RADIATION EFFECTS ON THE PROPAGATION OF PLANE PERPENDICULAR MAGNETOGASDYNAMIC SHOCK IN A PLASMA

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The general properties of a plane perpendicular magnetogasdynamic shock are studied with the inclusion of radiation pressure, radiation energy density and radiation flux escape. The medium is assumed to be a fully ionized perfect gas and the radiation flux escape parameter is taken independent of magnetic field. The effect of radiation is to enhance the compression ratio and to reduce the temperature of the downstream gas. The latter result is similar to that of magnetic field. Further, the isothermal radiative shocks are not possible for low β-plasmas. For a strong radiative shock with very high temperature, the compression ratio is independent of magnetic field and tends to a limiting value of seven.

1. INTRODUCTION

The interaction between hydrodynamic motion and magnetic field in plasma or in conducting fluids is of considerable significance in problems of plasma confinement and other laboratory experiments; and it has applications in astrophysical and geophysical situations and in the study of the behaviour of interstellar masses. Considering the significance of this interaction, magnetogasdynamic shocks have recently been discussed in detail by various authors (Hoffmann and Teller (1950); Helfer (1953); Lüst (1953, 1955); Bazer and Ericksons (1959, 1960); Shercliff (1960); Chang (1964)). In general, shock propagation involves very high temperatures where the radiation effects might play a significant role through the coupling of radiation and magnetogasdynamic fields. In the absence of magnetic fields such effects have been considered by Sachs (1946) and by Guess and Sen (1958). We, in this paper, investigate the effects of radiative transfer of energy and of radiation pressure and energy density on the propagation of plane perpendicular magnetogasdynamic shock in a plasma. The general properties of magnetogasdynamic shocks in presence of radiative terms may be useful in the heating of plasma for controlled thermonuclear devices and in certain astrophysical applications, e.g. double spectral lines in Cephied Variables, large density fluctuations in interstellar clouds, amplification of magnetic field, etc.

2. THE MODIFIED RANKINE-HUGONIOT CONDITIONS

Consider a plane perpendicular magnetogasdynamic shock in plasma propagating non-relativistically along the x-axis. The plasma may, however, be assumed to be of infinite conductivity. Further, the flow is made time-dependent by referring to a coordinate system moving with the shock front. We shall further use suffixes 1 and 2 to denote the physical variables (velocity U, material gas pressure p, magnetic field B, density ρ and temperature T) in front and back of the shock respectively. The following magnetogasdynamic equations will then describe the flow:

$$\rho_1 U_1 = \rho_2 U_2 = m, \quad \text{say} \qquad (1)$$

$$\rho U \frac{dU}{dx} = -\frac{d}{dx}\left(p+\frac{B^2}{8\pi}+\frac{a}{3}T^4\right) \qquad (2)$$

$$\rho U \frac{d}{dx}\left(E+\frac{B^2}{8\pi\rho}+\frac{aT^4}{\rho}\right) = -\left(p+\frac{B^2}{8\pi}+\frac{aT^4}{3}\right)\frac{dU}{dx}-\frac{dF}{dx}. \qquad (3)$$

These represent respectively the laws of conservation of mass, of momentum and of energy as applied to one-dimensional flow. Further, since the magnetic lines of force are frozen with the matter on account of infinite conductivity we shall have

$$\frac{B_1}{\rho_1} = \frac{B_2}{\rho_2}$$

or as a consequence of (1)

$$B_1 U_1 = B_2 U_2. \qquad (4)$$

$\frac{aT^4}{3}$ and $\frac{aT^4}{\rho}$ are the radiation pressure and radiation energy per unit mass and are added to the material pressure p and internal energy E respectively. F is the radiative flux and a is the radiation constant. In the absence of any known effects of the magnetic field on the opacity of the matter specially in plasma, we have assumed the above forms for the radiation pressure and energy density. These forms were obtained by Ledoux and Walraven (1958) for the hydrodynamic system.

In eqns. (2) and (3), it is assumed that we have thermodynamic equilibrium. In the absence of thermodynamic equilibrium, which in most of the cases of practical situations represents the state of affairs in the plasma, radiative flux and black body radiation cannot exist together until the medium is transparent to a wavelength region very far removed from the wavelength for maximum black body radiation (Guess and Sen 1958).

Integrating eqns. (2) and (3) and assuming uniform conditions in front and back of the shock, we obtain the equations

$$p_1+\frac{B_1^2}{8\pi}+\frac{aT_1^4}{3}+\rho_1 U_1^2 = p_2+\frac{B_2^2}{8\pi}+\frac{aT_2^4}{3}+\rho_2 U_2^2 \qquad (5)$$

and

$$E_1+\frac{p_1}{\rho_1}+\frac{B_1^2}{4\pi\rho_1}+\frac{4aT_1^4}{3\rho_1}+\tfrac{1}{2}U_1^2 = E_2+\frac{p_2}{\rho_2}+\frac{B_2^2}{4\pi\rho_2}+\frac{4aT_2^4}{3\rho_2}+\tfrac{1}{2}U_2^2+\frac{F_E}{\rho_1 U_1}, \qquad (6)$$

where $F_E = F_2 - F_1$ is the net radiative transport of energy from the shock front. We have not assumed any specific mechanism for the radiation escape. In case of shocks with strong magnetic field, one should include the radiation escape through synchrotron radiation, etc. The actual form of the radiation escape parameter, F_E, is not well understood and may, however, be fairly complex in the presence of magnetic field. Equations (1), (4)–(6) thus represent the modified Rankine-Hugoniot conditions representing the plane perpendicular magnetogasdynamic shock in a plasma. So far we have made no assumptions on the equation of state and the treatment is valid for any equation of state. We shall, however, assume plasma to form an ideal gas and hence

$$p = R\rho T; \quad E = \frac{RT}{\gamma-1}, \qquad \qquad (7)$$

where γ is the ratio of the two specific heats and is a fixed number. In the absence of magnetic field, the modified equations are (1), (5) and (6) with $B = 0$ and were earlier obtained by Guess and Sen (1958). For $F_E = 0$, the equations reduced to one obtained by Sachs (1946). When the radiation effects are neglected ($F = a = 0$) in eqns. (1), (4)–(6) we arrive at the same equation for magnetogasdynamic shock as were first obtained by Hoffmann and Teller (1950).

The modified Rankine-Hugoniot conditions can be written in a form exactly resembling the classical Rankine-Hugoniot conditions for the gas-dynamic shocks through the introduction of total pressure p^*, of total specific internal energy E^* and of the total specific enthalpy h^* defined as

$$\left.\begin{aligned} p^* &= p + \frac{B^2}{8\pi} + \frac{aT^4}{3} \\ E^* &= E + \frac{B^2}{8\pi\rho} + \frac{aT^4}{\rho} \\ h^* &= E^* + \frac{p^*}{\rho} \end{aligned}\right\} . \qquad \qquad (8)$$

and

Apart from the three Rankine-Hugoniot equations we have one additional condition on magnetic flux, viz. eqn. (4), and an additional term $\frac{F_E}{\rho_1 U_1}$ in eqn. (6).

We shall now obtain an expression for the modified sound velocity of the medium which determines the condition for shock propagation. Since in our case the magnetic field lines are normal to the direction of propagation of the waves, only the longitudinal modified magneto-acoustic wave will exist. Following Hoffmann and Teller (1950) and assuming $dF/dU = 0$, we get

$$v^2 = \frac{p}{\rho}\left[\frac{\gamma+20(\gamma-1)\alpha+16(\gamma-1)\alpha^2}{1+12(\gamma-1)\alpha}+\frac{2}{\beta}\right], \qquad \qquad (9)$$

where v is the effective velocity of the modified magneto-acoustic wave and

$$\alpha = \frac{aT^4}{3p} = \frac{\text{radiation pressure}}{\text{material pressure}}, \quad \ldots \quad (10a)$$

$$\beta = \frac{p}{B^2/8\pi} = \frac{\text{material pressure}}{\text{magnetic pressure}}. \quad \ldots \quad (10b)$$

As $\alpha \to 0$, we get

$$v^2 = S^2+A^2, \quad \ldots \quad (11)$$

where $S\ \{= \surd(\gamma p/\rho)\}$ is the velocity of sound in the medium and $A\ (= B\surd/4\pi\rho)$ is the Alfvén wave velocity. For very high β-plasma we shall have

$$v^2 = \frac{p}{\rho}\left[\frac{\gamma+20(\gamma-1)\alpha+16(\gamma-1)\alpha^2}{1+12(\gamma-1)\alpha}\right] \quad \ldots \quad (12)$$

which is the same as obtained by Sachs (1946). Thus the shock condition will not be affected in the presence of very small magnetic fields.

In case of low β-plasma, with dominating radiation effects ($\alpha \gg 1$), we will have

$$v^2 = S^2_{\text{rad}}+A^2, \quad \ldots \quad (13)$$

where $S_{\text{rad}}\left(= \sqrt{\frac{4}{3}\frac{\frac{aT^4}{3}}{\rho}}\right)$, and showing thereby that radiation behaves like a perfect gas with $\gamma_{\text{rad}} = \frac{4}{3}$. In the presence of very high magnetic fields, e.g.

$$B \gg \sqrt{\frac{16\pi}{9}aT^4}, \quad \ldots \quad (14)$$

the wave propagation will be dominatingly governed by the Alfvén wave velocity. On the other hand, if the radiation pressure is very much higher than the magnetic pressure, the radiation field will govern the wave propagation, and its speed will then be

$$v^2 = S^2_{\text{rad}}. \quad \ldots \quad (15)$$

In such situations the plasma will behave like ordinary gas with $\gamma_{\text{rad}} = \frac{4}{3}$ and thus represent the case of ordinary gasdynamics. In fact Chandrasekhar (1939) has pointed out that for a gasdynamic system radiation behaves like a perfect gas with $\gamma_{\text{rad}} = \frac{4}{3}$ at very high temperatures. Thus we conclude that at extremely high temperatures, the gas pressure as well as the magnetic pressure has no influence on the propagation velocity in the plasma.

3. Radiation Escape from a Plane Perpendicular Shock Propagating in a Plasma

The effect of radiation escape is of great importance in the study of controlled thermonuclear devices and also in the study of solar flares. In this treatment we shall neglect the relativistic effects and also the one arising from radiation pressure and energy density. Further, the actual mechanism of radiation escape is not well understood and thus we will not invoke any specific mechanism. Escape through synchrotron radiation, etc., cannot, however, be ignored.

Under these restricted conditions we can rewrite the modified Rankine-Hugoniot conditions as follows:

$$\rho_1 U_1 = \rho_2 U_2 \quad \ldots \quad (16)$$

$$B_1 U_1 = B_2 U_2 \quad \ldots \quad (17)$$

$$p_1 + \frac{B_1^2}{8\pi} + \rho_1 U_1^2 = p_2 + \frac{B_2^2}{8\pi} + \rho_2 U_2^2 \quad \ldots \quad (18)$$

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{B_1^2}{4\pi\rho_1} + \frac{U_1^2}{2} = \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{B_2^2}{4\pi\rho_2} + \frac{U_2^2}{2} + \frac{F_E}{\rho_1 U_1}. \quad \ldots \quad (19)$$

We can now define the shock strength parameter as

$$\chi^* = \frac{p_2^*}{p_1^*}, \quad \ldots \quad (20)$$

where p^* $(= p + B^2/8\pi)$ is the total pressure of the plasma. Further, the compression ratio $\eta(= \rho_2/\rho_1)$ gives

$$\eta = \frac{U_1}{U_2} = \frac{B_2}{B_1}. \quad \ldots \quad (21)$$

We also define the effective Mach number (M_e) as follows:

$$M_e^2 = \frac{U^2}{v^2}, \quad \ldots \quad (22)$$

where v^2 is given by eqn. (11). Thus we can write

$$\frac{1}{M_e^2} = \frac{1}{M^2} + \frac{1}{M_A^2}, \quad \ldots \quad (23)$$

where M $(= U/S)$ is the usual gasdynamic Mach number and M_A $(= U/A)$ is the Alfvén Mach number. The condition that the shock will propagate in the plasma, viz. $M_{e_1} > 1$, leads to $M_1 > 1$ and $M_{A_1} > 1$. Thus, *a necessary condition for the existence of a magnetogasdynamic shock is that the kinetic energy density of the flow upstream with respect to the shock exceeds* $\gamma(\gamma-1)$ *times the internal energy of the plasma and its magnetic energy density.*

In terms of the shock strength and the compression ratio we can write the momentum and energy eqns. (18) and (19) in the following form:

$$\eta - 1 = (\chi^* - 1)\eta \frac{p_1^*}{\rho_1 U_1^2} \quad \ldots \quad (24a)$$

$$\tfrac{1}{2}(\eta^2 - 1) + \frac{2-\gamma}{2(\gamma-1)}\eta^2(\eta-1)\frac{1}{M_{A_1}^2} - \eta^2\frac{F_E}{\rho_1 U_1^3} = \frac{\gamma}{\gamma-1}(\chi^* - \eta)\eta\frac{p_1^*}{\rho_1 U_1^2} \quad (24b)$$

or as

$$\eta-1=\left(1-\frac{1}{\chi^*}\right)\frac{p_2^*}{\rho_2 U_2^2} \quad \ldots \quad (25a)$$

$$\frac{\gamma-1}{2\gamma}(\eta^2-1)+\frac{2-\gamma}{2\gamma}\eta^2(\eta-1)\frac{1}{M_{A_1}^2}-\frac{\gamma-1}{\gamma}\eta^2\frac{F_E}{\rho_1 U_1^3}=\left(1-\frac{\eta}{\chi^*}\right)\frac{p_2^*}{\rho_2 U_2^2}. \quad (25b)$$

Eliminating χ^* between eqns. (24*a*, *b*), we obtain

$$\left[\frac{2-\gamma}{2(\gamma-1)}\frac{\eta}{M_{A_1}^2}+\frac{1}{\gamma-1}\frac{1}{M_{e_1}^2}+\frac{1}{2}\right](\eta-1)-\eta^2\frac{F_E}{(\eta-1)\rho_1 U_1^3}=\frac{1}{\gamma-1}\frac{M_{e_1}^2-1}{M_{e_1}^2}. \quad (26)$$

Similarly, from eqns. (25*a*, *b*), we have

$$\left[\frac{\gamma+1}{2}+\frac{2-\gamma}{2}\frac{\eta^2}{M_{A_1}^2}\right](\eta-1)+\frac{\gamma-1}{\eta-1}\eta^2\frac{F_E}{\rho_1 U_1^3}=\frac{1-M_{e_2}^2}{M_{e_2}^2}. \quad \ldots \quad (27)$$

In the absence of any radiation flux escape we have for $M_{e_1} > 1$, $\eta > 1$ from eqn. (26) and $M_{e_2} < 1$ from eqn. (27). These results were earlier obtained by Chang (1964). However, for the same gas with the same M_{e_1} the presence of radiation escape will enhance the compression ratio. The other effect could have been to decrease η to less than one for very high values of F_E. In such a case, F_E may not remain independent of other physical parameters and hence one should consider the problem *de novo*.

Now, eqn. (24*b*) could be solved to give the expression in terms of temperature ratio. Thus, after a little algebra, we get

$$\frac{T_2-T_1}{T_1}=\frac{\gamma-1}{2}\left[\left(1-\frac{1}{\eta^2}\right)M_1^2-\frac{4}{\gamma}\frac{\eta-1}{\beta_1}-\frac{2F_E}{\rho_1 U_1^3}M_1^2\right]. \quad \ldots \quad (28)$$

In the absence of radiation flux escape, $\eta > 1$ and the temperature downstream will be smaller than that upstream for a low upstream gasdynamic Mach number but may become large at very high upstream gasdynamic Mach numbers. For very large β_1, the downstream temperature will be larger than the upstream temperature at comparatively lower gasdynamic Mach numbers. Further, Chang (1964) has shown that the magnetic field has little effect on the compression ratio for strong shocks ($M_1 > 50$) and that above this Mach number they closely approach the usual values for strong gasdynamic shocks. The latter results indicate that in the shock speed range (of interest) in problems of controlled thermonuclear fusion the magnetic field has almost no direct influence as far as heating of the gas is concerned. Further, they may indirectly worsen the situation because of radiation losses from synchrotron radiation, etc., since the energy loss from the shock wave region will change the usual magnetic gasdynamic Rankine-Hugoniot conditions.

However, when there is radiation escape a similar situation will arise, i.e. the temperature downstream will be less than the upstream one since the radiation escape too will, like the magnetic field, reduce the temperature of the downstream gas.

Further, from eqn. (24*a*) we see that

$$\chi^* \gtrless 1 \quad \text{for } \eta \gtrless 1. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

Thus, when there is radiation escape there will be increments in the pressure, in the magnetic field and in the density of the plasma behind the shock since both the shock strength parameter χ^* and the compression ratio η are greater than unity.

Following Guess and Sen (1958), we now obtain an absolute upper limit for the magnitude of the radiation flux escape by using the isothermality condition wherein the radiation escaping through the shock front reduces the temperature behind it to such an extent that there is essentially no temperature jump across the shock front. However, in the presence of the magnetic field, this condition may not necessarily be obtaining at all Mach numbers. The physical realization of an isothermal shock may not be possible; but, as will be shown later, the isothermality may be approached in certain extreme cases. The usual magnetic gasdynamic shock may be termed as an adiabatic shock.

Assuming the isothermality condition, viz. T_2-T_1, we get from (28)

$$\frac{2F_E}{\rho_1 U_1^3} = 1 - \frac{1}{\eta^2} - \frac{2(\eta-1)}{M_{A_1}^2}, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

where

$$\frac{1}{\eta} = \frac{1+\dfrac{1}{\beta_1} \pm \sqrt{\left(1+\dfrac{1}{\beta_1}\right)^2 + \dfrac{4}{\beta_1}}}{2\gamma M_1^2}. \qquad \ldots \quad \ldots \quad \ldots \quad (31)$$

Here, we have made use of eqns. (10*b*), (23), (24*b*) and (30).

In the limit of very high β_1, we get

$$\eta = \gamma M_1^2 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32a)$$

and

$$\frac{2F_E}{\rho_1 U_1^3} = 1 - \frac{1}{\gamma^2 M_1^4}. \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32b)$$

These are the usual gasdynamic limits. But for very low β_1 values we have

$$\eta \to \gamma \beta_1 M_1^2 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33a)$$

and

$$\frac{2F_E}{\rho_1 U_1^3} = -3 + \frac{2}{M_{A_1}^2}\left(1 - \frac{1}{8M_{A_1}^2}\right). \qquad \ldots \quad \ldots \quad (33b)$$

From (33*a*) it is evident that isothermality cannot be attained for low β-plasma in the case of low upstream gasdynamic Mach number. Further, for large values of upstream Alfvén Mach number, we get a negative value of F_E indicating again that the isothermality condition cannot be achieved. Thus for a low β-plasma, it can be inferred that a radiative shock will always be adiabatic. However, a radiative shock may be isothermal in the intermediate range of Mach number for $\beta > 1$.

4. Strong Radiative Plane Perpendicular Magnetogasdynamic Shock in Plasma

We shall now study the properties of strong plane perpendicular magnetogasdynamic shock in plasma which is radiating. Following the close analogy of such a shock with ordinary gasdynamic shock we may consider a strong radiative shock as the one in which the total pressure is negligible in comparison with the dynamic pressure $\rho_1 U_1^2$ and the total specific enthalpy h_1^* is negligible in comparison with the kinetic energy density $\frac{U_1^2}{2}$ in the flow upstream of the shock. The total pressure, p^*, and the total specific enthalpy, h^*, are defined, *vide* eqn. (8).

Under these conditions, the Rankine-Hugoniot conditions (5) and (6) can be written as

$$\rho_1 U_1^2 = p_2 + \frac{B_2^2}{8\pi} + \rho_2 U_2^2 + \frac{aT_2^4}{3} \qquad \text{(34)}$$

and

$$\frac{U_1^2}{2} = \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{B_2^2}{4\pi\rho_2} + \frac{4aT_2^4}{3\rho_2} + \frac{U_2^2}{2} + \frac{F_E}{\rho_1 U_1}. \qquad \text{(35)}$$

The condition (34) in terms of the compression ratio η and the shock strength parameter χ^*, defined as

$$\chi^* = \frac{p_2 + \frac{B_2^2}{8\pi} + \frac{aT_2^4}{3}}{p_1 + \frac{B_1^2}{8\pi} + \frac{aT_1^4}{3}} = \frac{p_2^*}{p_1^*}, \qquad \text{(36)}$$

can be written as

$$\chi^* = \frac{p_1}{p_1^*}\left(1 - \frac{1}{\eta}\right)\gamma M_1^2. \qquad \text{(37)}$$

Thus the strength of the shock is reduced by a factor of the ratio of the upstream gas pressure to the total pressure—a result obtained by Chang (1964) in the absence of radiation effects. Further, from (34) and (35) after a little algebra, we get

$$\eta = \frac{\dfrac{\gamma+1}{\gamma-1}p_2 + \dfrac{3B_2^2}{8\pi} + \dfrac{7aT_2^4}{3}}{p_2 + \dfrac{B_2^2}{8\pi} + \dfrac{aT_2^4}{3} - \dfrac{2F_E}{U_1}}, \qquad \text{(38)}$$

which in the absence of radiation effects reduces to

$$\eta = \frac{\gamma+1}{\gamma-1} - \frac{2(2-\gamma)}{\gamma-1}\,\frac{B_2^2/8\pi}{p_2+(B_2^2/8\pi)}. \qquad \text{(39)}$$

Thus, the compression ratio in the presence of magnetic field is less than that in the analogous gasdynamic case and depends on the ratio of the magnetic pressure to the total pressure of the downstream plasma. Chang (1964) has, however, concluded that the compression ratio is independent of magnetic field—a conclusion which is not valid.

On the other hand, in the limit of very high temperatures, terms of the order of aT_2^4 may dominate and we are led to the result

$$\eta \rightarrow 7, \qquad \text{(40)}$$

which could be written, in close analogy to gasdynamics, as

$$\eta = \frac{\gamma_{\text{rad}}+1}{\gamma_{\text{rad}}-1}, \qquad \text{(41)}$$

since at very high temperatures the plasma will behave like a perfect gas with $\gamma_{\text{rad}} = \frac{4}{3}$. The expression (41) clearly indicates the importance of radiation pressure and density at very high temperatures. Similar conclusions were obtained by Guess and Sen (1958).

In this expression γ refers to the ratio of the two specific heats in the downstream plasma. Further, if one may assume γ to be variable on account of the increase in the number of active internal degrees of freedom of the plasma particles in the downstream gas, then from (38) we conclude that

$$\text{when } \lim \gamma \rightarrow 1, \quad \lim \eta \rightarrow \infty.$$

This means that in such cases the shock will behave like a snow-plough.

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