ON THE INITIAL STAGE OF BURNING OF MULTITUBULAR POWDERS IN A RCL HIGH-LOW PRESSURE GUN AND ITS DEPENDENCE ON THE RATIO OF CHAMBER VOLUMES FOR MULTITUBULAR POWDERS

PART I. ISOTHERMAL MODEL

by T. C. JAIN, *Department of Mathematics, University of Rajasthan, Jaipur*

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In the existing theory of a RCL high-low pressure gun it is assumed that the ratio of the pressure in the second chamber to that in the first chamber attains a critical value from the beginning. In the present paper it is assumed, following Ray (1962), that the pressure ratio falls from a supercritical value to the critical value before the shot begins to move and as a necessary condition for this, upper and lower limits for the ratio of chamber volumes of RCL high-low pressure guns using multitubular powders have been obtained. It is further assumed that the ratio of the atmospheric pressure to the pressure in the first chamber is less than the critical value which satisfies the physical conditions. The theory developed here may be treated as an extension of that of Ray (1962).

NOTATIONS

P_1 = Mean pressure in the first chamber

P_2 = Mean pressure in the second chamber

V_1 = Volume of the first chamber

V_2 = Volume of the second chamber

λ = Mean value of RT, i.e. force constant

C = Mass of the charge

δ = Density of the charge

N_1 = Fraction of the charge remaining at time t in the first chamber in the gaseous state

N_2 = Fraction of the charge remaining at time t in the second chamber in the gaseous state

f = Fraction of the web-size of the charge remaining at time t

D = Web-size of the charge

$\epsilon = \dfrac{\eta\rho'}{1-\eta\rho'}$

η = Co-volume of the propellant gases

ρ' = Density of the gases

S_1 = Area of the exit nozzle
S_2 = Area of the middle nozzle.

Introduction

Kapur (1957) developed the theory for a RCL high-low pressure gun and later Gupta (1960) worked on RCL high-low pressure guns using heptatubular powders. They had assumed basically the presence of steady state of mass flow when the shot begins to move and also that the rate of flow is settled by the pressure in the first chamber alone. According to the classical theory of nozzles (Corner 1950) steady state of flow is reached when $\omega = P_2/P_1$ (i.e. the ratio of the pressures in the second and in the first chambers) attains a value which is equal to or less than the critical value ω^*. Corner found this critical value to be 0·555 taking into account co-volume correction. No definite information is available as to how the pressures in the two chambers develop and how the pressure ratio attains a critical value. Ray (1962) has found an upper and a lower limit for the ratio of chamber volumes of a high-low pressure gun and has obtained an upper limit for the fraction of the charge burnt when the steady state of flow is reached in the case of propellants which have a quadratic formfunction under the following assumptions: (i) that there exists roughly a closed vessel condition in the second chamber and a nearly symmetric flow through the nozzle; (ii) that immediately after ignition, before the shot starts pressure has been attained, the pressure ratio P_2/P_1 remains supercritical; but as burning continues the pressure ratio rapidly decreases, attains a critical value and afterwards it becomes subcritical; (iii) that the shot starts when the pressure ratio attains the critical or subcritical value.

In the present paper we have developed the theory for RCL high-low pressure guns using propellants having general cubic formfunction $z = (1-f)(a-bf-cf^2)$ (Jain 1964) and have (i) obtained an upper and a lower limit for the ratio of chamber volumes in order that the value of ω falls from a supercritical value to the critical value ω^* before the shot begins to move; (ii) estimated z^*, the fraction of the charge burnt, when the pressure ratio attains the critical value.

We have obtained these results under the same assumptions as those of Ray but with the additional assumption that the ratio of the atmospheric pressure to the pressure in the first chamber is less than the critical value. The results obtained here hold good for all types of multitubular powders having the cubic formfunction.

K_1 and K_2, the lower and the upper limits for the ratio of chamber volumes, have been evaluated for a heptatubular powder (Tavernier 1956) with $m = 7$, $\rho = 9/4$ and for different values of ϕ_1, ϕ_2 and z_s (fraction of the charge burnt when the shot starts). To have a comparative study with Ray's results, we have also calculated the values of K_1 and K_2 for high-low pressure guns (in which case $\phi_1 = 0$) for propellants having quadratic formfunctions and it has been found that though K_1 has the same values our K_2 is nearer to K_1 than that of Ray for the corresponding values of the variables involved. The equations of state for the first and the second chambers are respectively

$$P_1\left[V_1-\frac{C}{\delta}(1-z)-CN_1\eta\right]=CN_1\lambda \quad \ldots \quad (1)$$

and

$$P_2[V_2-CN_2\eta]=CN_2\lambda. \quad \ldots \quad (2)$$

The equation of continuity for the first chamber ($w \geqslant w^*$) is

$$C\frac{dz}{dt}=C\frac{dN_1}{dt}+\frac{\psi S_1P_1}{\sqrt{\lambda}}+\frac{\psi' S_2P_1}{\sqrt{\lambda}}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\sqrt{\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad \ldots \quad (3)$$

and that for the second chamber ($w \geqslant w^*$) is

$$C\frac{dN_2}{dt}=\frac{\psi' S_2P_1}{\sqrt{\lambda}}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\sqrt{\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]} \quad \ldots \quad (4)$$

where

$$\psi=\gamma^{\frac{1}{2}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad \ldots \quad (5)$$

and

$$\psi'=\left(\frac{2\gamma}{\gamma-1}\right)^{\frac{1}{2}}. \quad \ldots \quad (6)$$

The equation for rate of burning is

$$D\frac{df}{dt}=-\beta P_1. \quad \ldots \quad (7)$$

Also the formfunction is

$$z=(1-f)(a-bf-cf^2). \quad \ldots \quad (8)$$

The critical value w^* of w, taking into account the co-volume correction, is given by

$$\omega^*=\left(\frac{P_2}{P_1}\right)^*=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}[1-0{\cdot}248\epsilon+0{\cdot}117\epsilon^2+o(\epsilon^3)]. \quad \ldots \quad (9)$$

The eqns. (1) and (2) can be written as

$$\frac{P_1}{\delta}(\alpha+z-\delta' N_1)=N_1\lambda \quad \ldots \quad (10)$$

and

$$\frac{P_2}{\delta}(\beta-\delta' N_2)=N_2\lambda \quad \ldots \quad (11)$$

where

$$\alpha = \frac{V_1-(C/\delta)}{(C/\delta)} \quad \ldots \quad (12)$$

$$\beta = \frac{V_2}{(C/\delta)} \quad \ldots \quad (13)$$

$$\delta' = \eta\delta \quad \ldots \quad (14)$$

$$\therefore \qquad \omega = \frac{P_2}{P_1} = \frac{N_2(\alpha+z-N_1\delta')}{N_1(\beta-N_2\delta')}.$$

Introducing

$$\xi_1 = z-N_1\delta' \quad \ldots \quad (15)$$

and

$$\xi_2 = -N_2\delta' \quad \ldots \quad (16)$$

we get

$$\omega = \frac{\xi_2(\alpha+\xi_1)}{(\xi_1-z)(\beta+\xi_2)}. \quad \ldots \quad (17)$$

From eqns. (3) and (7) we have

$$\frac{dz}{df} = \frac{dN_1}{df} - \frac{\psi S_1 D}{C\beta\sqrt{\lambda}} - \frac{\psi' S_2 D}{C\beta\sqrt{\lambda}}\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}$$

$$= \frac{dN_1}{df} - \phi_1 - \phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)} \quad \ldots \quad (18)$$

where

$$\phi_1 = \frac{\psi S_1 D}{\beta C\sqrt{\lambda}} \quad \ldots \quad (19)$$

$$\phi_2 = \frac{\psi' S_2 D}{\beta C\sqrt{\lambda}}. \quad \ldots \quad (20)$$

From (8) we get

$$\frac{dz}{df} = -(a'-b'f-c'f^2) \quad \ldots \quad (21)$$

where

$$a' = a+b \quad \ldots \quad (22)$$

$$b' = 2(b-c) \quad \ldots \quad (23)$$

$$c' = 3c. \quad \ldots \quad (24)$$

Also, (15) gives

$$\delta'\frac{dN_1}{df} = \frac{dz}{df} - \frac{d\xi_1}{df}. \quad \ldots \quad (25)$$

Using the relations (18), (21) and (25), we get

$$\frac{d\xi_1}{df} = (\delta'-1)(a'-b'f-c'f^2) - \delta'\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]. \quad \ldots \quad (26)$$

Again, from (4), (7) and (16) we get

$$\frac{d\xi_2}{df} = \delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}. \qquad (27)$$

Now

$$\frac{dz}{d\xi_1} = \frac{dz}{df}\cdot\frac{df}{d\xi_1} = \frac{(a'-b'f-c'f^2)}{\delta'\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]-(\delta'-1)(a'-b'f-c'f^2)} \qquad (28)$$

and

$$\frac{dz}{d\xi_2} = \frac{dz}{df}\cdot\frac{df}{d\xi_2} = -\frac{(a'-b'f-c'f^2)}{\delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}}. \qquad (29)$$

We assume that $\frac{d\xi_1}{dz}$, i.e. $1-\delta'\frac{dN_1}{dz}$, is positive or, in other words, $\frac{dN_1}{dz} < \frac{1}{\delta'}$, meaning thereby that the flow of the propellant gases from the first chamber is so regulated that the rate of increase of N_1 with z in the first chamber does not exceed a certain limit, viz. $\frac{1}{\delta'}$. The eqns. (28) and (29) give

$$\frac{d\xi_2}{d\xi_1} = -\frac{\delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}}{\delta'\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]-(\delta'-1)(a'-b'f-c'f^2)}. \qquad (30)$$

Differentiating the relation (17) logarithmically with respect to ξ_1 and simplifying, we have

$$\frac{d\omega}{d\xi_1} = -\frac{\omega}{\xi_2(\alpha+\xi_1)}\left[-(1-\omega)\xi_1\frac{d\xi_2}{d\xi_1}-\xi_2\left(1-\omega+\omega\frac{dz}{d\xi_1}\right)-\omega z\frac{d\xi_2}{d\xi_1}\right.$$
$$\left.-\left(\beta\omega\frac{dz}{d\xi_1}-\beta\omega+\alpha\frac{d\xi_2}{d\xi_1}\right)\right]. \qquad (31)$$

We have assumed that $1 \geqslant \omega \geqslant \omega^*$. Now since $\xi_2 < 0$ and $\frac{d\xi_2}{d\xi_1} < 0$,

$$\therefore\quad -\xi_1(1-\omega)\frac{d\xi_2}{d\xi_1}-\xi_2\left(1-\omega+\omega\frac{dz}{d\xi_1}\right)-\omega z\frac{d\xi_2}{d\xi_1} > 0. \qquad (32)$$

Also from (17) we get

$$\frac{d\omega}{d\xi_2} = -\frac{\omega}{\xi_2(\alpha+\xi_1)}\left[-\left\{\xi_1(1-\omega)+\xi_2\left((1-\omega)\frac{d\xi_1}{d\xi_2}+\omega\frac{dz}{d\xi_2}\right)+\omega z\right\}\right.$$
$$\left.+\left(\omega\beta\frac{d\xi_1}{d\xi_2}-\omega\beta\frac{dz}{d\xi_2}-\alpha\right)\right]. \qquad (33)$$

Since $1 \geqslant \omega \geqslant \omega^*$ and $\frac{dz}{d\xi_2} < 0$, $\frac{d\xi_1}{d\xi_2} < 0$,

$$\therefore \quad \xi_1(1-\omega)+\xi_2\left[(1-\omega)\frac{d\xi_1}{d\xi_2}+\omega\frac{dz}{d\xi_2}\right]+\omega z > 0 \quad \ldots \quad \ldots \quad (34)$$

which after some simplification becomes the same as (32).

Relation between the Constants of the Gun and the Charge

The steady state of flow is reached when w attains the critical value and also when its value falls further below the critical value, i.e. when $\omega \leqslant \omega^*$.

Since $d\xi_1 > 0$, $\frac{d\omega}{d\xi_1} < 0$ for $\omega = \omega^*$,

therefore, from eqn. (31) we get (for $\omega = \omega^*$)

$$\left[-\xi_1(1-\omega)\frac{d\xi_2}{d\xi_1}-\xi_2\left(1-\omega+\omega\frac{dz}{d\xi_1}\right)-\omega z\frac{d\xi_2}{d\xi_1}\right]$$

$$< \left(\omega\beta\frac{dz}{d\xi_1}-\omega\beta+\alpha\frac{d\xi_2}{d\xi_1}\right). \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

Since L.H.S. of the above is positive by (32),

$$\therefore \quad \omega\beta\frac{dz}{d\xi_1}-\omega\beta+\alpha\frac{d\xi_2}{d\xi_1} > 0, \quad \text{for } \omega = \omega^* \quad \ldots \quad \ldots \quad (36)$$

while from (33), since $d\xi_2 < 0$ and $\frac{d\omega}{d\xi_2} > 0$ for $\omega = \omega^*$, we obtain

$$\omega\beta\frac{d\xi_1}{d\xi_2}-\omega\beta\frac{dz}{d\xi_2}-\alpha > \xi_1(1-\omega)+\xi_2\left[(1-\omega)\frac{d\xi_1}{d\xi_2}+\omega\frac{dz}{d\xi_2}\right]+\omega z$$

which in view of (34) gives

$$\left(\omega\beta\frac{d\xi_1}{d\xi_2}-\omega\beta\frac{dz}{d\xi_2}-\alpha\right) > 0. \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

The inequalities (36) and (37) are the same which can be easily verified. Using (29) and (30) in (37), we obtain

$$\omega^*\frac{\beta}{\alpha} > \frac{\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}}{(a'-b'f^*-c'f^{*2})-\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right]} \quad \ldots \quad \ldots \quad (38)$$

if

$$a'-b'f^*-c'f^{*2}-\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right] > 0. \quad \ldots \quad \ldots \quad (39)$$

Now

$$a'-b'f^*-c'f^{*2} < a' \text{ for } b' > 0$$

and

$$a'-b'f^*-c'f^{*2} < a'-b' \text{ for } b' \leqslant 0,$$

since c' is always positive.

$$\therefore \qquad a'-b'f^*-c'f^{*2} < d'$$

where

$$\left.\begin{aligned} d' &= a' \text{ for } b' > 0 \\ &= a'-b' \text{ for } b \leqslant 0 \end{aligned}\right\}. \qquad (40)$$

Hence the above inequality (38) becomes

$$\frac{\omega^*\beta}{\alpha} > \frac{\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}}{d'-\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right]} \qquad (41)$$

provided

$$d'-\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right] > 0. \qquad (42)$$

Taking

$$(V_2/V_1) = K,$$

we get

$$\beta = K(\alpha+1). \qquad (43)$$

Putting $\omega^* = 0{\cdot}555$, the value attained by w at the steady state of flow, and using *(43) in (41)*, we get

$$K > \frac{\alpha\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}}{(1+\alpha)\omega^*\left\{d'-\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{(1-\omega^{*\frac{\gamma-1}{\gamma}})}\right]\right\}}$$

$$= \frac{1{\cdot}8081\,\phi_2\alpha}{(1+\alpha)[(d'-\phi_1)4{\cdot}8054-\phi_2]} = K_1. \qquad (44)$$

The inequality (44) gives the lower limit for the ratio of chamber volumes independent of z^*, which must be satisfied in a RCL high-low pressure gun.

Case I. RCL high-low pressure gun using charges having quadratic form-function: If we put $a = 1$, $b = -\theta$ and $c = 0$, then $a' = 1-\theta$, $b' = -2\theta$ and $c' = 0$ and the formfunction is

$$z = (1-f)(1+\theta f).$$

Here

$$\begin{aligned} d' &= 1-\theta \text{ for } b' > 0, \text{ i.e. for } \theta < 0 \\ &= 1+\theta \text{ for } b' \leqslant 0, \text{ i.e. for } \theta \geqslant 0 \end{aligned}$$

and the lower limit for K can be obtained from (44).

Case II. High-low pressure gun: If we put $S_1 = 0$, i.e. $\phi_1 = 0$, the RCL high-low pressure gun becomes a high-low pressure gun and (44) gives

$$K > \frac{1{\cdot}8081\phi_2\alpha}{(1+\alpha)(4{\cdot}8054d'-\phi_2)} = K_1. \quad .. \quad .. \quad .. \quad (45)$$

Case III. High-low pressure gun using charges having quadratic form-function: If we put $\phi_1 = 0$, $a' = 1-\theta$, $b' = -2\theta$ and $c' = 0$, the inequality (44) corresponds to that obtained by Ray (1962).

Estimation of z^*

From (17) we have

$$\xi_1 = \frac{\alpha\xi_2+z\omega\beta+z\omega\xi_2}{\omega\beta-(1-\omega)\xi_2} \quad .. \quad .. \quad .. \quad .. \quad (46)$$

$\therefore$ (35) leads to, for $w = \omega^*$,

$$\left[-(1-\omega)\left(\frac{\alpha\xi_2+z\omega\beta+z\omega\xi_2}{\omega\beta-(1-\omega)\xi_2}\right)\frac{d\xi_2}{d\xi_1}-\xi_2\left(1-\omega+\omega\frac{dz}{d\xi_1}\right)\right.$$

$$\left.-\omega z\frac{d\xi_2}{d\xi_1}\right] > \left(\omega\beta\frac{dz}{d\xi_1}-\omega\beta+\alpha\frac{d\xi_2}{d\xi_1}\right)$$

or

$$-\xi_2\left[2\omega\beta-\xi_2(1-\omega)+\frac{\omega^2\beta}{1-\omega}\frac{dz}{d\xi_1}-\omega\xi_2\frac{dz}{d\xi_1}\right]$$

$$< \frac{1}{1-\omega}\left[\omega\beta(z+\alpha)\frac{d\xi_2}{d\xi_1}+\omega^2\beta^2\frac{dz}{d\xi_1}-\omega^2\beta^2\right]-\xi_2\omega\beta\frac{dz}{d\xi_1}. \quad .. \quad (47)$$

Since in (47), L.H.S. > 0, R.H.S. must be > 0,
$\therefore$ for $\omega = \omega^*$

$$\left[\omega\beta(z+\alpha)\frac{d\xi_2}{d\xi_1}+\omega^2\beta^2\frac{dz}{d\xi_1}-\omega^2\beta^2\right]-(1-\omega)\xi_2\omega\beta\frac{dz}{d\xi_1} > 0$$

as $1-\omega > 0$, for $\omega = \omega^*$
or

$$-(1-\omega)\xi_2\frac{dz}{d\xi_1}+\omega\beta\frac{dz}{d\xi_1} > \omega\beta-(z+\alpha)\frac{d\xi_2}{d\xi_1}, \text{ for } \omega = \omega^*. \quad .. \quad (48)$$

As $\xi_1 > 0$, (46) gives

$$-\xi_2 < \frac{\beta\omega z}{\alpha+\omega z} \quad .. \quad .. \quad .. \quad .. \quad (49)$$

using (28), (30) and (49) in (48), for $\omega = \omega^*$, we get

$$\left[(1-\omega)\frac{\beta\omega z}{\alpha+\omega z}+\omega\beta\right]\left\{\frac{a'-b'f-c'f^2}{\delta'\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]-(\delta'-1)(a'-b'f-c'f^2)}\right\}$$

$$>\omega\beta+(z+\alpha)\left\{\frac{\delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}}{\delta'\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]-(\delta'-1)(a'-b'f-c'f^2)}\right\}$$

which on putting

$$A = \delta'\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)} \quad \ldots \quad (50)$$

$$B = \delta'\phi_1 \quad \ldots \quad (51)$$

and simplifying, becomes

$$\omega^*\beta(a'-b'f^*-c'f^{*2})[(1-\omega^*)z^*+(\alpha+\omega^*z^*)\delta']$$
$$>[\omega^*\beta(\alpha+\omega^*z^*)(A+B)+A(\alpha+z^*)(\alpha+\omega^*z^*)].$$

Since $a'-b'f^*-c'f^{*2} < d'$, we get

$$\omega^*\beta d'[(1-\omega^*)z^*+(\alpha+\omega^*z^*)\delta']$$
$$>[\omega^*\beta(\alpha+\omega^*z^*)(A+B)+A(\alpha+z^*)(\alpha+\omega^*z^*)]$$

or

$$A\omega^*z^{*2}+z^*[A\alpha(1+\omega^*)+\omega^{*2}\beta\{A+B-(\delta'-1)d'\}-\omega^*\beta d']$$
$$<\omega^*\alpha\beta(\delta'd'-A-B)-A\alpha^2 \quad \ldots \quad (52)$$

which can be written as

$$z^{*2}+Mz^* < N \quad \ldots \quad (53)$$

where

$$A\omega^*M = A\alpha(1+\omega^*)+\omega^{*2}\beta\{A+B-(\delta'-1)d'\}-\omega^*\beta d' \quad \ldots \quad (54)$$

and

$$A\omega^*N = \omega^*\alpha\beta(\delta'd'-A-B)-A\alpha^2 > 0. \quad \ldots \quad (55)$$

From (53) we get

$$z^* < \tfrac{1}{2}[\sqrt{(M^2+4N)}-M]. \quad \ldots \quad (56)$$

This inequality is non-trivial only if $N < M+1$.

Also it gives an upper limit for the fraction of the charge which must burn before the pressure ratio becomes critical.

Condition for z^* to be less than z_s

From (52) we find that if

$$\omega^*\alpha\beta(\delta'-A-B)-A\alpha^2$$
$$<A\omega^*z_s^2+z_s[A\alpha(1+\omega^*)-\omega^{*2}\beta\{A+B-(\delta'-1)d'\}-\omega^*\beta d'] \quad \ldots \quad (57)$$

then

$$A\omega^*z^{*2}+z^*[A\alpha(1+\omega^*)-\omega^{*2}\beta\{A+B-(\delta'-1)d'\}-\omega^*\beta d']$$
$$<A\omega^*z_s^2+z_s[A\alpha(1+\omega^*)-\omega^{*2}\beta\{A+B-(\delta'-1)d'\}-\omega^*\beta d'].$$

Now if

$$A\alpha(1+\omega^*)+\omega^{*2}\beta\{A+B-(\delta'-1)d'\}-\omega^*\beta d'>0, \quad .. \quad .. \quad (58)$$

all the coefficients of z^{*2}, z^*, z_s^2 and z_s are positive and we have

$$Z^* < Z_s.$$

Hence, for $z^* < z_s$, we get from (57) that

$$\omega^*\beta[\alpha(\delta'd'-A-B)+z_s\{d'(\delta'-1)\omega^*+d'-\omega^*(A+B)\}]$$
$$<A(\alpha+z_s)(\alpha+\omega^* z_s) \quad .. \quad .. \quad .. \quad .. \quad (59)$$

or

$$K < \frac{A(\alpha+z_s)(\alpha+\omega^* z_s)}{\omega^*(1+\alpha)[\alpha(\delta'd'-A-B)+z_s\{d'(\delta'-1)\omega^*+d'-\omega^*(A+B)\}]} = K_2 \quad (60)$$

giving an upper limit for the ratio of chamber volumes. The inequality (60) also gives a sufficient condition for z^* to be less than z_s.

Hence from (44) and (60) we get

$$K_1 < K < K_2. \quad .. \quad .. \quad .. \quad .. \quad (61)$$

Thus we have obtained an upper and a lower limit for the ratio of chamber volumes in the case of a RCL high-low pressure gun in order that the steady state of flow may be reached before the shot starts.

Evaluation of K_1 and K_2

The values of K_1 and K_2 for tubular and multitubular propellants using α as a parameter have been calculated. Tables I and II give the values of K_1 and K_2 for high-low pressure guns (i.e. when $\phi_1 = 0$). It is interesting to note that the values of K_1 in Table I (for tubular propellants) are the same as those obtained by Ray but those of K_2 are nearer to K_1 for the corresponding values of α and z_s. Table III gives the values of K_1 and K_2 for the case of a RCL high-low pressure gun, with $\phi_1 = 0{\cdot}25$ and $\phi_2 = 1$, using multitubular propellants.

TABLE I

$\delta' = 1, \quad a = 1, \quad b = 0, \quad c = 0, \quad \phi_1 = 0, \quad \phi_2 = 1$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·0430	0·0517	0·0631	0·0734	0·0838
0·2	0·0789	0·0874	0·0967	0·1062	0·1155
0·3	0·1092	0·1170	0·1254	0·1340	0·1428
0·4	0·1353	0·1426	0·1502	0·1581	0·1660
0·5	0·1578	0·1645	0·1718	0·1790	0·1964
0·6	0·1775	0·1838	0·1906	0·1973	0·2043
0·7	0·1949	0·2009	0·2072	0·2135	0·2200
1·0	0·2367	0·2417	0·2470	0·2524	0·2578

TABLE II

$\delta' = 1, \quad a = 0{\cdot}8216, \quad b = 0{\cdot}0824, \quad \phi_1 = 0, \quad \phi_2 = 1$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·0491	0·0586	0·0707	0·0828	0·0944
0·2	0·0901	0·0992	0·1095	0·1201	0·1305
0·3	0·1247	0·1329	0·1421	0·1517	0·1614
0·4	0·1544	0·1620	0·1704	0·1791	0·1880
0·5	0·1802	0·1869	0·1950	0·2030	0·2112
0·6	0·2027	0·2089	0·2164	0·2238	0·2315
0·7	0·2226	0·2284	0·2353	0·2423	0·2495
1·0	0·2703	0·2749	0·2806	0·2867	0·2926

TABLE III

$\delta' = 1, \quad a = 0{\cdot}8216, \quad b = 0{\cdot}0824, \quad \phi_1 = 0{\cdot}25, \quad \phi_2 = 1$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·0764	0·0829	0·0958	0·1117	0·1230
0·2	0·1400	0·1458	0·1459	0·1683	0·1768
0·3	0·1939	0·1984	0·2058	0·2172	0·2245
0·4	0·2401	0·2466	0·2524	0·2621	0·2677
0·5	0·2801	0·2836	0·2892	0·2980	0·3026
0·6	0·3151	0·3183	0·3231	0·3310	0·3352
0·7	0·3460	0·3491	0·3533	0·3605	0·3639
1·0	0·4202	0·4225	0·4257	0·4317	0·4338

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REFERENCES

Corner, J. (1950). Theory of Internal Ballistics of Guns. John Wiley & Sons, New York.
Gupta, M. C. (1960). *Def. Sci. J.*, **10** (2), 59–83.
Jain, T. C. (1964). *Proc. natn. Inst. Sci. India*, A **30**, 18–46.
———— (1964). *Proc. natn. Inst. Sci. India*, A **30**, 715–735.
Kapur, J. N. (1957). *Proc. natn. Inst. Sci. India*, A **23**, 312–321.
Ray, A. (1962). *Proc. natn. Inst. Sci. India*, A **28**, 795–805.
Tavernier, P. (1956). *Méml. Artill. fr.*, **30** (1), 117–165.