A NOTE ON THE ANALYTICAL SOLUTION OF THE EQUATIONS OF INTERNAL BALLISTICS FOR A TAPERED-BORE GUN

by J. N. Kapur, *Hindu College, Delhi University, Delhi*

(Communicated by P. L. Bhatnagar, F.N.I.)

(*Received June 11 ; read August 30, 1958*)

Abstract

In the present paper, the solution of the equations of internal ballistics for a tapered-bore gun has, under one assumption, been reduced to the solution of the system of equations for an orthodox gun.

Introduction

Corner (1950) discussed the basic equations for a tapered-bore gun and tried to show that these could be reduced to the equations for an orthodox gun by a simple transformation. Later Kapur (1957*a*) showed that the reduction was not complete and gave two analytical methods for solving the basic equations. These methods are applicable to guns of any tapering. Recently Jain (1957) has given another alternative method for a conical-bored gun of small taper. In addition to the usual simplifying assumptions for the orthodox gun, he makes two further assumptions about the tapering : (i) in the dynamical equation, he assumes that the variable area of cross-section can be replaced by a mean value, (ii) in the final integration of the velocity-space differential equation, he assumes that the powers of a higher than the second can be neglected, where a is given by

$$A = A_0\left(1-\frac{ax}{l}\right)^2 = A_0[1-a(\xi-1)]^2. \qquad (1)$$

In the present note, it is shown that if we are prepared to make assumption (i) above, it is not necessary to make the second approximation. This is achieved by making a transformation similar to the one used by Corner.

Basic Equations and their Integration

In the usual notation, the basic equations are

$$FCz = p\left[\int_0^x Adx + A_0 l - Cz\left(b-\frac{1}{\delta}\right)\right] + \tfrac{1}{2}(\gamma-1)w_1 v^2 \qquad (2)$$

$$D\frac{df}{dt} = -\beta p \qquad (3)$$

$$z = (1-f)(1+\theta f) \qquad (4)$$

$$w_1 v\frac{dv}{dx} = Ap \qquad (5)$$

Making the substitutions

$$\xi = \frac{\int_0^x A dx + A_0 l}{A_0 l} = \frac{V}{V_0} \quad \ldots \quad (6)$$

$$\zeta = \frac{A_0 l}{FC} p = \frac{V_0}{FC} p \quad \ldots \quad (7)$$

$$\eta = \frac{A_0 D}{FC\beta} v \quad \ldots \quad (8)$$

$$M = \frac{A_0^2 D^2}{FC\beta^2 w_1} \quad \ldots \quad (9)$$

$$B = \frac{\left(b - \frac{1}{\delta}\right) C}{A_0 l}, \quad \ldots \quad (10)$$

we get

$$z = \zeta(\xi - Bz) + \tfrac{1}{2}(\gamma - 1)\frac{\eta^2}{M} \quad \ldots \quad (11)$$

$$\eta \frac{d\eta}{d\xi} = M\zeta \quad \ldots \quad (12)$$

$$\eta \frac{df}{d\xi} = -\frac{A_0}{A}\zeta \quad \ldots \quad (13)$$

$$z = (1-f)(1+\theta f) \quad \ldots \quad (14)$$

From (12) and (13)

$$\frac{d\eta}{df} = -M\frac{A}{A_0} \ldots \quad \ldots \quad (15)$$

Assuming that in this equation A can be replaced by its mean value $\bar{A}$, we get

$$\frac{d\eta}{df} = -M\frac{\bar{A}}{A_0} = -\bar{M} \text{ [say]} \quad \ldots \quad (16)$$

which gives, on integration,

$$\eta = \bar{M}(f_0 - f). \quad \ldots \quad (17)$$

From (11), (12), and (17)

$$z = z_0 + (1 - \theta + 2\theta f_0)\frac{\eta}{\bar{M}} - \theta\frac{\eta^2}{\bar{M}^2}, \quad \ldots \quad (18)$$

and

$$\frac{\eta d\eta}{(a-\eta)(b+\eta)} = \frac{d\xi}{N(\xi - Bz)}, \quad \ldots \quad (19)$$

where

$$N = \frac{M}{\theta'} \quad \ldots \quad (20)$$

$$\theta' = \theta\frac{A_0^2}{\bar{A}^2} + \tfrac{1}{2}(\gamma - 1)M \quad \ldots \quad (21)$$

$$ab = MNz_0 \quad \ldots \quad (22)$$

$$a-b = N(1-\theta+2\theta f_0)\frac{A_0}{\bar{A}} \quad \ldots \quad (23)$$

The integration of (19) has been discussed in the H.M.S.O. publication (1951) and can be taken over in the present case ; only some constants have to be adjusted. Its integration would determine ξ as a function of η, and then (11) and (18) would give ζ as a function of η. The maximum pressure and all-burnt positions are then obtained in the usual way. After all-burnt, $z = 1$, and (11) and (12) give

$$\zeta(\xi-B)^{\gamma} = \text{const.} = \zeta_2(\xi_2-B)^{\gamma} \quad \ldots \quad (24)$$

and

$$\eta^2 = \frac{2M}{\gamma-1}\,[1-\zeta(\xi-B)], \quad \ldots \quad (25)$$

so that the muzzle velocity is given by

$$\eta_3^2 = \frac{2M}{\gamma-1}\left[1-\frac{\zeta_2(\xi_2-B)^{\gamma}}{(\xi_3-B)^{\gamma-1}}\right] \quad \ldots \quad (26)$$

Remarks on the Solution

(*a*) The variable area of cross-section enters in two of the four basic equations of internal ballistics, viz. in the dynamical equation and the energy equation. We eliminate it from the first equation by taking its mean value and from the second by means of a suitable transformation which effectively means using ' volume behind the shot ' as variable instead of the usual shot-travel.

(*b*) The present method reduces the solution of the equations of internal ballistics of a tapered-bore gun to that of the orthodox gun with suitable adjustments of constants, but it is based on a basic assumption, which appears plausible for small tapering, but whose validity can be tested only by comparison with the more accurate solutions obtained numerically by the methods of Jain and Sodha (1957) or Kapur (1957*a*).

(*c*) The numerical example, discussed by Jain to illustrate his method for composite charges, in reality refers to a single charge only ; since according to the result proved by Kapur (1956*a*), in the case of simultaneous burning out of component charges, the composite charge behaves as a single charge with mass C_1+C_2, force-constant $\frac{C_1F_1+C_2F_2}{C_1+C_2}$ and form-factor $\frac{C_1\theta_1+C_2\theta_2}{C_1+C_2}$.

Advantages of the Present Method

(i) The second assumption of Jain has not to be made.
(ii) Shot-start pressure is not to be taken as zero.
(iii) Co-volume term is not to be neglected and standard tables for evaluating it can be used.
(iv) No separate discussion for composite charges is necessary.
(v) The equation can also be easily solved for the general linear and power laws of burning (Clemmow, 1956 ; Kapur, 1956*b*, 1958).

Acknowledgements

The author is grateful to Dr. J. Corner for discussions regarding his treatment of the tapered-bore gun and to Dr. P. L. Bhatnagar and Dr. R. S. Varma for their constant encouragement.

REFERENCES

Clemmow, C. A. (1928). Theory of internal ballistics based on a pressure-index law of burning for propellants. *Phil. Trans. Roy. Soc. London*, **227**, A, 345.

Corner, J. (1950). Theory of Interior Ballistics of Guns. John Wiley & Sons, New York, pp. 334–338.

H.M.S.O. (1951). Internal Ballistics, London, pp. 84–100.

Jain, V. K. (1957). On internal ballistics of a tapered-bore gun using composite charges. *Proc. Nat. Inst. Sci. Ind.*, **23**, A, 108–118.

Jain, V. K., and Sodha, M. S. (1957). On internal ballistics of a tapered-bore gun. *In course of publication.*

Kapur, J. N. (1956*a*). Equivalent charge method in the general theory of composite charges. *Proc. Nat. Inst. Sci. Ind.*, **22**, A, 63–81.

——— (1956*b*). Solution of the equations of internal ballistics when the rate of burning is a linear function of the pressure. *Trans. Nat. Inst. Sci. Ind.*, **3**, 257–290.

——— (1957*a*). Internal ballistics of a tapered-bore gun. *Proc. Nat. Inst. Sci. Ind.*, **23**, A, 438–467.

——— (1958). The solution of equations of internal ballistics for power law of burning. *Ibid.*, **24**, 15–30.