LAMINAR FLOW THROUGH A UNIFORM CIRCULAR PIPE WITH SMALL SUCTION

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In this paper an attempt has been made to find the solution of Navier-Stokes equations for the steady flow of a viscous incompressible fluid through a porous pipe of uniform circular cross-section with small suction at the porous wall. An exact solution of the dynamical equations reduced to third and second order non-linear differential equations with appropriate boundary conditions is obtained. It is found that the maximum velocity of the flow exists on the axis of the pipe and that at the centre of the mouth it is greater than the maximum velocity of the Poiseuille flow. The axial pressure gradient and the axial velocity decrease along the length of the channel and vanish simultaneously at a finite distance from the mouth of the pipe. It is noted that due to suction an adverse pressure gradient is developed beyond the point where both axial pressure gradient and axial velocity vanish, which causes a back flow from infinity to this point. The magnitude of the skin friction coefficient increases with the increase of the suction parameter λ, defined with reference to suction velocity and pipe radius. For $\lambda = 0$, the results transform to the known results of Hagen-Poiseuille flow.

Introduction

An analysis is presented for the flow of a viscous incompressible fluid through a porous pipe of uniform circular cross-section with small suction at the porous wall. An exact solution of the Navier-Stokes equations, reduced to third and second order non-linear differential equations with appropriate boundary conditions, is obtained. A perturbation method is used to solve the equations for small flow through the porous wall.

The exact solution of Navier-Stokes equations for the flow of a viscous incompressible fluid between two parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate, has been recently investigated by Verma and Bansal (1966). The Hagen-Poiseuille flow in a circular pipe is well known. Yuan and Finkelstein (1956) discussed the laminar pipe flow with injection and suction through a porous wall under the assumption that the maximum velocity of the Hagen-Poiseuille flow exists at the centre of the mouth of the channel. But it is found that the pressure gradient along the axis, at the mouth of the channel, is not the same as the pressure gradient of the Poiseuille flow.

In a recent paper Choudhary and Sinha (1964) have discussed the steady flow of a viscous incompressible fluid through a uniform circular pipe with small outward normal suction under the assumptions that the pressure is uniform over a cross-section and that the axial pressure gradient is uniform throughout the channel. The assumption on pressure seems to be very restricted. The uniform suction at the boundary may be very small; even then the change in the pressure gradient along the axis of the pipe cannot be neglected. Moreover, their final expressions for the axial velocity distribution and the radial velocity distribution do not satisfy the equation of continuity.

We have reconsidered this problem under the assumptions that the flow is due to the pressure gradient of the Hagen-Poiseuille flow at the mouth and that the suction at the wall is uniform and small. Moreover, a perturbation method is used to determine the motion instead of assuming the form of stream function as done by Yuan and Finkelstein.

It is found that the maximum velocity of the flow exists on the axis of the pipe and that at the centre of the mouth it is greater than the maximum velocity of the Poiseuille flow. The axial velocity distribution is parabolic. The axial pressure gradient and the axial velocity decrease along the length of the channel and vanish simultaneously at a finite distance from the mouth of the pipe. It is interesting to note that due to suction an adverse pressure gradient is developed beyond the point where both axial velocity and axial pressure gradient vanish, which causes a back flow from infinity to this point. In the present case the radial velocity, which vanishes in the Poiseuille flow, has a finite magnitude except at the axis of the pipe where it vanishes.

1. Formulation of the Problem

The Navier-Stokes equations of motion in cylindrical polar coordinates for a viscous incompressible steady flow are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}+\frac{\omega}{r}\frac{\partial u}{\partial \phi} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu[\nabla^2 u], \quad \ldots \quad (1.1)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}+\frac{\omega}{r}\frac{\partial v}{\partial \phi}-\frac{\omega^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left[\nabla^2 v-\frac{v}{r^2}-\frac{2}{r^2}\frac{\partial \omega}{\partial \phi}\right], \quad \ldots \quad (1.2)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial r}+\frac{w}{r}\frac{\partial w}{\partial \phi}+\frac{vw}{r} = -\frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \phi}+\nu\left[\nabla^2 w+\frac{2}{r^2}\frac{\partial v}{\partial \phi}-\frac{w}{r^2}\right], \quad \ldots \quad (1.3)$$

and the equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{1}{r}\frac{\partial w}{\partial \phi} = 0, \quad \ldots \quad (1.4)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}+\frac{\partial^2}{\partial x^2},$$

and x is taken in the direction of the flow, r is the radial direction, ϕ is the azimuthal angle and u, v, w are the velocity components along x, r, ϕ increasing.

If the axis of x is taken as the axis of the cylinder and r is measured at right angles to it, then for laminar flow through a circular pipe

$$w = 0, \quad \frac{\partial}{\partial \phi} = 0. \qquad (1.5)$$

Hence eqn. (1.3) identically vanishes and eqns. (1.1), (1.2) and (1.4) become

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial x^2}\right], \qquad (1.6)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right] \qquad (1.7)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r} = 0. \qquad (1.8)$$

The boundary conditions are

and

$$\left.\begin{array}{l} r = R;\ v = v_0,\ u = 0 \\ r = 0;\ v = 0,\ \dfrac{\partial u}{\partial r} = 0 \end{array}\right\}. \qquad (1.9)$$

Since there is uniform suction, $\frac{\partial v}{\partial x} = 0$, therefore v is a function of r only. It is evident from eqn. (1.8) that $\frac{\partial^2 u}{\partial x^2} = 0$.

Thus the eqns. (1.6), (1.7), (1.8) reduce to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right], \qquad (1.10)$$

$$v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right] \qquad (1.11)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r} = 0. \qquad (1.12)$$

Let us introduce the non-dimensional quantities as follows:

$$\bar{u} = \frac{u}{U_m},\ \bar{v} = \frac{v}{v_0},\ \bar{x} = \frac{x}{R},\ \eta = \frac{r}{R},\ \bar{p} = \frac{p}{\rho U_m^2},$$

$$\lambda = \frac{v_0 R}{\nu},\ R_e = \frac{U_m R}{\nu}, \qquad (1.13)$$

where R is the radius of the cylinder, v_0 is the suction velocity and U_m is the maximum velocity of the Poiseuille flow when there is no suction, and λ

and R_e are the suction parameter and Reynolds number for the axial flow respectively.

The eqns. (1.10), (1.11) and (1.12) in non-dimensional form are

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\lambda}{R_e}\bar{v}\frac{\partial \bar{u}}{\partial \eta} = -\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{1}{R_e}\left(\frac{\partial^2 \bar{u}}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial \bar{u}}{\partial \eta}\right), \quad \ldots \quad \ldots (1.14)$$

$$\bar{v}\frac{\partial \bar{v}}{\partial \eta} = -\frac{R_e^2}{\lambda^2}\frac{\partial \bar{p}}{\partial \eta}+\frac{1}{\lambda}\left(\frac{\partial^2 \bar{v}}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial \bar{v}}{\partial \eta}-\frac{\bar{v}}{\eta^2}\right) \quad \ldots \quad \ldots (1.15)$$

and

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\lambda}{R_e}\left(\frac{\partial \bar{v}}{\partial \eta}+\frac{\bar{v}}{\eta}\right) = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (1.16)$$

The boundary conditions (1.9) with the aid of (1.13) are

$$\left.\begin{aligned} &\eta = 1\,;\ \bar{v} = 1,\quad \bar{u} = 0 \\ &\text{and} \\ &\eta = 0\,;\ \bar{v} = 0,\ \frac{\partial \bar{u}}{\partial \eta} = 0 \end{aligned}\right\}. \quad \ldots \quad \ldots \quad \ldots (1.17)$$

Let

$$\left.\begin{aligned} \bar{p}(\bar{x}, \eta) &= p_0+p'(\bar{x}, \eta) \\ \bar{u}(\bar{x}, \eta) &= u_0+u'(\bar{x}, \eta) \\ \bar{v} &= v'(\eta) \end{aligned}\right\}, \quad \ldots \quad \ldots \quad \ldots (1.18)$$

where the primed quantities are the perturbations caused by the suction and p_0 and u_0 are the known quantities for flow when there is no suction (i.e. the Hagen-Poiseuille flow in a circular pipe (Schlichting 1960)), satisfying the equations

$$\left.\begin{aligned} &\frac{\partial p_0}{\partial \eta} = 0,\ \frac{\partial u_0}{\partial \bar{x}} = 0 \\ &\text{and} \\ &\frac{\partial p_0}{\partial \bar{x}} = \frac{1}{R_e}\left(\frac{\partial^2 u_0}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial u_0}{\partial \eta}\right) \end{aligned}\right\}. \quad \ldots \quad \ldots \quad \ldots (1.19)$$

We have p_0 independent of η and

$$u_0 = (1-\eta^2). \quad \ldots \quad \ldots \quad \ldots \quad \ldots (1.20)$$

Substituting (1.18) in eqns. (1.14), (1.15) and (1.16), we get

$$u_0\frac{\partial u'}{\partial \bar{x}}+\frac{\lambda}{R_e}v'\frac{\partial u_0}{\partial \eta}+u'\frac{\partial u'}{\partial \bar{x}}+\frac{\lambda}{R_e}v'\frac{\partial u'}{\partial \eta} = -\frac{\partial p'}{\partial \bar{x}}+\frac{1}{R_e}\left[\frac{\partial^2 u'}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial u'}{\partial \eta}\right], \quad (1.21)$$

$$v'\frac{\partial v'}{\partial \eta} = -\frac{R_e^2}{\lambda^2}\frac{\partial p'}{\partial \eta}+\frac{1}{\lambda}\left[\frac{\partial^2 v'}{\partial \eta^2}+\frac{1}{\eta}\frac{\partial v'}{\partial \eta}-\frac{v'}{\eta^2}\right] \quad \ldots \quad \ldots \quad \ldots \quad \ldots (1.22)$$

and

$$\frac{\partial u'}{\partial \bar{x}}+\frac{\lambda}{R_e}\left(\frac{\partial v'}{\partial \eta}+\frac{v'}{\eta}\right) = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (1.23)$$

The boundary conditions (1.17) with the aid of (1.18) are

and
$$\left.\begin{aligned} \eta = 1\,;\ v' = 1, \quad u' = 0 \\ \eta = 0\,;\ v' = 0,\ \frac{\partial u'}{\partial \eta} = 0 \end{aligned}\right\} \qquad (1.24)$$

2. Method of Solution

Let
$$v' = \frac{1}{\eta} f(\eta), \qquad (2.1)$$

therefore from eqn. (1.23) we have

$$u' = -\frac{\lambda}{R_e}\bar{x}\frac{1}{\eta}f'(\eta) + F(\eta), \qquad (2.2)$$

where $f(\eta)$ and $F(\eta)$ are the unknown functions to be determined.

Substituting (2.1), (2.2) in eqns. (1.21) and (1.22), we get

$$\frac{\partial p'}{\partial \bar{x}} = \frac{1}{R_e}\left\{F'' + \frac{1}{\eta}F' + \frac{\lambda}{\eta}[(1-\eta^2)f' + 2\eta f + Ff' - fF']\right\}$$
$$-\frac{\lambda}{R_e^2}\frac{\bar{x}}{\eta^3}[\eta^2 f''' - \eta f'' + f' + \lambda(\eta f'^2 - \eta f f'' + ff')], \qquad (2.3)$$

$$\frac{\partial p'}{\partial \eta} = \frac{\lambda}{R_e^2}\frac{1}{\eta^3}[\eta^2 f'' - \eta f' - \lambda(\eta f f' - f^2)]. \qquad (2.4)$$

At the mouth of the channel the pressure gradient along the axis of the channel is the same as the pressure gradient of the Poiseuille flow.

Therefore, at $\bar{x} = 0$, $\dfrac{\partial p'}{\partial \bar{x}} = 0$.

Hence from (2.3)
$$F'' + \frac{1}{\eta}F' + \frac{\lambda}{\eta}\{(1-\eta^2)f' + 2\eta f + Ff' - fF'\} = 0. \qquad (2.5)$$

Differentiating (2.4) with respect to $\bar{x}$, we have

$$\frac{\partial^2 p'}{\partial \bar{x}\partial \eta} = 0. \qquad (2.6)$$

Now differentiating (2.3) with respect to η and using (2.5) and (2.6), we get
$$\frac{d}{d\eta}\left\{\frac{1}{\eta^3}[\eta^2 f''' - \eta f'' + f' + \lambda(\eta f'^2 - \eta f f'' + ff')]\right\} = 0 \qquad (2.7)$$

which is to be satisfied for all x.

Integrating eqn. (2.7), we have

$$\eta^3 f''' - \eta^2 f'' + \eta f' + \lambda(\eta^2 f'^2 - \eta^2 f f'' + \eta f f') = c\eta^4, \qquad (2.8)$$

where c is the constant of integration to be determined.

Now we have to obtain the solutions of (2.5) and (2.8) with the help of the following boundary conditions:

$$\left.\begin{aligned} &f(1) = 1,\ f'(1) = 0,\ F(1) = 0 \\ &f(0) = 0,\ \lim_{\eta\to 0}\frac{d}{d\eta}\left\{\frac{f'(\eta)}{\eta}\right\} = 0 \text{ and } F'(0) = 0 \end{aligned}\right\}. \quad \ldots \quad (2.9)$$

3. Solution for Small Suction Parameter

The solution for eqn. (2.8) can be expressed for small values of λ by a power series developed near $\lambda = 0$ as follows:

$$f = f_0+\lambda f_1+\lambda^2 f_2+\ldots+\lambda^n f_n \quad \ldots \quad \ldots \quad \ldots \quad (3.1)$$

and

$$c = c_0+\lambda c_1+\lambda^2 c_2+\ldots+\lambda^n c_n, \quad \ldots \quad \ldots \quad \ldots \quad (3.2)$$

where the f_n's and c_n's are taken to be independent of λ.

By substituting eqn. (3.1) and (3.2) in eqn. (2.8) and equating coefficients of like powers of λ, we get the following set of equations:

$$\eta^3 f_0''' - \eta^2 f_0'' + \eta f_0' = c_0\eta^4, \quad \ldots \quad (3.3)$$

$$\eta^3 f_1''' - \eta^2 f_1'' + \eta f_1' + \eta^2 f_0'^2 - \eta^2 f_0 f_0'' + \eta f_0 f_0' = c_1\eta^4 \quad \ldots \quad (3.4)$$

and

$$\eta^3 f_2''' - \eta^2 f_2'' + \eta f_2' + 2\eta^2 f_0' f_1' - \eta^2 f_1 f_0'' - \eta^2 f_0 f_1'' + \eta f_1 f_0' + \eta f_0 f_1' = c_2\eta^4. \quad \ldots \quad (3.5)$$

The boundary conditions to be satisfied by the f_n's are

$$\left.\begin{aligned} &f_n(0) = 0,\ f_n'(1) = 0,\ \lim_{\eta\to 0}\frac{d}{d\eta}\left\{\frac{f_n'}{\eta}\right\} = 0, \quad \text{for } n \geqslant 0 \\ &\text{also} \\ &f_0(1) = 1 \text{ and } f_n(1) = 0, \quad \text{for } n \geqslant 1 \end{aligned}\right\}. \quad \ldots \quad (3.6)$$

From eqns. (3.3) to (3.5) the second order perturbation solution of eqn. (2.8) is

$$f(\eta) = (2\eta^2-\eta^4)+\frac{\lambda}{36}(4\eta^2-9\eta^4+6\eta^6-\eta^8)$$

$$+\frac{\lambda^2}{5400}(166\eta^2-380\eta^4+275\eta^6-75\eta^8+15\eta^{10}-\eta^{12}), \quad \ldots \quad (3.7)$$

and

$$c = -16+12\lambda+\frac{88}{135}\lambda^2. \quad \ldots \quad (3.8)$$

It is seen from the foregoing equations that the second order perturbation solution is sufficiently accurate even for $\lambda = 1$.

Now, for the solution of eqn. (2.5), let

$$F(\eta) = -(1-\eta^2)+\frac{\phi(\eta)}{\eta}. \quad \ldots \quad (3.9)$$

The boundary conditions to be satisfied by the function $\phi(\eta)$ are

$$\phi(1) = 0 \text{ and } \lim_{\eta \to 0} \frac{d}{d\eta}\left\{\frac{\phi(\eta)}{\eta}\right\} = 0. \qquad \text{(3.10)}$$

Substituting the value of $F(\eta)$ from (3.9) in eqn. (2.5), we have

$$4\eta^3 + \eta^2\phi'' - \eta\phi' + \phi + \lambda[\eta\phi f' - \eta f\phi' + \phi f] = 0. \qquad \text{(3.11)}$$

It can be easily seen from (2.8) and (3.11) that

$$\phi(\eta) = -\frac{4}{c}f'(\eta). \qquad \text{(3.12)}$$

Hence

$$F(\eta) = -(1-\eta^2) - \frac{4}{c}\frac{f'(\eta)}{\eta}. \qquad \text{(3.13)}$$

Now, with the help of (1.18), (1.19), (2.3), (2.5) and (2.8), the axial pressure gradient is given by

$$-\frac{\partial \bar{p}}{\partial \bar{x}} = \frac{4}{R_e} + \frac{\lambda \bar{x}}{R_e^2}c, \qquad \text{(3.14)}$$

where c has the value given in (3.8).

The velocity components in the axial and in the radial directions obtained with the help of (1.18), (1.20), (2.1), (2.2), (3.7), (3.13) and (3.14) are

$$\bar{u} = \frac{R_e}{c}\frac{\partial \bar{p}}{\partial \bar{x}}\frac{1}{\eta}f'(\eta) \qquad \text{(3.15)}$$

and

$$\bar{v} = \frac{1}{\eta}f(\eta), \qquad \text{(3.16)}$$

where $f(\eta)$ is given by (3.7) and $\frac{\partial \bar{p}}{\partial \bar{x}}$ by (3.14). The pressure distributions in the axial and in the radial directions are obtained by the substitution of eqns. (2.5) and (2.8) in (2.3) and (2.4). On integrating, we have

$$\bar{p}(0, 0) - \bar{p}(\bar{x}, \eta) = \frac{4\bar{x}}{R_e} + \frac{\lambda c \bar{x}^2}{2R_e^2} - \frac{\lambda}{R_e^2}\frac{f'}{\eta} + \frac{\lambda^2}{2R_e^2}\frac{f^2}{\eta^2} + \frac{\lambda}{R_e^2}\left(4 + \frac{8\lambda}{36} + \frac{332}{5400}\lambda^2\right). \qquad \text{(3.17)}$$

The pressure drop in the flow direction can be readily obtained from (3.17), i.e.

$$\bar{p}(0, \eta) - \bar{p}(\bar{x}, \eta) = \frac{4\bar{x}}{R_e} + \frac{\lambda c \bar{x}^2}{2R_e^2}. \qquad \text{(3.18)}$$

The shearing stress at the wall is

$$\tau_0 = -\frac{\mu U_m}{R}\left(\frac{\partial \bar{u}}{\partial \eta}\right)_{\eta=1},$$

i.e.

$$\frac{\tau_0 R}{\mu U_m} = \frac{R_e}{c}\frac{\partial \bar{p}}{\partial \bar{x}}\left(8 - \frac{2\lambda}{3} - \frac{26}{135}\lambda^2\right). \qquad \text{(3.19)}$$

The coefficient of skin friction is directly obtained from (3.19) and can be written as

$$c_f = \frac{\tau_0}{\rho U_m^2/2} = \frac{2}{c}\frac{\partial \bar{p}}{\partial \bar{x}}\left(8 - \frac{2\lambda}{3} - \frac{26}{135}\lambda^2\right). \qquad (3.20)$$

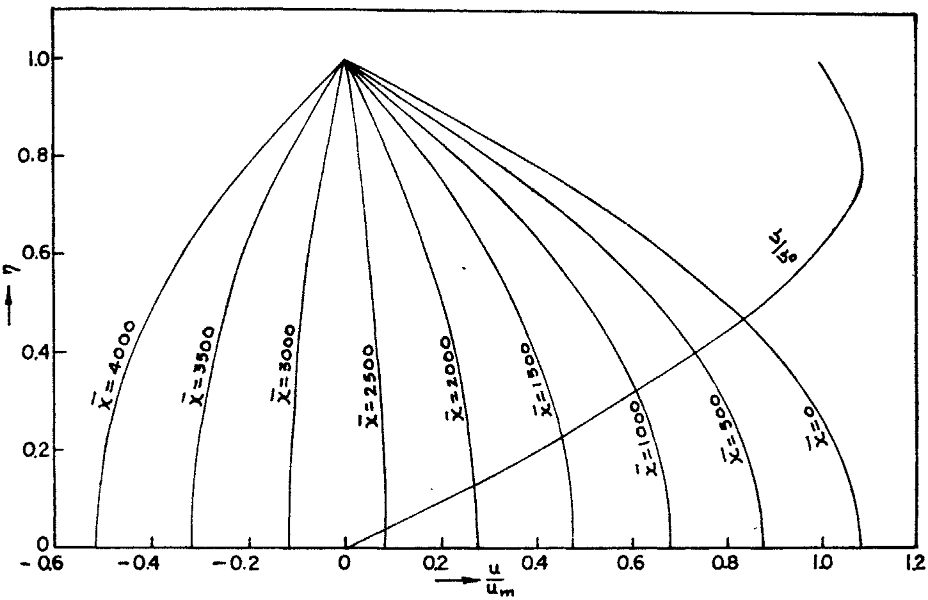

FIG. 1. Axial and radial velocity profiles plotted against $\eta(\lambda = 0{\cdot}1,\ R_e = 10^3)$.

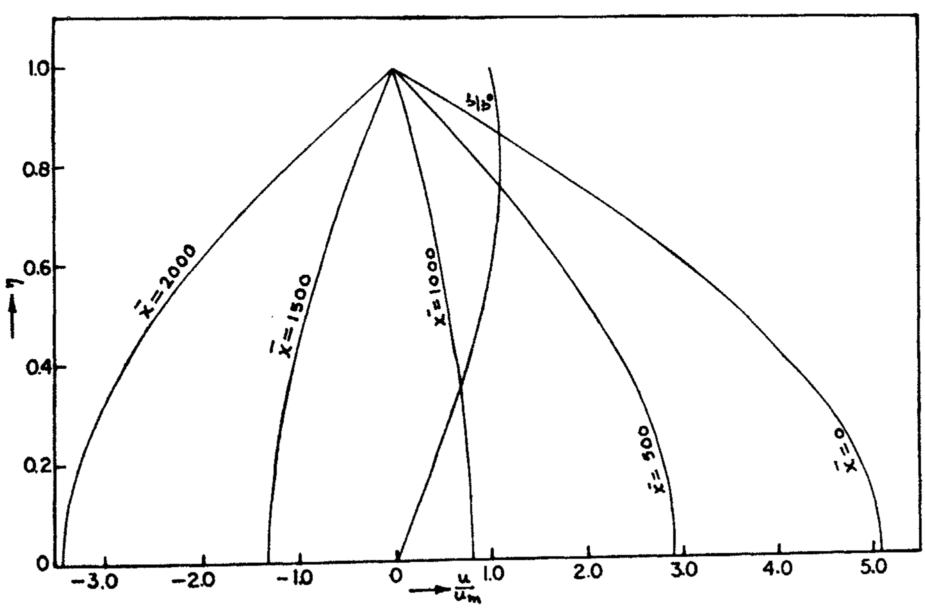

FIG. 2. Axial and radial velocity profiles plotted against $\eta(\lambda = 1,\ R_e = 10^3)$.

The flux across a section at $\bar{x}$ is

$$Q = 2\pi R^2 U_m \int_0^1 \bar{u}\eta \, d\eta,$$

$$= 2\pi R^2 U_m \frac{R_e}{c} \frac{\partial \bar{p}}{\partial \bar{x}}. \quad \ldots \quad (3.21)$$

The discharge for solid wall ($\lambda = 0$) is

$$Q_0 = \frac{\pi R^2 U_m}{2}. \quad \ldots \quad (3.22)$$

The discharge coefficient is

$$c_Q = \frac{Q}{Q_0} = \frac{4R_e}{c} \frac{\partial \bar{p}}{\partial \bar{x}}. \quad \ldots \quad (3.23)$$

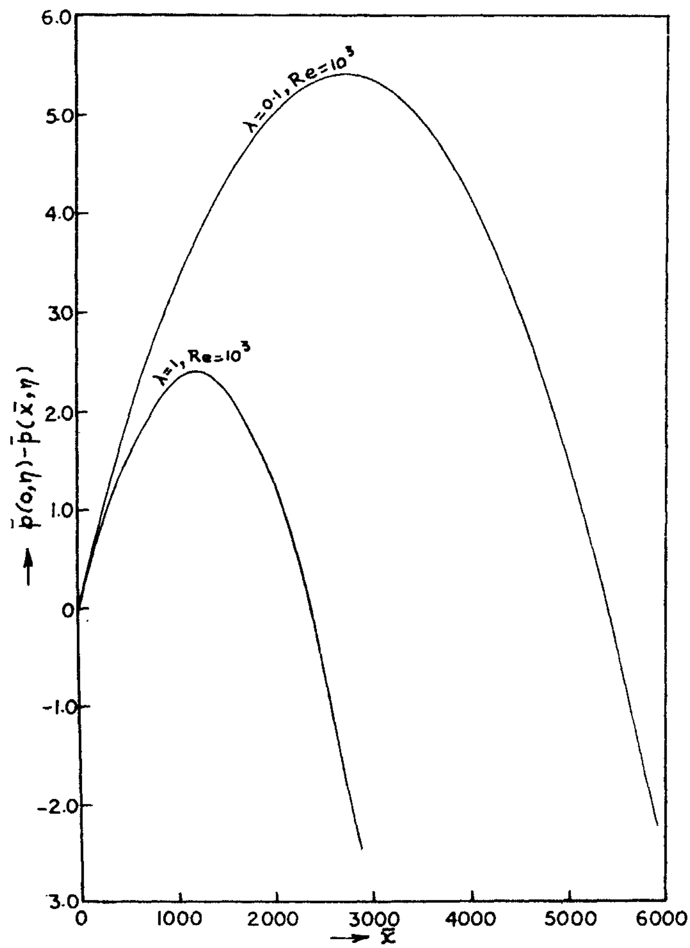

FIG. 3. Axial pressure drop *versus* length in flow direction for $\lambda = 0{\cdot}1$, 1 and $R_e = 10^3$.

4. Numerical Discussion

The axial and the radial velocity profiles for various values of $\bar{x}$ along the axis of the pipe are calculated from eqns. (3.15) and (3.16) for $\lambda = 0{\cdot}1$, 1 and $R_e = 10^3$, and are shown in Figs. 1 and 2. It is noted that the maximum velocity of the flow exists on the axis of the pipe and that at the centre of the mouth it is greater than the maximum velocity of the Poiseuille flow. The

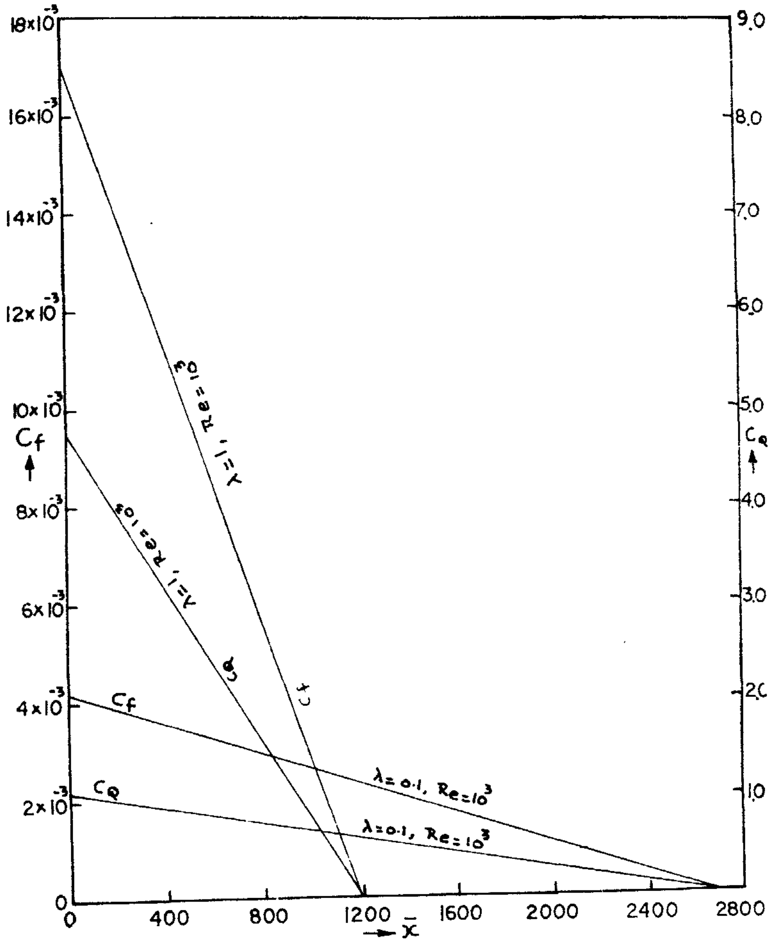

Fig. 4. Variation of coefficient of friction and flow coefficient versus length in flow direction for $\lambda = 0{\cdot}1$, 1 and $R_e = 10^3$.

axial velocity distribution is parabolic and decreases along the length of the channel. For $\lambda = 0{\cdot}1$, $R_e = 10^3$ at $\bar{x} = 2702{\cdot}7$ and for $\lambda = 1$, $R_e = 10^3$ at $\bar{x} = 1194{\cdot}4$, the axial velocity vanishes with the axial pressure gradient. It is interesting to note that due to suction an adverse pressure gradient is developed, beyond $\bar{x} = 2702{\cdot}7$ in case of $\lambda = 0{\cdot}1$ and $\bar{x} = 1194{\cdot}4$ in case of $\lambda = 1$, which causes a back flow from infinity to these points respectively. This phenomenon follows from the law of conservation of matter. In the present case the radial velocity, which vanishes in Poiseuille case, has a finite magnitude except at the axis of the pipe where it vanishes. It is interesting to note that the maximum radial velocity exists at $\eta = 0{\cdot}8$ and not at $\eta = 1$. The pressure drop along the axis of the channel is parabolic and is shown in Fig. 3 for $\lambda = 0{\cdot}1$, 1 and $R_e = 10^3$. The pressure increases beyond $\bar{x} = 5405{\cdot}4$ for $\lambda = 0{\cdot}1$ and $\bar{x} = 2388{\cdot}8$ for $\lambda = 1$ due to the occurrence of back flow.

The skin friction coefficient C_f, in the Poiseuille flow at the wall, has a constant value of $\frac{4}{R_e}$. In the present investigation the wall friction coefficient is calculated from (3.20) for $\lambda = 0{\cdot}1$, 1 and $R_e = 10^3$ and is shown in Fig. 4. It is found that due to suction the coefficient of wall friction decreases along the length of the channel and that it changes sign in the region of back flow. The magnitude of C_f increases with the increase of suction parameter λ. The flow coefficient is also calculated from (3.23) for $\lambda = 0{\cdot}1$, 1 and $R_e = 10^3$ and is shown in Fig. 4. It is noted that the flow coefficient C_Q behaves similar to C_f the coefficient of skin friction.

Proceeding to the limit $\lambda = 0$, we have

$$\bar{u} = (1-\eta^2),\ \bar{v} = 0,\ C_f = \frac{4}{R_e} \text{ and } C_Q = 1,$$

as in the case of the Hagen-Poiseuille flow.

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