STATISTICAL DISTRIBUTION OF COSMIC RAY STARS IN NUCLEAR EMULSIONS

by INDER SAIN MITTRA *and* P. S. GILL, F.N.I., *Gulmarg Research Observatory, Gulmarg, Kashmir*

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ABSTRACT

A study of statistical distribution of cosmic ray stars in nuclear emulsions is made in order to find out any interrelationship amongst them. It has been found that the number of close-pairs of stars separated by small mutual distances is much more than what is expected on the basis of random distribution. There is some evidence for the existence of two maxima in the distribution of close-pairs, which have been attributed to two different agencies giving rise to the close-pair effect. The effect of the lead absorber has also been studied. The results have been analysed in the light of the various mechanisms which can give rise to the anomalous distribution of stars.

1. INTRODUCTION

It is important to know whether all the stars in nuclear emulsions exposed to cosmic rays are produced by independent primaries, or whether they are interdependent. Certain phenomena point out that at least a small percentage of stars is interdependent. For instance, this is clearly shown by the phenomenon of 'double stars'—in which two stars are found to be connected by a charged particle. There is always a finite probability that the particles (charged or uncharged), emitted from nuclear disintegrations, may further interact with emulsion nuclei and give rise to the secondary stars. The stars thus produced can be distinguished from the primary stars, only if a charged particle produces them. There are also other cases in which the related stars cannot be found out, for instance, when a neutral star producing radiation gives rise to two or more than two stars in its passage through the emulsion; or when some stars are produced by particles forming part of a narrow angle sheaf. The distribution of primary stars is necessarily random in nature. If the number of secondary stars thus produced be sufficient then they can tend to make the overall distribution non-random. The effect produced by the secondary stars can therefore be studied by making a statistical analysis of the distribution of all the stars produced in emulsion plates.

The problem of non-random distribution of stars has been variously termed as 'double stars', or 'close-pair effect', or 'interdependence of stars' in literature. Earliest studies have been made by Leprince-Ringuet and Heidmann (1948) and Li and Perkins (1948), in which the authors found an excess of observed number of close-pairs over the expected number, at small mutual distances. Later studies by Li (1950), Brown and Masket (1952, 1953), Corato *et al.* (1954), Davis *et al.* (1952), Vorisek (1955) and Skinner (1956) confirmed the existence of close-pair effect and threw some light on its detailed structure. On the other hand, the results obtained by Salant *et al.* (1948), Silva *et al.* (1952), G. Davis (1952), Lohrmann (1955) and Gramenitskii *et al.* (1954) show no evidence for the existence of close-pair effect.

In view of the conflicting results reported by various workers the present work was undertaken to make an extensive study of this problem and thus provide definite evidence for, or against, the existence of any correlation amongst stars which may be giving rise to the close-pair effect. It has been found that the distribution of stars at mountain altitudes is definitely non-random. The positive close-pair effect tends to increase when two or/and three pronged stars are removed. There is some evidence for the existence of two broad maxima in the distribution of close-pairs—one in the interval 0·0–0·5 mm. and the other in 2·0–2·5 mm. The two maxima can be considered to be due to two different agencies which give rise to the observed non-randomness in the distribution of stars.

II. Experimental Details

The samples of stars studied were taken at random from three different batches of plates exposed under different conditions at mountain altitudes. Batch one was exposed at Gulmarg (altitude ~ 9,000 ft.) for about 48 days. The emulsions, type Ilford G5, 200μ thick, were placed vertically with 6 cm. of lead on top and on the sides. Out of these 1,950 stars, contained in a surface area 180·64 cm.^{2}, were studied. The numbers of 2-pronged and 3-pronged stars were 623 and 402 respectively. These stars have been referred to, hereafter, as 'Stars under Pb absorber'.

A second batch of plates of the same type was exposed at Apharvat (altitude ~ 13,500 ft.) for about 10 days, under no absorber. These plates were placed vertically and in different directions, viz., 30°, 60°, east and west. Out of these 597 stars, contained in a scanned area 75·00 cm.^{2}, were taken for study. There were 59 two-pronged and 208 three-pronged stars in it. A third batch of emulsion plates type Ilford G5, 400μ thick, was exposed in Lahoul Valley (altitude ~ 14,000 ft.) for about four weeks. The number of stars studied was 821, taken from a scanned area 36·00 cm.^{2} This sample contained 135 two-pronged and 208 three-pronged stars.

For the purpose of this study, the stars taken from batches two and three were combined, and have been referred to, hereafter, as 'Stars under no absorber'. The close-pair effect was also studied for 'All stars' taken together. This gave the number of stars studied as 3,368, 2,541, and 1,733 with prongs $\geqslant 2$, $\geqslant 3$, and $\geqslant 4$ respectively.

Scanning:

It was of very great importance in the present study that the scanning efficiency should be as high as possible. To achieve this, scanning was done at a very slow rate of 0·2 cm.^{2}/hr. at a magnification of $\times 450$. By checking and rechecking by different persons, it was estimated that the scanning efficiency was not less than 95 per cent. As the star density near the edges was found to be markedly less than the average star density in the plate, portions of width 5 mm. from the edges were left out of consideration. For unambiguous selection of stars, great care was taken not to mistake certain scattering events and unstable particle decays as two-pronged stars; and the knock-on events as the three-pronged stars.

Observed Number of Close-pairs:

The major requirement in the present experiment was to determine the positions of the stars accurately and thus to find out the mutual separations of them. This was done in most cases with the help of Cooke Research Microscope Series M4005, which is provided with vernier having least count of 10μ. The distances between the centres of those stars which were separated by $<$ 0·5 mm. were read directly from a scale in the eye-piece of the microscope. For greater distances, the projected co-ordinates of all the stars were plotted on a graph paper having hundred times the scanned area of the plate. This gave the idea of the overall distribution of the stars in the plate. In particular, it was noted that at places there were clusters of stars—two or more stars grouped together—and at other places, there were open spaces. This feature of star distribution seemed to indicate the non-random distribution of stars.

Now from every star on the graph paper, distances of all other stars, which were within a radius of 5·0 mm. (on the graph 5·0 cm.) from it, were accurately measured. From these distances, the frequencies of close-pairs with centres separated by a distance lying in a certain interval, 0·0–0·5, 0·5–1·0,........4·5–5·0 mm., were determined. This gave us the 'differential distribution' of close-pairs of separation 'R' ($r_1 < R < r_2$) with $r_1 - r_2 = 0{\cdot}5$ mm. This procedure was repeated for all the plates and the number of close-pairs in the corresponding intervals added up. In this way, a distribution of close-pairs was obtained for 'All stars', for 'Stars under Pb absorber' and for 'Stars under no absorber'.

Expected Number of Close-pairs:

The next step was to determine the expected number of close-pairs in a given interval on the basis of random distribution. Let us consider an emulsion plate of sides a, b ($a \geqslant b$). In this rectangular zone, let there be N stars distributed at random. Then the mean number of pairs of stars having mutual distances lying between r and $r + dr$ is given by (Corato *et al.*, 1954; Skinner, 1956)

$$Q(r)\,dr = \tfrac{1}{2}N(N-1)f(r)\,dr \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where

$$f(r) = 2r/ab[\pi - 2r(a+b)/ab + r^2/ab] \quad \ldots \quad \ldots \quad (2)$$

Integrating equation (1) between r_i and r_{i+1} and adding the results relative to all the zones, we obtain the number 'Q' of pairs of stars which have the distance between their centres lying in the interval r_i and r_{i+1}.

If 'P' is the observed number of close-pairs in a given interval, then $(P-Q)$ gives the observed deviation, whereas $\sqrt{Q}$ is the expected deviation. The ratio $(P-Q)/\sqrt{Q}$ gives a measure of the degree of non-randomness in the distribution of stars. If this ratio is positive and greater than unity then it shows that there are significant deviations from the random distribution.

III. Observations and Results

The analysis was made to determine the differential distribution of close-pairs, that is, those having a separation 'R' ($r_1 \leqslant R \leqslant r_2$) with the interval $r_1 - r_2 = 0{\cdot}5$ mm. up to $r = 5{\cdot}0$ mm. In order to see the effect of the lead absorber, the analysis was also made separately for 'Stars under Pb absorber' and for 'Stars under no absorber'. The case of 'All stars' taken together was also studied. The ratio $(P-Q)/\sqrt{Q}$ which represents the degree of non-randomness was determined in each case. The significance of the observed deviations was examined with the help of χ^2-test. In order to find out the effect of the star prongs, the process was repeated for stars with prongs $\geqslant 2$, $\geqslant 3$ and $\geqslant 4$ for all the above categories of stars.

Results on 'All Stars':

Results on 'All stars' have been displayed in Table I. Errors given therein are just statistical fluctuations. A study of this table reveals that there are significant deviations between the observed and expected distributions of close-pairs. χ^2-test gives a probability of less than 1 per cent that the deviations are just due to chance.

The variation of the ratio $(P-Q)/\sqrt{Q}$ with 'r' is shown in Fig. 1 for different star types. Curves I, II and III are for stars with prongs $\geqslant 2$, $\geqslant 3$ and $\geqslant 4$ respectively. The curves seem to indicate the presence of two maxima, one of these between 0·0 and 0·5 mm., and the other between 1·5 and 2·0 mm. After the second maximum, the curves show a downward trend. When all stars with prongs $\geqslant 2$ are considered,

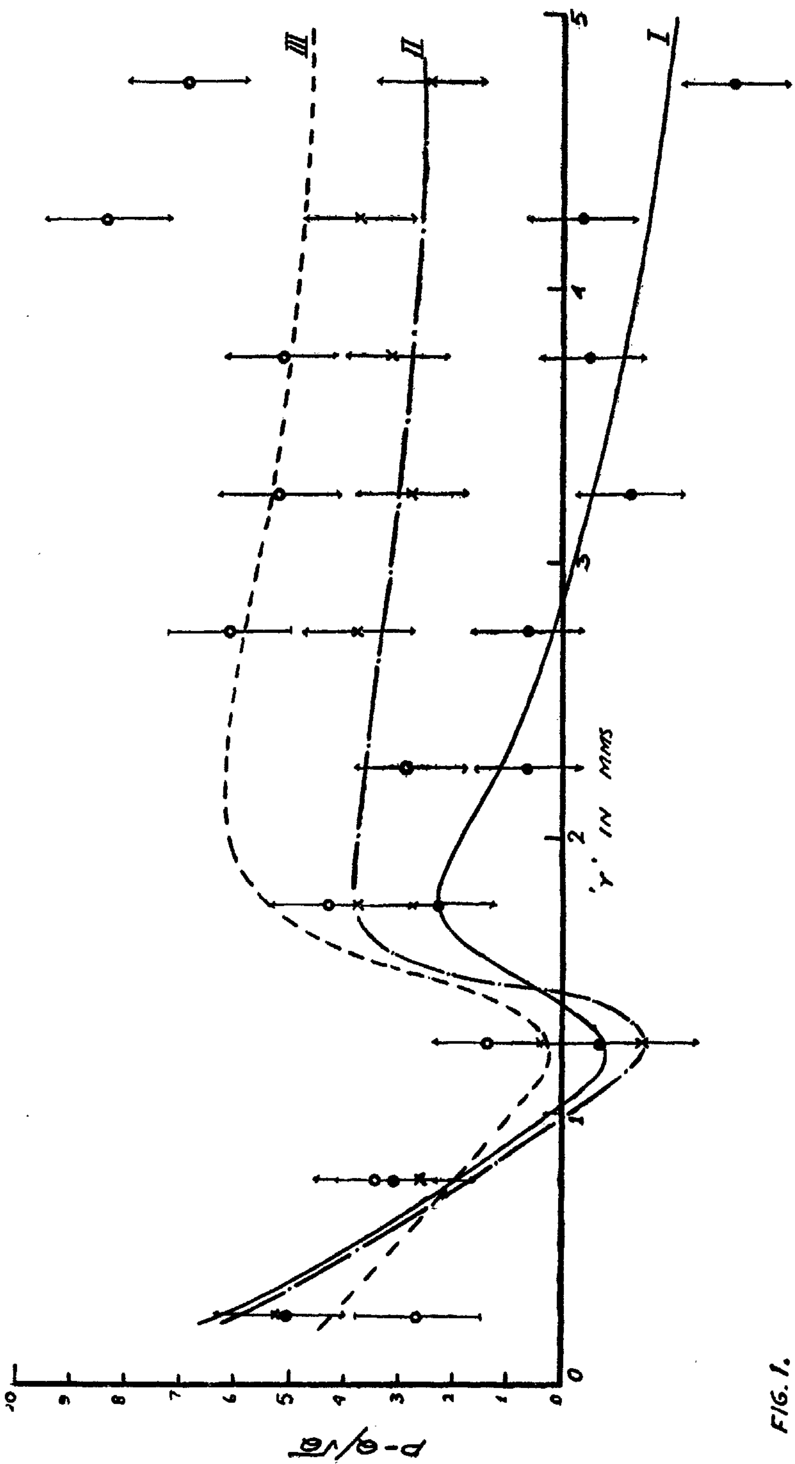

FIG. 1.

VARIATION OF P-Q/√Q WITH 'r' FOR 'ALL STARS' WITH DIFFERENT No. OF PRONGS

CURVE I for Stars with prongs ≥ 2 ; CURVE II for Stars with prongs ≥ 3 ; CURVE III for Stars with prongs ≥ 4

TABLE I

'All stars'

in mms.	Prongs ⩾ 2			Prongs ⩾ 3			Prongs ⩾ 4		
	P	Q	$(P-Q)/\sqrt{Q}$	P	Q	$(P-Q)/\sqrt{Q}$	P	Q	$(P-Q)/\sqrt{Q}$
0·0–0·5	233 ± 15·26	168·37	4·98 ± 1·18	148 ± 12·00	97·31	5·14 ± 1·23	64 ± 8·00	46·12	2·63 ± 1·18
0·5–1·0	568 ± 23·83	499·84	3·05 ± 1·07	335 ± 18·30	291·38	2·56 ± 1·07	178 ± 13·34	138·06	3·40 ± 1·14
1·0–1·5	779 ± 27·91	823·31	−1·54 ± 0·97	459 ± 21·42	471·13	−0·56 ± 0·99	243 ± 15·59	223·25	1·32 ± 1·04
1·5–2·0	1214 ± 34·85	1138·78	2·23 ± 1·03	736 ± 27·13	664·95	2·75 ± 1·05	391 ± 19·77	315·21	4·27 ± 1·11
2·0–2·5	1470 ± 38·34	1446·24	0·62 ± 1·01	904 ± 30·07	825·22	2·74 ± 1·05	446 ± 21·12	391·13	2·78 ± 1·07
2·5–3·0	1771 ± 42·08	1745·93	0·60 ± 1·01	1139 ± 33·75	1018·78	3·77 ± 1·06	616 ± 24·82	483·08	6·05 ± 1·13
3·0–3·5	1982 ± 44·52	2037·40	−1·23 ± 0·99	1268 ± 35·61	1172·96	2·77 ± 1·04	678 ± 26·04	556·20	5·17 ± 1·10
3·5–4·0	2297 ± 47·93	2321·42	−0·51 ± 0·99	1457 ± 38·17	1342·80	3·12 ± 1·04	765 ± 27·66	636·28	5·10 ± 1·10
4·0–4·5	2577 ± 50·77	2597·11	−0·39 ± 1·00	1644 ± 40·55	1500·41	3·71 ± 1·05	933 ± 30·55	711·77	8·29 ± 1·15
4·5–5·0	2698 ± 51·95	2867·39	−3·16 ± 0·97	1755 ± 41·89	1655·59	2·44 ± 1·03	977 ± 31·26	785·50	6·83 ± 1·12

the curve II crosses the r-axis between r=2·8–3·0 mm., thus giving negative values of $(P-Q)/\sqrt{Q}$ afterwards. For the case of stars with prongs ⩾ 3 and ⩾ 4, this ratio does not become zero up to the largest distances measured, but slowly tends to reduce to zero.

These results indicate the presence of two types of effects. One exists for short distances only and rapidly becomes zero near about the interval 1·0–1·5 mm. The second is more pronounced at larger distances. When 2-pronged stars are included in the data, this extends up to $r \sim$ 3·0 mm., but for stars with prongs ⩾ 4, it extends to distances larger than 5·0 mm., and is very strong. In fact, in this case, it overlaps the first effect and the deviation $(P-Q)$ never becomes zero. The two maxima shown by the $(P-Q)/\sqrt{Q}$ *vs.* 'r' curve may be the result of these two effects. Hereafter, these two effects will be designated as the 'first effect' and the 'second effect' respectively.

Results on 'Stars under Pb Absorber':

Results obtained for 'Stars under Pb absorber' are similar to those found on 'All stars'. These are displayed in Table II. In this case also, we find that the deviations are much more marked when 2-pronged stars are excluded from the data than otherwise.

The variation of the ratio $(P-Q)/\sqrt{Q}$ with 'r' is shown in Fig. 2. Curves I, II and III are for stars with prongs ⩾ 2, ⩾ 3 and ⩾ 4 respectively. There is a minimum in the interval 1·0–1·5 mm., for the case of stars with prongs ⩾ 2 and ⩾ 3. In all the three cases, two maxima can be noticed, one of these in the interval 0·0–0·5 mm., the other in the interval 1·5–2·0 mm. For stars with prongs ⩾ 2, there is a sharp fall after the second maximum, the ratio becoming zero between 2·2 and 2·3 mm. When the 2-pronged stars are not included in the data, the fall after the second maximum is not so sharp; the ratio does not become zero up to 5·0 mm., but shows a slow

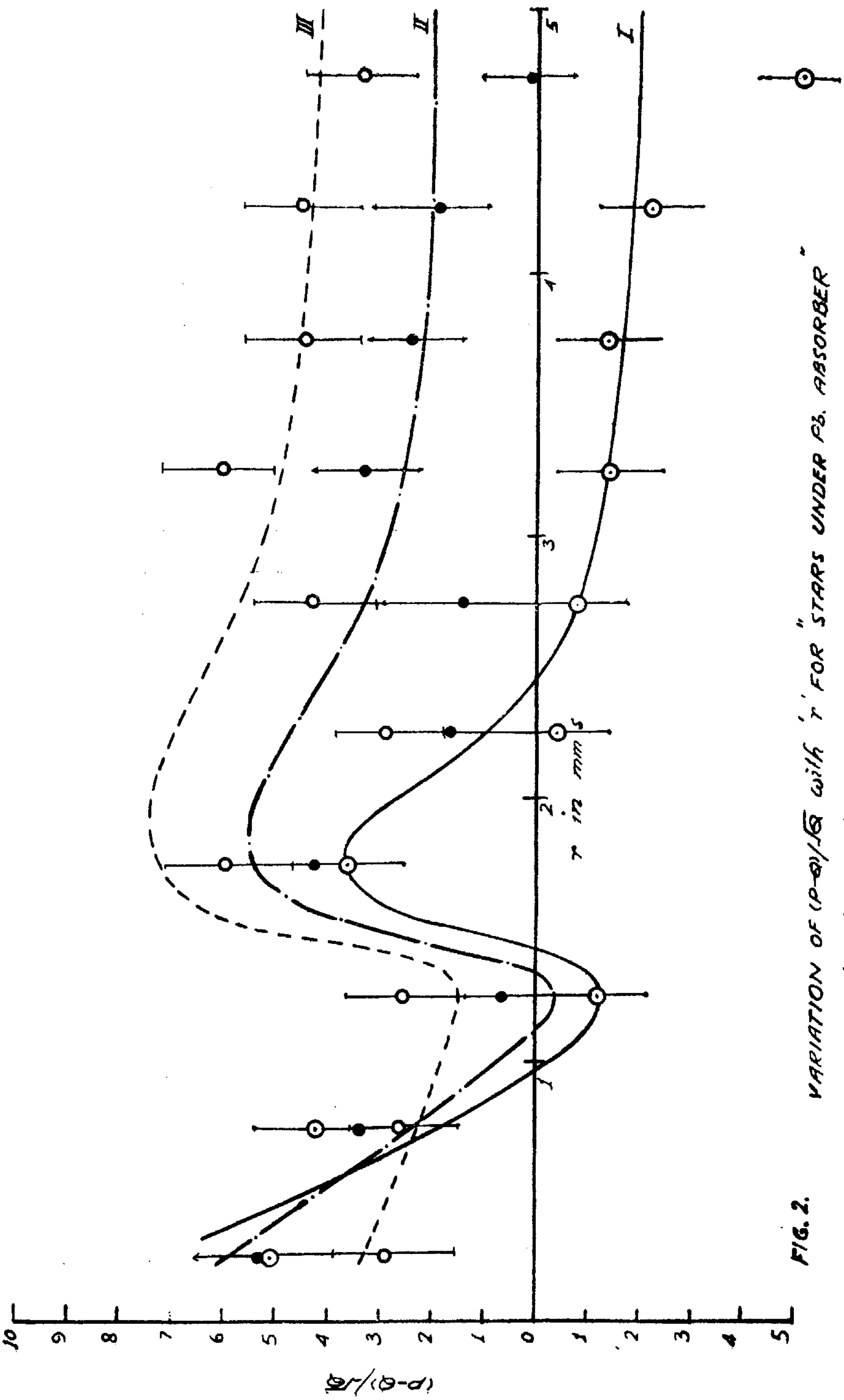

FIG. 2. VARIATION OF $(P-Q)/\sqrt{Q}$ with 'r' FOR "STARS UNDER Pb. ABSORBER"

CURVE I for stars with prongs ≥ 2
CURVE II for stars with prongs ≥ 3
CURVE III for stars with prongs ≥ 4.

TABLE II

'Stars under Pb absorber'

'r' in mms.	Prongs ≥ 2			Prongs ≥ 3			Prongs ≥ 4		
	P	Q	$(P-Q)/\sqrt{Q}$	P	Q	$(P-Q)/\sqrt{Q}$	P	Q	$(P-Q)/\sqrt{Q}$
0·0–0·5	133 ± 11·4	86·21	5·04 ± 1·22	72 ± 8·5	38·94	5·30 ± 1·36	31 ± 5·57	18·68	2·85 ± 1·29
0·5–1·0	323 ± 18·0	255·77	4·20 ± 1·13	154 ± 12·4	117·92	3·32 ± 1·14	76 ± 8·72	56·57	2·58 ± 1·16
1·0–1·5	397 ± 20·0	421·56	−1·20 ± 0·97	194 ± 14·0	185·71	0·61 ± 1·03	113 ± 10·63	89·08	2·53 ± 1·12
1·5–2·0	670 ± 25·9	583·43	3·58 ± 1·07	340 ± 18·4	270·45	4·23 ± 1·12	197 ± 14·04	129·73	5·91 ± 1·23
2·0–2·5	729 ± 27·0	741·37	−0·45 ± 0·99	354 ± 18·8	324·55	1·63 ± 1·04	190 ± 13·78	155·69	2·75 ± 1·10
2·5–3·0	872 ± 29·5	895·35	−0·78 ± 0·99	443 ± 21·1	414·69	1·39 ± 1·04	259 ± 16·09	198·94	4·26 ± 1·14
3·0–3·5	999 ± 31·6	1045·63	−1·44 ± 0·98	540 ± 23·2	468·78	3·29 ± 1·07	315 ± 17·75	224·86	6·01 ± 1·18
3·5–4·0	1144 ± 33·7	1191·95	−1·39 ± 0·98	596 ± 24·4	540·90	2·37 ± 1·05	331 ± 18·19	259·47	4·42 ± 1·13
4·0–4·5	1252 ± 35·3	1334·25	−2·25 ± 0·97	649 ± 25·5	604·01	1·87 ± 1·04	366 ± 19·13	289·74	4·48 ± 1·12
4·5–5·0	1276 ± 35·7	1474·65	−5·17 ± 0·93	665 ± 25·8	667·12	0·08 ± 0·99	379 ± 19·47	320·02	3·30 ± 1·09

tendency downwards. The results can again be interpreted as being due to the presence of two separate effects, the two maxima corresponding to each of them. The 'second effect' is much stronger when 2-pronged and 3-pronged stars are removed and almost overlaps the 'first effect'.

Results on 'Stars Under No Absorber':

The results on 'Stars under no absorber' are given in Table III. Fig. 3 gives the variation of the ratio $(P-Q)/\sqrt{Q}$ with 'r' for different star types. Curves I, II and III correspond to stars with prongs $\geqslant 2$, $\geqslant 3$ and $\geqslant 4$ respectively.

An examination of Fig. 3 shows that, as in the previous cases, the two maxima can still be considered to be there, the second maximum, however, lies in the interval 2·0–2·5 mm. and is quite broad. The ratio $(P-Q)/\sqrt{Q}$ does not become zero after the second maximum for all the three cases, but slowly tends to become zero. Moreover, unlike the case of 'All stars' and 'Stars under Pb absorber', the ratio becomes zero after the first maximum, in the interval 0·5–1·0 mm., even for the stars with prongs $\geqslant 4$.

Effect of the Pb Absorber:

In order to compare the results on 'Stars under Pb absorber' and 'Stars under no absorber', the total number of stars in the two cases has been normalized to 1,000. The resulting values of $(P-Q)$ have been plotted against 'r' for stars with prongs $\geqslant 2$, in the two cases in Fig. 4. Curve I is for stars under lead and curve II is for stars without lead.

The results in the two cases can be compared as follows:—

(i) Both have minimum values of $(P-Q)$ in the interval 1·0–1·5 mm.

(ii) Both can be considered to have two maxima. The first maximum lies in the interval 0·0–0·5 mm. The second maximum in curve I

TABLE III

'Stars under no absorber'

r' in mms.	Prongs ⩾ 2			Prongs ⩾ 3			Prongs ⩾ 4		
	P	Q	$(P-Q)/\sqrt{Q}$	P	Q	$(P-Q)/\sqrt{Q}$	P	Q	$(P-Q)/\sqrt{Q}$
0·0–0·5	100 ± 10·00	82·16	1·96 ± 1·10	76 ± 8·72	58·37	2·30 ± 1·14	33 ± 5·75	27·44	1·06 ± 1·10
0·5–1·0	245 ± 15·65	244·07	0·06 ± 1·00	181 ± 13·45	173·46	0·57 ± 1·02	102 ± 10·10	81·49	2·27 ± 1·10
1·0–1·5	382 ± 19·54	401·75	−0·99 ± 0·98	265 ± 16·28	285·42	−1·21 ± 0·96	130 ± 11·40	134·17	−0·36 ± 0·98
1·5–2·0	544 ± 23·32	555·35	−0·48 ± 0·99	396 ± 19·90	394·50	0·76 ± 1·00	194 ± 13·93	185·48	0·63 ± 1·02
2·0–2·5	741 ± 27·22	704·87	1·36 ± 1·03	550 ± 23·45	500·67	2·21 ± 1·05	256 ± 16·00	235·44	1·34 ± 1·04
2·5–3·0	899 ± 29·98	850·58	1·66 ± 1·03	696 ± 26·38	604·09	3·74 ± 1·07	357 ± 18·89	284·14	4·32 ± 1·12
3·0–3·5	983 ± 31·35	991·77	−0·28 ± 0·99	728 ± 26·98	704·18	0·90 ± 1·02	363 ± 19·05	331·34	1·74 ± 1·05
3·5–4·0	1153 ± 33·96	1129·47	0·70 ± 1·01	861 ± 29·34	801·90	2·09 ± 1·04	434 ± 20·83	377·41	2·91 ± 1·07
4·0–4·5	1325 ± 36·39	1262·86	1·75 ± 1·02	995 ± 31·54	896·40	3·29 ± 1·05	567 ± 23·81	422·03	7·06 ± 1·16
4·5–5·0	1422 ± 37·71	1392·74	0·78 ± 1·01	1090 ± 33·02	988·47	3·23 ± 1·05	598 ± 24·45	465·48	6·14 ± 1·13

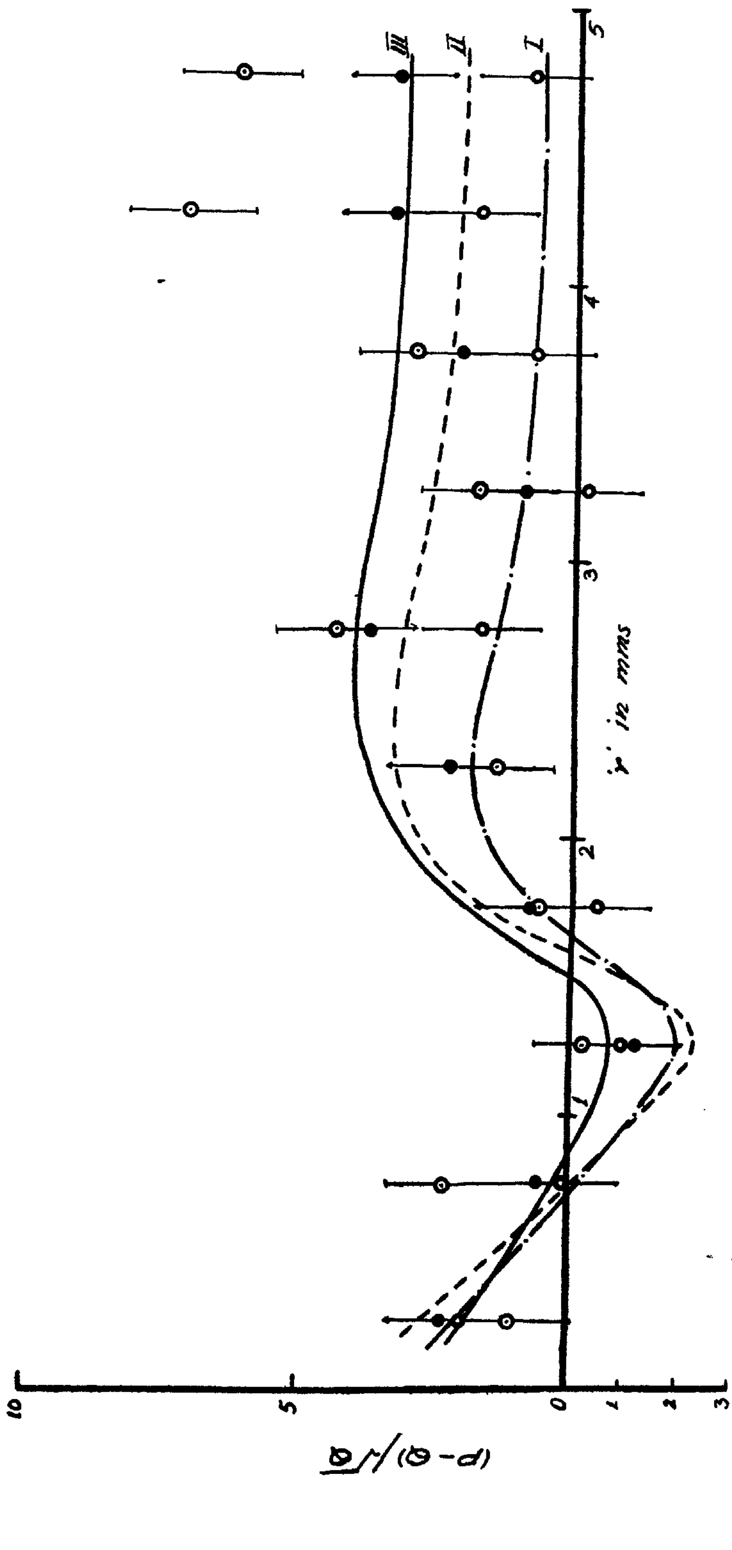

FIG 3.

VARIATION OF P-Q/√Q WITH 'r' FOR "STARS UNDER NO ABSORBER"

CURVE I (○) FOR STARS WITH PRONGS ≥ 2
CURVE II (●) FOR STARS WITH PRONGS ≥ 3
CURVE III (⊙) FOR STARS WITH PRONGS ≥ 4

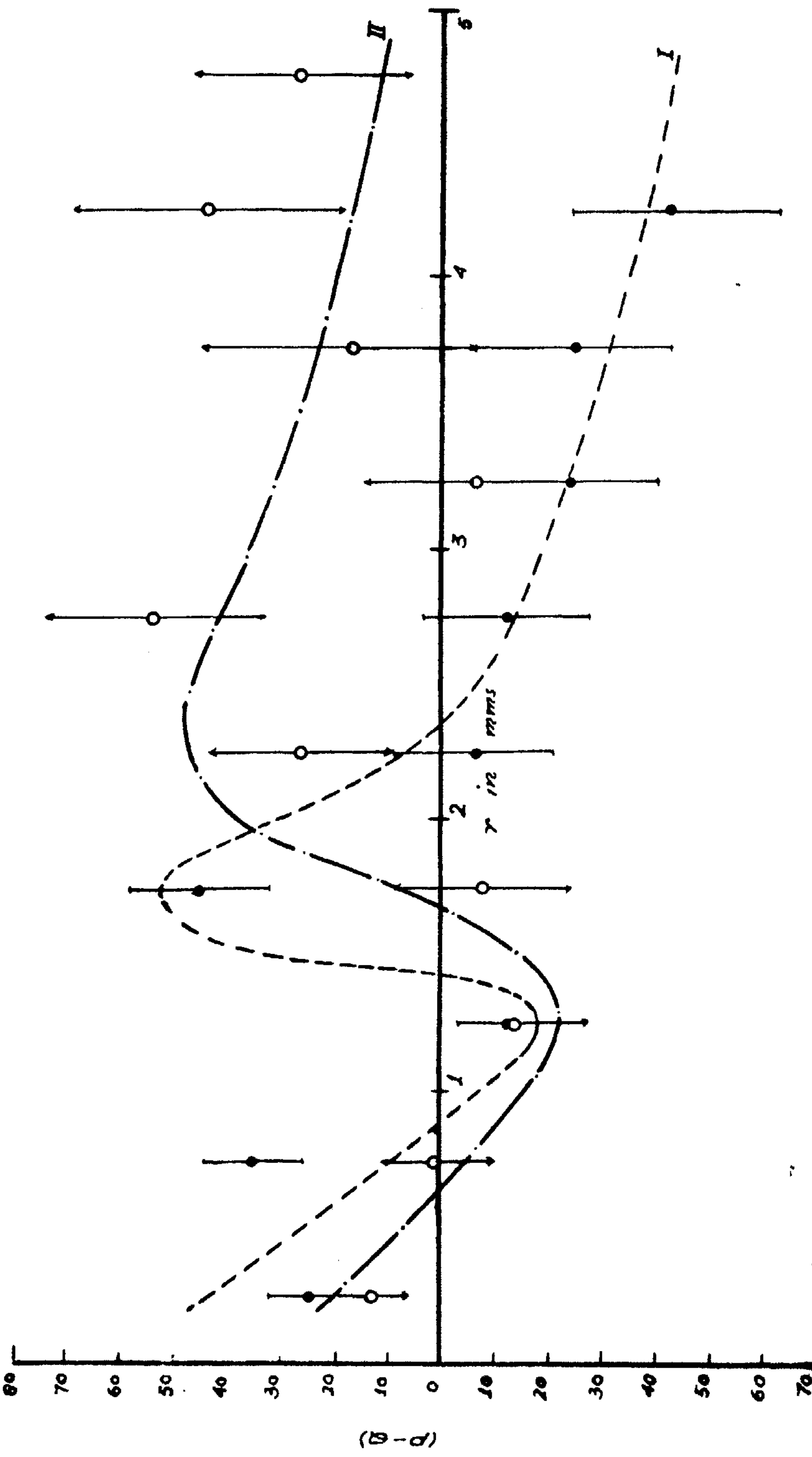

FIG. 4. Comparison of Excess of Close Pairs of Separation R ($r_1 \leq R \leq r_2$) for Stars Under Pb Absorber (CURVE I) and Under No Absorber (CURVE II)

(P−Q) normalized to 1000 stars in each case.

lies in the interval 1·5–2·0 mm., but in the case of curve II it lies in the interval 2·0–2·5 mm.

(iii) For 'Stars under Pb absorber', the curve has a sharp fall after the second maximum, crosses the r-axis at $r \sim 2{\cdot}3$ mm. and then has negative values up to the largest values of 'r' measured. For the case of 'Stars under no absorber', the curve falls after the second maximum, but slowly; tends to become zero but does not do so till $r=5{\cdot}0$ mm.

(iv) The stars under lead show larger effect at small distances, whereas the 'second effect' is greater in magnitude for stars under no absorber.

From the above analysis we conclude that the effect of the lead absorber is to increase the 'first effect' which extends only up to 2·0 mm. or so; and to decrease the 'second effect' which is more pronounced for comparatively higher energy stars.

Comparison with the Results of Other Workers:

The results of the present investigation have been compared with those obtained by previous workers by studying the variation of $(P-Q)$ with 'r' for integral distribution of close-pairs, that is, those with separation 'R' $(0 \leqslant R \leqslant r)$. The total number of stars in all the cases have been normalized to 1,000. Fig. 5 shows the plot of $(P-Q)$ *vs.* 'r' for all these experiments. A comparison of the curves reveals the following facts immediately:—

(i) The close-pair effect as observed in the present experiment is very much larger than in any of the other experiments.

(ii) The trend of the 'first effect' is in general conformity with that obtained by Leprince-Ringuet and Heidmann (1948), Li and Perkins (1948) and Davis *et al.* (1952). However, the magnitude in the present case is much larger.

(iii) Li (1950) obtained a curve which shows a broad maximum at 'r' $\sim 2{\cdot}5$ mm. This corresponds to the second maximum in the present curve.

(iv) The results of Corato *et al.* (1954) differ from all other workers; their curve shows a steady fall, it crosses the r-axis at $r \sim 1{\cdot}7$ mm., and then shows negative values of $(P-Q)$. According to their results, the close-pair effect exists only up to $\sim 2{\cdot}0$ mm. Moreover, they found that even this effect vanishes when 2-pronged stars are removed from the data. However, in the present work it has been found that the effect actually becomes larger when 2-pronged and 3-pronged stars are removed from the data. This should be considered in the light of the results obtained by Brown and Masket (1952, 1953) who report that the close-pair effect is only due to stars with prongs > 6.

(v) The present investigation shows the presence of the two maxima, corresponding to the probable existence of two types of agencies which bring about the close-pair effect. Not many other workers have obtained such a result. This may be due to the fact that they did not extend their observations to larger distances. However Vorisek (1955) and Brown and Masket (1952) report the existence of two maxima, one near $r \sim 0{\cdot}5$ mm. and the other at $r \sim 1{\cdot}3$ mm.

IV. Discussion of Results

Before the results are examined in the light of the various mechanisms which can give rise to the close-pair effect, it has to be noted that there are certain factors which can lead to spurious excess of close-pairs. Important amongst these are inefficient scanning, observer bias, edge effect, uneven thickness of emulsion and

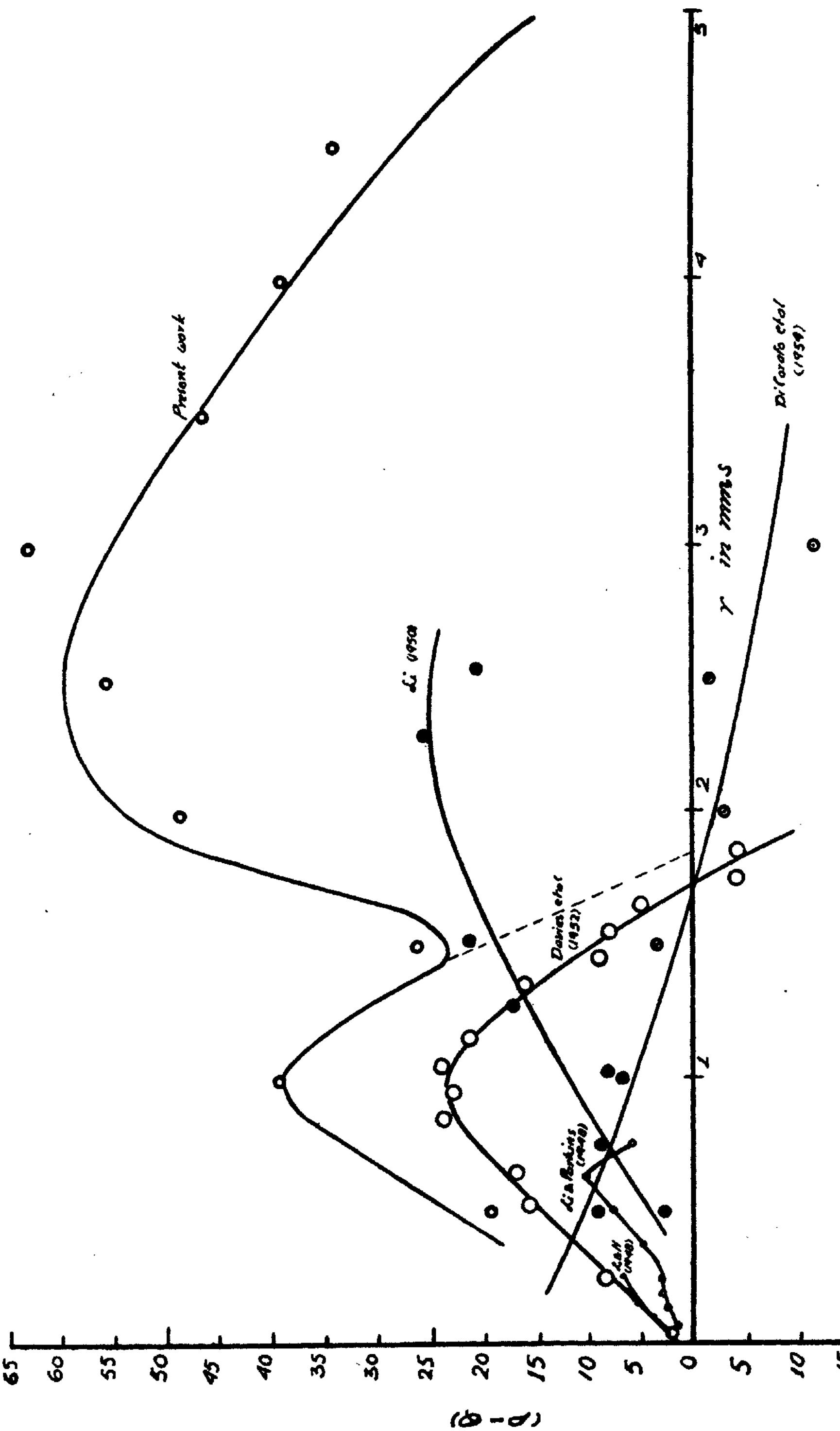

FIG. 5. Comparison of the results with those of other workers by studying the variation of excess of close-pairs of separation $R(0 \le R \le r)$ with 'r'. $(p-q)$ is normalized to 1000 stars in each Experiment.

fading. In the present investigation, all these factors were properly taken care of and the results are therefore substantially free from spurious effects.

The statistical analysis carried out on the distribution of stars has shown that there is positive evidence in favour of close-pair effect. This evidence together with the presence of maxima in the distribution of close-pairs, and the variation in the observed effect with star type and absorber, goes to show that some sort of interrelationship amongst the stars exists. However, it is very difficult to explain these results in a consistent manner because of a large number of unknown factors, the correct evaluation of which, in general, is not possible. For instance, emulsions being continuously sensitive, we cannot have any idea about the simultaneity of the events produced. Also, emulsion being a complex mixture of many different nuclei, it is difficult to ascertain the exact nature of the disintegrating nucleus, the nature of short tracks coming out and the distribution of the emitted neutrons. Apart from this, in the analysis carried out, we can only have an idea about the nature and magnitude of the overall effect. No detailed study of the individual so-called close-pair is possible, for it is not possible to distinguish a real pair from a fortuitous one. Also, if the connecting particles are neutral, we cannot know the energies and momenta carried by these particles and the way they are distributed.

Considering the above uncertainties, a satisfactory explanation of close-pair effect is not easy to achieve. However, certain mechanisms of correlation of stars can be put forward and the results discussed in the light of these predictions. These processes are: (*a*) Regenerative Process, (*b*) Single Penetrating Particles and (*c*) Sheafs of Particles.

(*a*) *Regenerative Process:*

In the Regenerative Process it is assumed that the close-pair effect is produced by the particles given out from the stars themselves. Let us suppose that the primary stars on the average give out 'q' neutral particles, capable of further producing stars. Let 'σ' be their cross-section of production of stars and $F(r)\,dr$ be the probability that the secondary star is produced within its own zone at a distance lying between r and $r+dr$. If $(Q_{q\sigma})_i$ be the total number of pairs formed by different possible association in a particular zone and $(Q)_i$ be the number only on the basis of random distribution, then it was shown by Corato *et al.* (1954) that

$$(Q_{q\sigma})_i-(Q)_i = (Nq\sigma)/(1+Kq\sigma)\,.\,K_i \qquad \ldots \quad \ldots \quad \ldots \quad (3)$$

where 'N' is the total number of observed stars and K, K_i are defined as

$$K = \int_0^{\sqrt{a^2+b^2}} F(r)\,dr\,; \qquad K_i = \int_{r_i}^{r_{i+1}} [F(r)-Kf(r)]\,dr \qquad \ldots \quad (4)$$

a, b ($a > b$) are the dimensions of the plate and $f(r)$ is defined by (2).

Assuming this excess to be equal to the observed excess (P_i-Q_i), we can determine the most probable value of

$$\alpha = (Nq\sigma)/(1+Kq\sigma) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

by means of the evaluation of the weighted means of the value of $\alpha_i = (P_i-Q_i)/K_i$. The weighted means of α_i, that is, $\bar{\alpha}$ and $\Delta\bar{\alpha}$ are given by

$$\bar{\alpha} = \frac{\sum_i [(P_i-Q_i)/K_i]K_i^2/P_i}{\sum_i K_i^2/P_i}\,; \qquad \Delta\bar{\alpha} = \left[\sum_i (K_i^2/P_i)\right]^{-\frac{1}{2}} \qquad \ldots \quad (6)$$

From $\bar{\alpha}$ and $\Delta\bar{\alpha}$, corresponding values of $\overline{q\sigma}$ and $\Delta\overline{q\sigma}$ can be determined.

Carrying out the above analysis, the values of $\overline{q\sigma}$ given in Table IV were obtained for different categories of stars. The contribution of the 'Regenerative Process' to the excess of close-pairs is given by (3). If the values obtained agree with $(P_i - Q_i)$ within statistical fluctuations, then the hypothesis that this process is mainly responsible for the non-random distribution of stars will be justified. A comparison of the values of $[(Q_{q\sigma})_i - Q_i]$ with $(P_i - Q_i)$ is made for the case of 'Stars under Pb absorber' in Figs. 6, 7 and 8 for stars with prongs $\geqslant 2$, $\geqslant 3$ and $\geqslant 4$ respectively. The curves show the variation of $[(Q_{q\sigma})_i - Q_i]$ with 'r'. These figures reveal that this process explains the observed effect at very short distances, but is grossly insufficient to explain the large excess of close-pairs at greater distances. The values of $[(Q_{q\sigma})_i - Q_i]$ are directly proportional to K_i which is a rapidly decreasing function of 'r'; hence the contribution of the Regenerative Process to the excess of close-pairs is negligible after 1·0 mm. or so. Similar results hold in the case of 'All stars' and 'Stars under Pb absorber'.

TABLE IV

Star Type	$\overline{q\sigma}$ in 10^{-24} cm.^{2}
1. 'All stars' with prongs $\geqslant 2$	16·93 ± 4·74
2. 'All stars' with prongs $\geqslant 3$	19·69 ± 4·20
3. 'All stars' with prongs $\geqslant 4$	12·84 ± 3·88
4. 'Stars under Pb absorber' with prongs $\geqslant 2$	24·39 ± 5·79
5. 'Stars under Pb absorber' with prongs $\geqslant 3$	28·08 ± 6·39
6. 'Stars under Pb absorber' with prongs $\geqslant 4$	17·91 ± 5·71
7. 'Stars under no absorber' with prongs $\geqslant 2$	8·58 ± 5·16
8. 'Stars under no absorber' with prongs $\geqslant 3$	11·33 ± 5·38
9. 'Stars under no absorber' with prongs $\geqslant 4$	8·13 ± 5·20

The application of the 'Regenerative Process' to the close-pair effect shows very clearly that this is not the only process which is operative in giving rise to the non-random distribution of stars. At the most it can explain the 'first effect' which extends up to 2·0 mm. or so. The values of '$\overline{q\sigma}$' required to explain the 'first effect' are also very high. If it is assumed that the particles responsible for the production of secondary stars are neutrons, then from the data of emission of first protons in the nuclear disintegrations at the mountain altitudes, we can expect, on the average, not more than 4 neutrons per star. If 'σ' is geometrical, that is, $0{\cdot}88 \times 10^{-24}$ cm.^{2}, the value of $\overline{q\sigma} \leqslant 3{\cdot}52 \times 10^{-24}$ cm.^{2} This value is considerably lower than the values of $\overline{q\sigma}$ required to explain the observed magnitude at short distances. However, it is instructive to compare the values of $\overline{q\sigma}$ with those obtained by the previous authors. Such a comparison shows that the present values are not incomparable with those obtained by them. These values are given in Table V.

TABLE V

Authors	$\overline{q\sigma}$ in 10^{-24} cm.^{2}
1. Leprince-Ringuet and Heidmann (1948)	7·5
2. Li and Perkins (1948)	6·5 ± 2·4
3. Li (1950)	13·8 ± 5·4
4. Davis *et al.* (1952)	28·0 ± 5·3
5. Corato *et al.* (1954)	4·4 ± 2·0

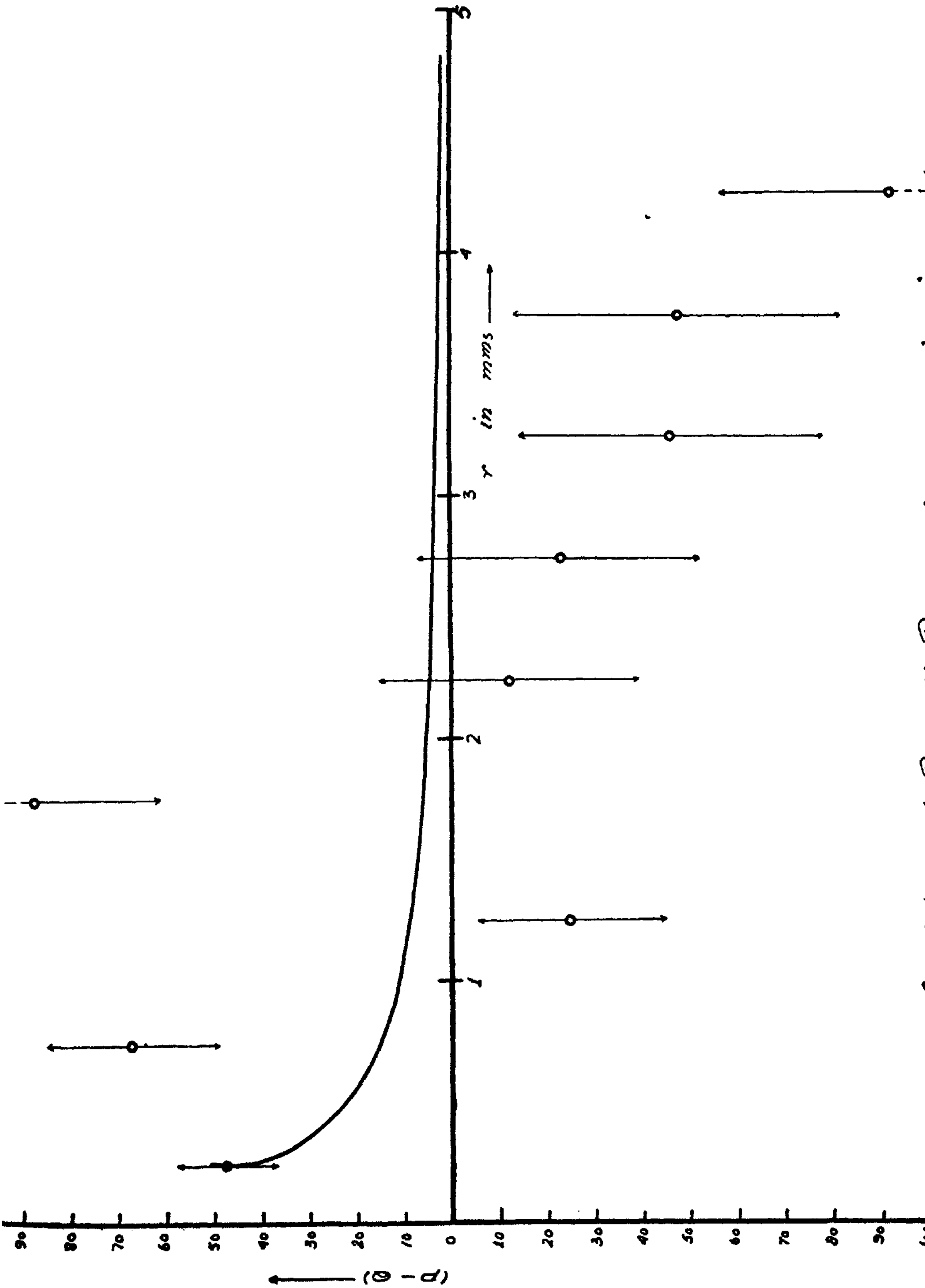

FIG. 6. Contribution of the Regenerative Process to the excess of close-pairs and its comparison with the observed excess for "Stars Under Pb Absorber" with prongs ≥ 2

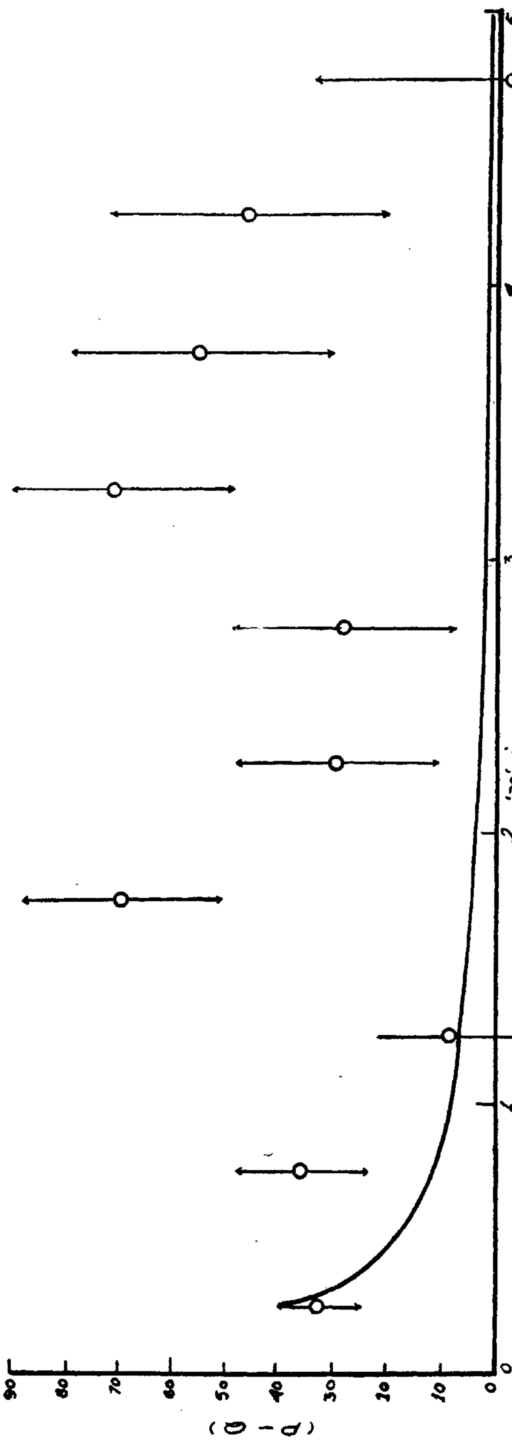

FIG. 7. Contribution of the Regenerative Process to the excess of close-pairs and its comparison with the observed excess for 'Stars Under Pb Absorber' with prongs ≥ 3

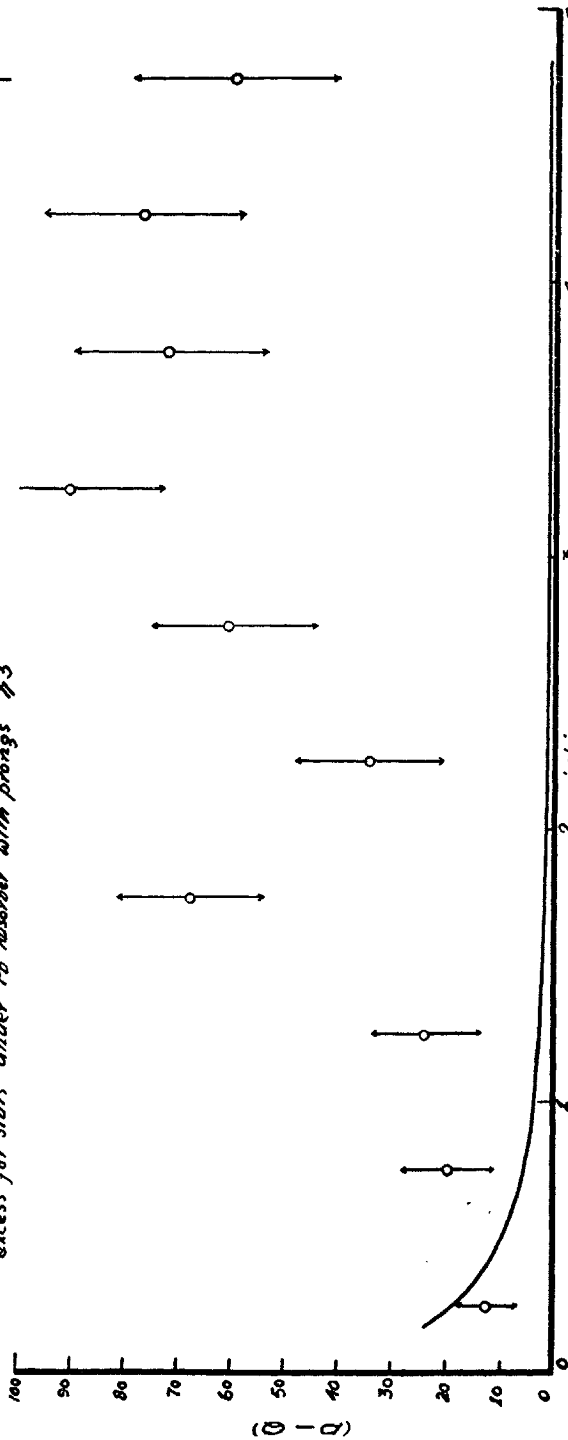

FIG. 8. Contribution of the Regenerative Process to the excess of close-pairs and its comparison with the observed excess for 'Stars Under Pb Absorber' with prongs' ≥ 4

(b) Single Penetrating Particles:

In this process it is assumed that 'Single Penetrating Particles' capable of producing two or more than two stars in their passage through the emulsion, give rise to the observed close-pair effect. A certain directional distribution of these particles has to be assumed. If it is assumed that these neutral penetrating particles are coming equally probably in all directions, then the number of close-pairs is the same as given by the 'Regenerative Process' except that 'q' is replaced by unity. From (3), therefore, we get the excess of close-pairs in the ith interval as

$$(Q\sigma)_i - Q_i = (N\sigma)/(1+K\sigma) \,.\, K_i. \qquad \qquad (7)$$

Assuming that the observed excess is entirely due to this process, then from (7) we can find out the cross-section of production of stars by taking $(Q_\sigma)_i = P_i$. In the case of 'All stars' with prongs $\geqslant 2$, $\sigma = 16{\cdot}94 \times 10^{-24}$ cm.^{2}; and in the case of 'Stars under no absorber' with prongs $\geqslant 2$, $\sigma = 8{\cdot}58 \times 10^{-24}$ cm.^{2} These values require that the particles should be highly interacting. Conversely, if it is assumed that the cross-section of production of stars is not greater than the geometrical value, then this process can explain only 5 per cent and 10 per cent of the total magnitude in the case of 'All stars' and 'Stars under no absorber' respectively. Similarly, in the case of other categories of stars, this process cannot explain more than a fraction of the total magnitude of the observed effect.

(c) Sheafs of Particles:

'Sheafs of Particles' produced in narrow angle cones in the material surrounding the plates and emitted isotropically in all downward directions can also give rise to non-randomness in the distribution of stars. Let 's' be the average number of particles in such a sheaf, having average conical angle 'α' and let λ, λ' be the mean free paths for the production of stars in emulsion, and absorption in the surrounding material respectively, then Li (1950) has shown that

$$\frac{P-Q}{N} = \left(\frac{s-1}{2}\,\frac{h}{\lambda}\right) \int_0^{\frac{\pi}{2}} [1 - \exp(-2r\cos\theta/\alpha\lambda')] \tan\theta \, d\theta$$
$$= C.g(r) \qquad \qquad (8)$$

where $(P-Q)$ is the excess of close-pairs of separation R $(0 \leqslant R \leqslant r)$, '$h$' is the thickness of the emulsion, $C = \frac{1}{2}(s-1)h/\lambda$ and $g(r)$ is the integral which is found out graphically by assuming different values of '$\alpha\lambda'$'.

In order to test this hypothesis, we can normalize $C.g(r)$ to the experimentally observed values of $(P-Q)/N$ and thus determine the values of 'C' for various values of $\alpha\lambda'$. From these values of 'C', average number of particles per sheaf 's' can be calculated for some assumed value of 'λ'. Table VI gives the values of 's' for some categories of stars for $\lambda = 18$ cm.

TABLE VI

$\alpha\lambda'$ (in cm.) =	1/20	1/200	1/2000
1. 'All stars' with prongs $\geqslant 2$	15	10	8
2. 'Stars under Pb absorber' with prongs $\geqslant 2$	14	9	8
3. 'Stars under no absorber' with prongs $\geqslant 2$	44	28	25

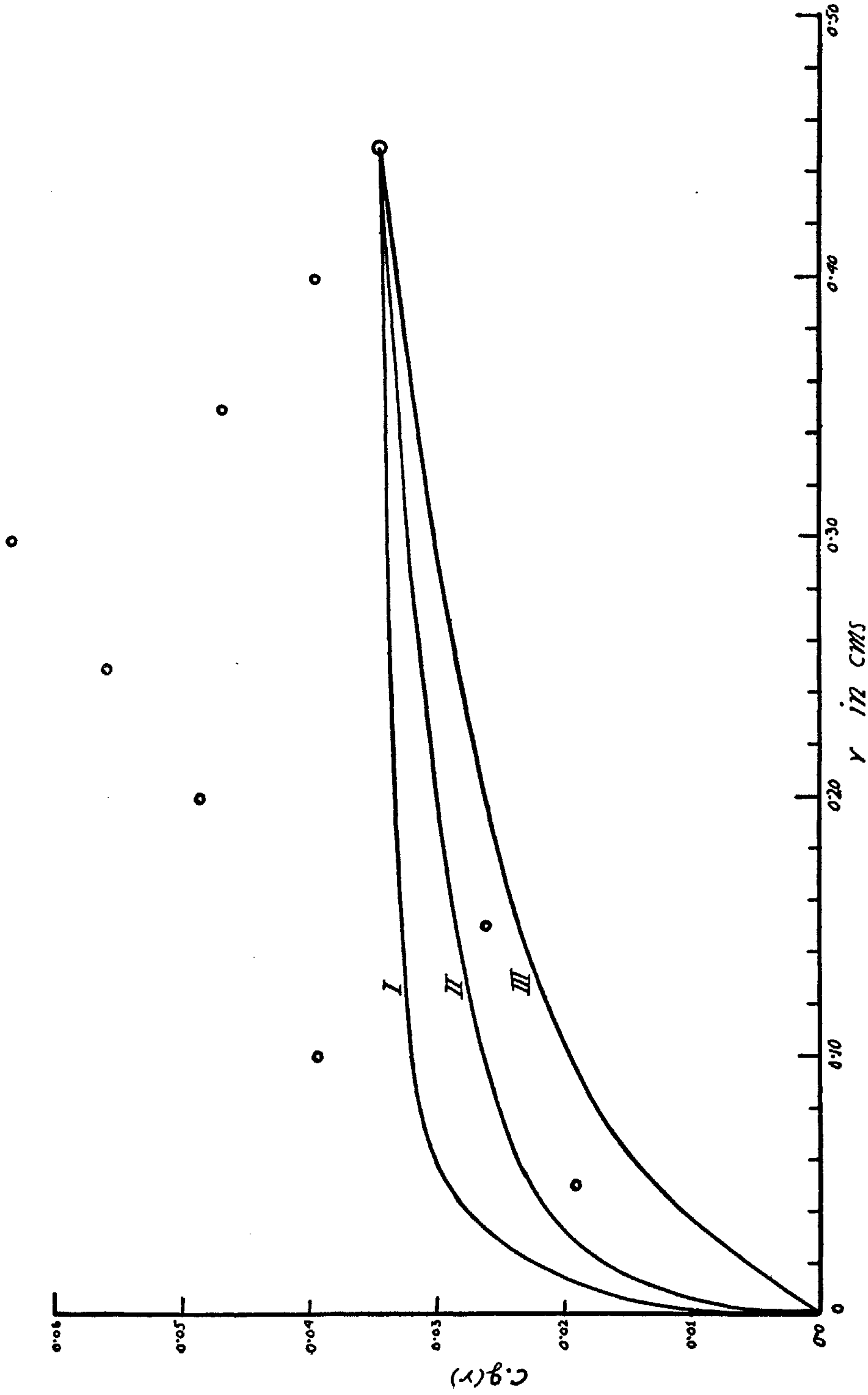

FIG. 9.

Contribution of 'Sheafs of Particles' to the excess of closed pairs and it's comparison with the observed excess for 'All Stars' with prongs ≯ 2

CURVE I for $\alpha\lambda' = 1/20$ cms
CURVE II for $\alpha\lambda' = 1/200$ cms
CURVE III for $\alpha\lambda' = 1/2000$ cms

The case of 'All stars' has been illustrated in Fig. 9 where the values of $C.g(r)$ normalized at $r = 0{\cdot}45$ cm. have been plotted along with the experimental values of $(P-Q)/N$ at different values of 'r'. From this figure it is clear that the value of $\alpha\lambda'$ which gives the best fit with the experimental results cannot be determined in this manner, as the $C.g(r)$ *vs.* 'r' curve does not change shape very much with $\alpha\lambda'$. However, some idea about the value of $\alpha\lambda'$ and hence of λ' can be had by taking a reasonably small value of 's'.

By choosing suitable values of α, λ, λ', this hypothesis can be made to explain the close-pair effect, especially the second part of it which remains positive up to 5·0 mm. or so. But it is unsatisfactory in the sense that rather high values of 's' and low values of 'λ'' have to be assumed to explain the observed magnitude.

V. Concluding Remarks

The discussion of the last section has shown that none of the mechanisms put forward to explain the close-pair effect can account for its total magnitude or detailed structure. This is not surprising in view of the many unknown factors, the correct evaluation of which, in general, is difficult to achieve.

Of the three mechanisms discussed, the 'Regenerative Process' has been more generally invoked to explain the close-pair effect. But because of the very nature of its theory, its contribution to the excess of close-pairs cannot be very large; after a few mms. it is almost negligible. It can be made to explain the 'first effect' by assuming rather large values of '$q\sigma$', which indicate the emission of large numbers of particles capable of producing secondary stars. However, it is difficult to visualize how so many particles can be emitted with energies large enough to produce further stars. Alternatively, large values of '$q\sigma$' indicate a very large interaction cross-section of these particles, which is again open to objection because no known particles have so large cross-section of production of stars. Similar difficulties arise when whole of the magnitude of close-pair effect is sought to be explained by 'Single Penetrating Particles'. The third mechanism of 'Sheafs of Particles' is based upon too many assumptions, but its parameters can be suitably adjusted to conform to the total magnitude of the effect.

There is no doubt that each of the above mechanisms contributes to some extent to bring about the non-random distribution of stars. But the effect of each of them cannot be separated and hence it is not possible to ascertain the exact contribution of each. The 'first effect' is probably brought about by the 'Regenerative Process', and the 'second effect' by the 'Sheafs of Particles'. But there is bound to be overlapping, the 'Regenerative Process' contributing to some extent to the 'second effect' and 'Sheafs of Particles' to the 'first effect'. 'Single Penetrating Particles' can be looked upon as contributing a little to both the processes.

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