d^4x \right| p_1\right)$$
$$= -\frac{1}{(2\pi)^2}\cdot\frac{e}{2m}\sqrt{\frac{m^2}{\epsilon_2\epsilon_1}}\, \delta(\epsilon_2-\epsilon_1)\,\eta(\boldsymbol{H}\times\boldsymbol{Q})\cdot(\boldsymbol{p}_1+\boldsymbol{p}_2)\,\bar{u}(\boldsymbol{p}_2)\,u(\boldsymbol{p}_1) \quad \ldots \qquad (9)$$

We also obtain

$$\left(p_2 \left| \frac{i\mu_0}{2}\int \bar{\psi}(\boldsymbol{\sigma}\,.\,\boldsymbol{H})\psi\, d^4x \right| p_1\right)$$
$$= \frac{i\mu_0}{(2\pi)^2}\sqrt{\frac{m^2}{\epsilon_2\epsilon_1}}\, \delta(\epsilon_2-\epsilon_1)\,\eta\bar{u}(\boldsymbol{p}_2)(\boldsymbol{\sigma}\,.\,\boldsymbol{H})\, u(\boldsymbol{p}_1) \quad \ldots \quad \ldots \qquad (10)$$

Therefore, from (6) we get

$$(p_2|S|p_1) = \delta(\epsilon_2-\epsilon_1)(p_2|M|p_1)$$
$$= \delta(\epsilon_2-\epsilon_1)\cdot\frac{\eta}{(2\pi)^2}\sqrt{\frac{m^2}{\epsilon_2\epsilon_1}}\left\{\frac{e}{2m}\bar{u}(\boldsymbol{p}_2)\,u(\boldsymbol{p}_1)(\boldsymbol{H}\times\boldsymbol{Q})(\boldsymbol{p}_1+\boldsymbol{p}_2) - i\mu_0\,\bar{u}(\boldsymbol{p}_2)(\boldsymbol{\sigma}\,.\,\boldsymbol{H})\,u(\boldsymbol{p})\right\} \quad .. \quad (11)$$

The matrix element (11) corresponds to the Feynman diagram in momentum space given in Fig. 1.

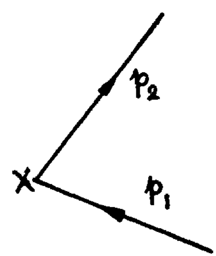

FIG. 1

It can be shown that the contributions of the first-order radiative corrections to this process, which do not vanish in the non-relativistic limit, can be taken into account, if one writes μ in place of μ_0 in (11), μ being given by Jauch (1955, p. 345)

$$\mu = \left(1+\frac{\alpha}{2\pi}\right)\mu_0, \quad .. \quad .. \quad .. \quad .. \quad (12)$$

where α is the fine-structure constant. Henceforth we shall therefore write μ in place of μ_0.

From the usual formula for the differential cross-section we have

$$d\sigma = \frac{(2\pi)^2}{\beta}\left[\int \delta(\epsilon_2-\epsilon_1)\,|\,(p_2|M|p_1)\,|^2\,|\boldsymbol{p}_2|\,\epsilon_2\,d\epsilon_2\right]d\Omega$$
$$= \frac{\eta^2 m^2}{(2\pi)^2}\left|\frac{e}{2m}\bar{u}(\boldsymbol{p}_2)\,u(\boldsymbol{p}_1)(\boldsymbol{H}\times\boldsymbol{Q})\,.\,(\boldsymbol{p}_1+\boldsymbol{p}_2) - i\mu_0\bar{u}(\boldsymbol{p}_2)(\boldsymbol{\sigma}\,.\,\boldsymbol{H})\,u(\boldsymbol{p}_1)\right|^2 d\Omega \quad .. \quad (13)$$

where $|\boldsymbol{p}_2| = |\boldsymbol{p}_1| = \beta\gamma m = \beta\epsilon$; β is the velocity of the electron in our system of units and $\gamma = (1-\beta^2)^{-\frac{1}{2}}$. Now

$$\left|\frac{e}{2m}\bar{u}(\boldsymbol{p}_2)\,u(\boldsymbol{p}_1)(\boldsymbol{H}\times\boldsymbol{Q})\,.\,(\boldsymbol{p}_1+\boldsymbol{p}_2) - i\mu_0\,\bar{u}(\boldsymbol{p}_2)(\boldsymbol{\sigma}.\,\boldsymbol{H})\,u(\boldsymbol{p}_1)\right|^2$$
$$= Tr[\xi^2 u(\boldsymbol{p}_2)\,\bar{u}(\boldsymbol{p}_2)\,u(\boldsymbol{p}_1)\,\bar{u}(\boldsymbol{p}_1) + i\xi\mu\,u(\boldsymbol{p}_2)\,\bar{u}(\boldsymbol{p}_2)\{(\boldsymbol{\sigma}\,.\,\boldsymbol{H})\,u(\boldsymbol{p}_1)\,\bar{u}(\boldsymbol{p}_1) - u(\boldsymbol{p}_1)\,\bar{u}(\boldsymbol{p}_1)(\boldsymbol{\sigma}\,.\,\boldsymbol{H})\} + \mu^2 u(\boldsymbol{p}_2)\,\bar{u}(\boldsymbol{p}_2)(\boldsymbol{\sigma}\,.\,\boldsymbol{H})\,u(\boldsymbol{p}_1)\,\bar{u}(\boldsymbol{p}_1)(\boldsymbol{\sigma}\,.\,\boldsymbol{H})] \quad .. \quad (14)$$

where

$$\xi = -\frac{e}{2m}(p_1+p_2)\cdot(H\times Q) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

The matrix $u(p_1)\bar{u}(p_1)$ is, in fact, the density matrix ρ of the electron in the momentum-state p_1. Explicit representation of the density matrix in terms of the energy-momentum 4-vector and the polarisation direction $\boldsymbol{\zeta}_1$ has been given by Tolhoek and Lipps (1953). If we define the unit vector $\boldsymbol{\zeta}$ to coincide with the direction of the spin-angular momentum of the electron in the Lorentz frame of reference in which it is at rest, then it can be shown that

$$u(p)\,\bar{u}(p) = \frac{1}{4}\left\{1+\frac{\epsilon}{m}\rho_3 - i\frac{(p\,.\,\rho_1\sigma)}{m} - i\frac{(p\,.\,\boldsymbol{\zeta})}{m}\rho_1 - i\frac{(p\times\boldsymbol{\zeta})\,.\,\rho_2\sigma}{m} + K\,.\,\rho_3\sigma + J.\sigma\right\} \qquad \ldots \quad \ldots \quad (16)$$

where

$$\left.\begin{aligned} K &= \boldsymbol{\zeta}+\frac{(\boldsymbol{\zeta}\,.\,p)p}{m(\epsilon+m)}, \\ J &= \frac{\epsilon}{m}\boldsymbol{\zeta}-\frac{(\boldsymbol{\zeta}\,.\,p)p}{m(\epsilon+m)} \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

and ϵ is the energy. Here we have made use of the standard representation of Dirac's γ-matrices, *viz.*, $\boldsymbol{\gamma}=\rho_1\sigma$, $\gamma^0 = -i\rho_3$. The density matrix for a partially polarised electron beam may be represented by the same expression as in (16) except that here one writes $\tau\boldsymbol{\zeta}$ instead of $\boldsymbol{\zeta}$, where τ ($\leqslant 1$) is the degree of polarisation. The justification for this procedure rests on the fact that a partially polarised state may be regarded as the incoherent superposition of electron-states with the same momentum but different polarisation directions. The density matrix of the resultant state is thus the average of the density matrices of the constituent states with different polarisation directions. From the linear dependence of ρ's on $\boldsymbol{\zeta}_i$'s it is thus evident one can write

$$\rho = \frac{1}{n}\sum_{i=1}^{n}\rho(\boldsymbol{\zeta}_i) = \rho\left(\frac{1}{n}\sum_{i=1}^{n}\boldsymbol{\zeta}_i\right) = \rho(\tau\boldsymbol{\zeta}).$$

In the present case the incident beam is taken to be longitudinally polarised with the degree of polarisation τ. So that from (16) and (17) we get

$$u(p_1)\,\bar{u}(p_1) = \tfrac{1}{4}\{1-i\beta\gamma(n_1\,.\,\rho_1\sigma)+\gamma\rho_3-i\beta\gamma\tau\rho_1+K_1\,.\,\rho_3\sigma+J\,.\,\sigma\} \quad \ldots \quad (18a)$$

with

$$K_1 = \tau\gamma n_1 \text{ and } J_1 = \tau n_1 \qquad \ldots \quad \ldots \quad \ldots \quad (18b)$$

where $p_1 = \beta\gamma m n_1$. The density matrix $u(p_2)\,\bar{u}(p_2)$ corresponds to that of the detector which receives the electrons scattered with momentum $p_2 = |p_2|\,n_2 = \beta\gamma m n_2$ and polarised in the direction $\boldsymbol{\zeta}_2$. We, therefore, have

$$u(p_2)\,\bar{u}(p_2) = \tfrac{1}{4}\{1+\gamma\rho_3-i\beta\gamma(n_2\,.\,\boldsymbol{\zeta}_2)\rho_1-i\beta\gamma(n_2\,.\,\rho_1\sigma) -i\beta\gamma(n_2\times\boldsymbol{\zeta}_2)\,.\,\rho_2\sigma+K_2\,.\,\rho_3\sigma+J_2\,.\,\sigma\} \quad \ldots \quad \ldots \quad (19a)$$

with

and

$$\left.\begin{aligned} K_2 &= \boldsymbol{\zeta}_2+(\gamma-1)(n_2\,.\,\boldsymbol{\zeta}_2)n_2, \\ J_2 &= \gamma\boldsymbol{\zeta}_2-(\gamma-1)(n_2\,.\,\boldsymbol{\zeta}_2)n_2 \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad (19b)$$

From (13) we now get

$$\frac{d\sigma}{d\Omega} = \left(\frac{\eta m}{4\pi}\right)^2 [\xi^2\{1+\gamma^2-\beta^2\gamma^2(\boldsymbol{n}_1 \,.\, \boldsymbol{n}_2)-\beta^2\gamma^2\tau(\boldsymbol{n}_2 \,.\, \boldsymbol{\zeta}_2)+(\boldsymbol{K}_1 \,.\, \boldsymbol{K}_2)+\boldsymbol{J}_1 \,.\, \boldsymbol{J}_2\}$$
$$+2\xi\mu\{\beta^2\gamma^2\boldsymbol{n}_2 \,.\, (\boldsymbol{H}\times\boldsymbol{n}_1)-\boldsymbol{K}_2 \,.\, (\boldsymbol{H}\times\boldsymbol{K}_1)-\boldsymbol{J}_2 \,.\, (\boldsymbol{H}\times\boldsymbol{J}_1)\}$$
$$+\mu^2\{H^2(1+\gamma^2+\beta^2\gamma^2(\boldsymbol{n}_1 \,.\, \boldsymbol{n}_2)-\beta^2\gamma^2\tau(\boldsymbol{n}_2 \,.\, \boldsymbol{\zeta}_2)-(\boldsymbol{K}_1 \,.\, \boldsymbol{K}_2)-(\boldsymbol{J}_1 \,.\, \boldsymbol{J}_2))$$
$$+2((\boldsymbol{J}_1 \,.\, \boldsymbol{H})(\boldsymbol{J}_2 \,.\, \boldsymbol{H})+(\boldsymbol{K}_1 \,.\, \boldsymbol{H})(\boldsymbol{K}_2 \,.\, \boldsymbol{H})-\beta^2\gamma^2(\boldsymbol{n}_1 \,.\, \boldsymbol{H})(\boldsymbol{n}_2 \,.\, \boldsymbol{H}))\}] \quad .. \quad (20)$$

The polarisation terms of $\frac{d\sigma}{d\Omega}$ in (20) depend linearly on the polarisations of both the incident and the scattered beam, so that if the incident beam is unpolarised the polarisation-dependent part of $\frac{d\sigma}{d\Omega}$ disappears. It is therefore clear that we cannot polarise an unpolarised electron beam by scattering in a magnetic field.

3. Discussions

We now specialise to the two cases, *viz.* (*a*) when the magnetic field is longitudinal and (*b*) when it is transverse with respect to the incident electrons.

(*a*) *Longitudinal magnetic field.*—We take the incident beam to be in the direction of the x_1-axis. From (20), remembering (18*b*) and (19*b*), we now get

$$\frac{d\sigma}{d\Omega} = \left(\frac{\eta m}{4\pi}\right)^2 (\xi^2+\mu^2H^2)[(1+\gamma^2-\beta^2\gamma^2n_{21})+w_L] \quad .. \quad .. \quad (21)$$

where

$$w_L = \tau\{\zeta_{21}(2\gamma+(\gamma-1)^2n_{21}^2-\beta^2\gamma^2n_{21})+\zeta_{22}((\gamma-1)^2n_{21}n_{22}-\beta^2\gamma^2n_{22})$$
$$+\zeta_{23}((\gamma-1)^2n_{21}n_{23}-\beta^2\gamma^2n_{23})\} \quad .. \quad .. \quad .. \quad .. \quad (22)$$

The polarisation-independent part of $\frac{d\sigma}{d\Omega}$ in (21) is positive definite. Therefore, $\frac{d\sigma}{d\Omega}$ is maximum or minimum when w_L is maximum or minimum subject to the condition $|\boldsymbol{\zeta}_2|^2 = 1$. By Lagrange's method of undetermined multiplier it is easy to see that w_L is maximum when

$$\left.\begin{aligned} \zeta_{21} &= \frac{\tau}{|\tau|}\frac{2\gamma+\{(\gamma-1)^2n_{21}-\beta^2\gamma^2\}n_{21}}{1+\gamma^2-\beta^2\gamma^2n_{21}}, \\ \zeta_{22} &= \frac{\tau}{|\tau|}\cdot\frac{\{(\gamma-1)^2n_{21}-\beta^2\gamma^2\}n_{22}}{1+\gamma^2-\beta^2\gamma^2n_{21}}, \\ \zeta_{23} &= \frac{\tau}{|\tau|}\cdot\frac{\{(\gamma-1)^2n_{21}-\beta^2\gamma^2\}n_{23}}{1+\gamma^2-\beta^2\gamma^2n_{21}}. \end{aligned}\right\} \quad .. \quad .. \quad (23)$$

Therefore (23) gives the direction of polarisation. The degree of polarisation P may be defined as

$$P = \left\{\left(\frac{d\sigma}{d\Omega}\right)_{\max} - \left(\frac{d\sigma}{d\Omega}\right)_{\min}\right\} \Big/ \left\{\left(\frac{d\sigma}{d\Omega}\right)_{\max} + \left(\frac{d\sigma}{d\Omega}\right)_{\min}\right\}$$

so that from (21), (22) and (23) we get

$$P = |\tau| \quad .. \quad .. \quad .. \quad .. \quad .. \quad (24)$$

The degree of polarisation is thus the same as that of the incident beam, whereas the polarisation direction changes. The polarisation direction of the scattered beam is independent of the magnetic field. Therefore we cannot get any information about the magnetic moment of the electron by analysing the polarisation of the scattered beam in this case.

From (23) we can show that

$$(\boldsymbol{n}_2.\boldsymbol{\zeta}_2) = 0,$$

when

$$n_{21} = \frac{\beta^2\gamma^2}{\gamma^2+1} = \frac{\gamma^2-1}{\gamma^2+1}.$$

This shows that electrons scattered at an angle $\cos^{-1}\frac{\gamma^2-1}{\gamma^2+1}$ are transversely polarised. It is thus possible to obtain transversely polarised electrons from a longitudinally polarised beam by scattering in a magnetic field.

(*b*) *Transverse magnetic field.*—In this case we take $\boldsymbol{H}$ in the direction of the x_3-axis and $\boldsymbol{n}_1$ in the direction of the x_1-axis as before. We consider scattering in the plane to which $\boldsymbol{H}$ is normal, *i.e.*, in the x_1-x_2 plane; so that $n_{21}^2+n_{22}^2 = 1$ and $n_{23} = 0$. From (20) we now get

$$\frac{d\sigma}{d\Omega} = \left(\frac{\eta m}{4\pi}\right)^2 . [\{\xi^2(1+\gamma^2-\beta^2\gamma^2 n_{21})+\mu^2H^2(1+\gamma^2+\beta^2\gamma^2 n_{21}) + 2\xi\mu H\beta^2\gamma^2 n_{22}\}+w_T] \quad .. \quad (25)$$

where

$$w_T = \tau[\zeta_{21}\{(2\gamma+(\gamma-1)^2n_{21}^2)(\xi^2-\mu^2H^2)-2\xi\mu H(\gamma-1)^2n_{21}n_{22} -(\xi^2+\mu^2H^2)\beta^2\gamma^2n_{21}\}+\zeta_{22}\{(\gamma-1)^2(\xi^2-\mu^2H^2)n_{21}n_{22} -2\xi\mu H(2\gamma+(\gamma-1)^2n_{22}^2)-(\xi^2+\mu^2H^2)\beta^2\gamma^2n_{22}\}] \quad .. \quad .. \quad (26)$$

The part of $\frac{d\sigma}{d\Omega}$ in (25) independent of polarisation is positive definite. Therefore $\frac{d\sigma}{d\Omega}$ is maximum or minimum when w_T is maximum or minimum subject to the condition that $|\boldsymbol{\zeta}_2|^2 = 1$. Proceeding as before and writing

$$w_T = \tau[A\zeta_{21}+B\zeta_{22}],$$

where

$$\left.\begin{aligned} A &= 2\gamma(\xi^2-\mu^2H^2)+n_{21}\chi,\\ B &= -4\xi\mu H\gamma+n_{22}\chi\\ \chi &= (\gamma-1)^2(\zeta^2-\mu^2H^2)n_{21}-2\xi\mu H(\gamma-1)^2n_{22}-(\xi^2+\mu^2H^2)\beta^2\gamma^2 \end{aligned}\right\} \quad .. \quad (27)$$

and

we find that w_T is maximum when

$$\left.\begin{aligned} \zeta_{21} &= \frac{\tau}{|\tau|}.\frac{A}{\sqrt{A^2+B^2}},\\ \zeta_{22} &= \frac{\tau}{|\tau|}.\frac{B}{\sqrt{A^2+B^2}}, \quad \zeta_{23} = 0. \end{aligned}\right\} \quad .. \quad .. \quad .. \quad (28)$$

The degree of polarisation P is then given by

$$P = \frac{|\tau|\sqrt{A^2+B^2}}{\xi^2(1+\gamma^2-\beta^2\gamma^2n_{21})+\mu^2H^2(1+\gamma^2+\beta^2\gamma^2n_{21})+2\xi\mu H\beta^2\gamma^2n_{22}}.$$

It can be shown that with A, B defined in (27)

$$P = |\tau| \quad \ldots \quad (29)$$

We thus see that both for transverse and longitudinal magnetic field the degree of polarisation of the scattered beam remains the same as that of the incident beam, while the orientation of the polarisation vector changes. This may be explained by arguing that the effect of the scattering consists essentially in turning the spin-axes of the electrons polarised parallel or antiparallel to the direction of momentum through the same angle.

Further, from (27) and (28) it may be shown that

$$(\boldsymbol{n}_2 \cdot \boldsymbol{\zeta}_2) = 0 \quad \ldots \quad (30)$$

if

$$\chi + 2\gamma(\xi^2 - \mu^2 H^2) n_{21} - 4\xi\mu H \gamma n_{22} = 0 \quad \ldots \quad (31)$$

Writing $n_{21} = \cos \vartheta$ and $n_{22} = \sin \vartheta$, the condition (31) can be expressed as

$$\begin{aligned} &\chi + 2\gamma(\xi^2 - \mu^2 H^2) n_{21} - 4\xi\mu H \gamma n_{22} \\ &= 2\left\{\xi\left(\cos\frac{\vartheta}{2} + \gamma \sin\frac{\vartheta}{2}\right) - \mu H\left(\sin\frac{\vartheta}{2} - \gamma\cos\frac{\vartheta}{2}\right)\right\} \\ &\quad \times \left\{\xi\left(\cos\frac{\vartheta}{2} - \gamma \sin\frac{\vartheta}{2}\right) - \mu H\left(\sin\frac{\vartheta}{2} + \gamma\cos\frac{\vartheta}{2}\right)\right\} \\ &= 0 \quad \ldots \quad (32) \end{aligned}$$

Solving equation (32) for ϑ we, therefore, get the direction of the scattered beam which according to (30) is transversely polarised. The solution, however, is quite complicated since ξ, as given by equations (15) and (8), also depends on ϑ. From (27) and (28) it is evident that the polarisation direction of the scattered beam depends on the magnetic moment μ of the electron. We can therefore determine μ by analysing the scattered beam by Coulomb scattering at heavy nuclei. Following Tolhoek (1956) we can write the intensity of the beam scattered by the nucleus in the form

$$I \sim \bar{I}(\theta)\left[1 + \frac{a(\theta)}{\sin\theta}|\tau|\, \boldsymbol{\zeta}_2 \cdot (\boldsymbol{n}_2 \times \boldsymbol{n}')\right] \quad \ldots \quad (33)$$

where $\boldsymbol{n}'$ is the direction of the finally scattered beam; and $\boldsymbol{\zeta}_2$, $\boldsymbol{n}_2$ are the polarisation and momentum directions of the incident beam, which are the same as those of the beam scattered by the magnetic field; and θ is the scattering angle, *i.e.*,

$$\theta = \sphericalangle \boldsymbol{n}_2, \boldsymbol{n}'.$$

Let

$$\boldsymbol{n}_2 \times \boldsymbol{n}' = \boldsymbol{n}_0 \sin\theta \quad \ldots \quad (34)$$

where $\boldsymbol{n}_0$ is the unit vector normal to the plane of scattering. The asymmetry factor δ in the intensity distribution is then given by

$$\delta = a(\theta)\,|\tau|\, \boldsymbol{\zeta}_2 . \boldsymbol{n}_0 \quad \ldots \quad (35)$$

If we take as axes of reference our original system of axes with respect to which the components of $\boldsymbol{\zeta}_2$ are given in (25), we get

$$\delta = a(\theta)\,|\tau|\,(\zeta_{21} n_{01} + \zeta_{22} n_{02}) \quad \ldots \quad (36)$$

Exact theoretical values of $a(\theta)$ for different values of the atomic number Z of the scatterer, for different angles of scattering θ, and for different energy values of the electron have been given by Sherman (1956). The present author has calculated in a paper (to be published in near future) the first contribution to $a(\theta)$ in Born's approximation in powers of (αZ) and his result is

$$a(\theta) = \alpha Z \, . \frac{\beta(1-\beta^2)^{\frac{1}{2}} \sin\theta \tan^2\frac{\theta}{2} \, ln\left(\sin^2\frac{\theta}{2}\right)}{2\left(1-\beta^2\sin^2\frac{\theta}{2}\right)} \quad \ldots \quad \ldots \quad (37)$$

where as usual α is the fine-structure constant. ζ_{21} and ζ_{22} are given in terms of the magnetic moment μ in (28) and (27). Therefore from observed values of δ it is possible to find μ with the help of (36).

This method of determining μ, however, fails when any one of $(p_2-p_1)_i l_i$ $(i = 1, 2, 3)$ is an exact multiple of π. For then Q, as given in (8), and hence ξ by (15) becomes infinite. But $\eta\xi$ remains finite, which is quite obvious from (8) and (15). Thus in the expression (17) for the differential cross-section the terms involving μH disappear.

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REFERENCES

Carrassi, M. (1957). The spin kinematics for a charged particle in a uniform magnetic field. *Nuovo Cimento.*, **5**, 955–960.
Huff, L. D. (1931). Motion of a Dirac electron in a magnetic field. *Phys. Rev.*, **38**, 501–512.
Jauch, J. M. and Rohrlich, F. (1955). The Theory of Photons and Electrons. Addison-Wesley Publishing Company, Inc. Cambridge.
Lee, T. D. and Yang, C. N. (1956). Question of parity conservation in weak interactions. *Phys. Rev.*, **104**, 254–258.
Mendlowitz, H. and Case, K. M. (1955). Double scattering of electrons with magnetic interaction. *Ibid.*, **97**, 33–38.
Sherman, N. (1956). Coulomb scattering of relativistic electrons by point nuclei. *Ibid.*, **103**, 1601–1607.
Tolhoek, H. A. (1956). Electron polarisation, theory and experiment. *Rev. Mod. Phys.*, **28**, 277–298.
Tolhoek, H. A. and De Groot, S. R. (1951). On the theory of beta-radioactivity III. *Physica,* **17**, 17–32.
Tolhoek, H. A. and Lipps, F. W. (1953). The Transformation of 4–vectors, Antisymmetric Tensors, etc. Report John Hopkins University. (Mimeographed.)
Wu, Ambler, Hayward, Hoppes and Hudson (1957). Experimental test of parity conservation in beta decay. *Phys. Rev.*, **105**, 1413–1415.