ON THE INITIAL STAGE OF BURNING OF MULTITUBULAR POWDERS IN A RCL HIGH-LOW PRESSURE GUN AND ITS DEPENDENCE ON THE RATIO OF CHAMBER VOLUMES FOR MULTITUBULAR POWDERS

PART II. NON-ISOTHERMAL MODEL

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In this paper the theory for a non-isothermal model has been developed following the same assumptions as in the case of the isothermal model (Part I), which may be treated as an extension of that of Ray (1962).

INTRODUCTION

The author (1966) has recently discussed the case of isothermal burning in a RCL high-low pressure gun. In the present paper the theory for non-isothermal burning of multitubular powders in a RCL high-low pressure gun has been developed and we have

(*a*) obtained an upper and a lower limit for the ratio of chamber volumes;

(*b*) estimated z^*, the fraction of the charge that must burn before the pressure ratio becomes critical.

It is interesting to note here also that the values of K_1, for $\phi_2\sqrt{T_1'^*} = 1$, in a H/L gun are the same as those given by Ray (1962); but the values of K_2 in our case are nearer to those of K_1 than to those obtained by Ray.

With the same notations as in the isothermal model (Jain 1966) the basic equations for a RCL high-low pressure gun are as follows:

The equation of state for the first chamber is

$$P_1\left[V_1-\frac{C}{\delta}(1-z)-CN_1\eta\right] = CN_1RT_1. \quad \ldots \quad \ldots \quad (1)$$

The equation of state for the second chamber is

$$P_2[V_2-CN_2\eta] = CN_2RT_2. \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The equation of continuity for the first chamber ($\omega \geqslant \omega^*$) is

$$C\frac{dz}{dt} = C\frac{dN_1}{dt}+\frac{\psi S_1P_1}{\sqrt{RT_1}}+\frac{\psi' S_2P_1}{\sqrt{RT_1}}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\sqrt{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} \quad \ldots \quad (3)$$

while that for the second chamber ($\omega \geqslant \omega^*$) is

$$C\frac{dN_2}{dt} = \frac{\psi' S_2 P_1}{\sqrt{RT_1}}\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} \sqrt{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}. \quad \cdots \quad (4)$$

The equation for rate of burning is

$$D\frac{df}{dt} = -\beta P_1. \quad \cdots \quad (5)$$

The formfunction is

$$z = (1-f)(a-bf-cf^2). \quad \cdots \quad (6)$$

The equations of energy (Kapur 1957) for the first and the second chambers are

$$\frac{d}{dt}(N_1T_1) = T_0\frac{dz}{dt} - \gamma T_1\frac{d}{dt}(z-N_1) \quad \cdots \quad (7)$$

and

$$\frac{d}{dt}(N_2T_2) = \gamma T_1\frac{dN_2}{dt}. \quad \cdots \quad (8)$$

Introducing

$$T_1' = \frac{T_1}{T_0} < 1, \quad T_2' = \frac{T_2}{T_0} < 1 \quad \cdots \quad (9)$$

$$z - \delta' N_1 = \xi_1 \quad \cdots \quad (10)$$

and

$$-\delta' N_2 = \xi_2 \quad \cdots \quad (11)$$

eqns. (7) and (8) can be written as

$$\frac{d}{d\xi_1}\left[(z-\xi_1)T_1'\right] = \left(\delta' - \gamma T_1'\delta' + \gamma T_1'\right)\frac{dz}{d\xi_1} - \gamma T_1' \quad \cdots \quad (7A)$$

and

$$\frac{d}{d\xi_2}\left(\xi_2 T_2'\right) = \gamma T_1'. \quad \cdots \quad (8A)$$

Putting

$$\alpha = \frac{V_1 - (C/\delta)}{(C/\delta)} \quad \cdots \quad (12)$$

$$\beta = \frac{V_2}{(C/\delta)} \quad \cdots \quad (13)$$

and

$$\delta' = \eta\delta \quad \cdots \quad (14)$$

in (1) and (2) and using (10) and (11) we get

$$P_1(C/\delta)(\alpha+\xi_1) = CN_1RT_1'T_0 \quad \cdots \quad (15)$$

and

$$P_2(C/\delta)(\beta+\xi_2) = CN_2RT_2'T_0, \quad \cdots \quad (16)$$

$$\therefore \quad \omega = \frac{P_2}{P_1} = \frac{T_2'(\alpha+\xi_1)\xi_2}{T_1'(\beta+\xi_2)(\xi_1-z)}. \quad \cdots \quad (17)$$

From (3) and (5) we get

$$\frac{dz}{df}=\frac{dN_1}{df}-\frac{\psi S_1 D}{\beta C\sqrt{RT_1}}-\frac{\psi' S_2 D}{\beta C\sqrt{RT_1}}\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}$$

$$=\frac{dN_1}{df}-\frac{\phi_1}{\sqrt{T_1'}}-\frac{\phi_2}{\sqrt{T_1'}}\omega^{\frac{1}{\gamma}}\sqrt{1-\omega^{\frac{\gamma-1}{\gamma}}}, \qquad (18)$$

where

$$\phi_1=\frac{\psi S_1 D}{\beta C\sqrt{RT_0}} \qquad (19)$$

and

$$\phi_2=\frac{\psi' S_2 D}{\beta C\sqrt{RT_0}}. \qquad (20)$$

From (6) we get

$$\frac{dz}{df}=-(a'-b'f-c'f^2), \qquad (21)$$

where

$$a'=a+b \qquad (22)$$

$$b'=2(b-c) \qquad (23)$$

$$c'=3c. \qquad (24)$$

Also from (10) we obtain

$$\delta'\frac{dN_1}{df}=\frac{dz}{df}-\frac{d\xi_1}{df}. \qquad (25)$$

Combining (18) and (25) we get

$$\frac{dz}{df}=\frac{1}{\delta'}\left(\frac{dz}{df}-\frac{d\xi_1}{df}\right)-\frac{\phi_1}{\sqrt{T_1'}}-\frac{\phi_2}{\sqrt{T_1'}}\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}$$

which on using (21) gives

$$\frac{d\xi_1}{df}=(\delta'-1)(a'-b'f-c'f^2)-\frac{\delta'}{\sqrt{T_1'}}\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]. \qquad (26)$$

Again

$$\frac{d\xi_2}{df}=\frac{d\xi_2}{dt}\cdot\frac{dt}{df}=\frac{\delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}}{\sqrt{T_1'}}, \qquad (27)$$

$$\therefore\quad \frac{dz}{d\xi_1}=\frac{dz}{df}\cdot\frac{df}{d\xi_1}=\frac{(a'-b'f-c'f^2)\sqrt{T_1'}}{\delta'\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]-\sqrt{T_1'}(\delta'-1)(a'-b'f-c'f^2)} \qquad (28)$$

$$\frac{dz}{d\xi_2}=\frac{dz}{df}\cdot\frac{df}{d\xi_2}=-\frac{(a'-b'f-c'f^2)\sqrt{T_1'}}{\delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}} \qquad (29)$$

and

$$\frac{d\xi_2}{d\xi_1} = -\frac{\delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}}{\delta'\left[\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{\left(1-\omega^{\frac{\gamma-1}{\gamma}}\right)}\right]-\sqrt{T'_1}(\delta'-1)(a'-b'f-c'f^2)}. \qquad (30)$$

Differentiating (17) logarithmically with respect to ξ_1 and using (7A) and (8A), we get

$$\frac{d\omega}{d\xi_1} = -\frac{\omega}{T'_2\xi_2(\alpha+\xi_1)}\left[-\xi_1(\gamma-\omega)T'_1\frac{d\xi_2}{d\xi_1}-\xi_2\left\{T'_2-\omega\gamma T'_1+\omega(\delta'-\gamma T'_1\delta'+\gamma T'_1)\frac{dz}{d\xi_1}\right\}\right.$$
$$\left.-\omega T'_1 z\frac{d\xi_2}{d\xi_1}-\left\{\gamma T'_1\alpha\frac{d\xi_2}{d\xi_1}+\omega\beta(\delta'-\gamma T'_1\delta'+\gamma T'_1)\frac{dz}{d\xi_1}-\omega\beta\gamma T'_1\right\}\right]. \quad .. \quad (31)$$

We have assumed that $1 \geqslant \omega \geqslant \omega^*$. Also $\xi_2 < 0$, $\frac{dz}{d\xi_1} > 0$ and $\frac{d\xi_2}{d\xi_1} < 0$. When $\omega = \omega^*$, the ratio $\frac{T'_2}{T'_1}$ takes the value $\frac{2}{\gamma+1}$ 0·889 (for $\gamma = 1{\cdot}25$), while $\omega\gamma = 0{\cdot}694$ for $\omega = \omega^*$. Hence

$$T'_2-\omega\gamma T'_1 > 0 \quad \text{for } \omega = \omega^*. \qquad .. \quad .. \quad .. \quad (32)$$

By taking $\gamma = 1{\cdot}25$ and $\omega^* = 0{\cdot}555$ (Corner 1950), we find that $\delta'-\gamma T'_1\delta'+\gamma T'_1$ is positive for $\omega = \omega^*$ and for the possible values of δ' which vary between 1·4 and 1·6 for the known propellants (H.M.S.O. 1951),

$$\therefore \quad \left[-\xi_1(\gamma-\omega)T'_1\frac{d\xi_2}{d\xi_1}-\xi_2\left\{T'_2-\omega\gamma T'_1+\omega(\delta'-\gamma T'_1\delta'+\gamma T'_1)\frac{dz}{d\xi_1}\right\}-\omega T'_1 z\frac{d\xi_2}{d\xi_1}\right] > 0$$
$$\text{for } \omega = \omega^*. \quad .. \quad (33)$$

Again differentiating (17) logarithmically with respect to ξ_2 and using (7A) and (8A), we get

$$\frac{d\omega}{d\xi_2} = -\frac{\omega}{\xi_2 T'_2(\alpha+\xi_1)}\left\{-\left[\xi_1(\gamma-\omega)T'_1+\xi_2\left((T'_2-\omega\gamma T'_1)\frac{d\xi_1}{d\xi_2}\right.\right.\right.$$
$$\left.\left.+\omega(\delta'-\gamma T'_1\delta'+\gamma T'_1)\frac{dz}{d\xi_2}\right)-\omega z T'_1\right]$$
$$+\left[\gamma\omega\beta T'_1\frac{d\xi_1}{d\xi_2}-\omega\beta(\delta'-\gamma T'_1\delta'+\gamma T'_1)\frac{dz}{d\xi_2}\right.$$
$$\left.\left.-\gamma T'_1\alpha\right]\right\}. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (34)$$

Since $1 \geqslant \omega \geqslant \omega^*$, $\frac{dz}{d\xi_2} < 0$, $\frac{d\xi_1}{d\xi_2} < 0$ and as explained above $T_2' > \omega\gamma T_1'$ and

$\delta' - \gamma T_1'\delta' + \gamma T_1' > 0$ for $\omega = \omega^*$,

$$\therefore \quad \left[\xi_1(\gamma-\omega)T_1' + \xi_2\left\{(T_2' - \omega\gamma T_1')\frac{d\xi_1}{d\xi_2} + \omega(\delta' - \gamma T_1'\delta' + \gamma T_1')\frac{dz}{d\xi_2}\right\} - \omega z T_1'\right] > 0$$

$$\text{for } \omega = \omega^*. \quad .. \quad (35)$$

The inequalities (33) and (35) are the same as can be easily verified.

Relation between the Constants of the Gun and the Charge

We assume that the steady state of flow is reached when ω attains the critical value and also when its value falls further below the critical value, i.e. when $\omega \leqslant \omega^*$.

For $\omega = \omega^*$, $d\xi_1 > 0$ and $\frac{d\omega}{d\xi_1} < 0$.

Hence from (31), for $\omega = \omega^*$, we get

$$\left\{-\xi_1(\gamma-\omega)T_1'\frac{d\xi_2}{d\xi_1} - \xi_2\left[T_2' - \omega\gamma T_1' + \omega(\delta' - \gamma T_1'\delta' + \gamma T_1')\frac{dz}{d\xi_1}\right] - \omega T_1' z\frac{d\xi_2}{d\xi_1}\right\}$$

$$< \omega\beta(\delta' - \gamma T_1'\delta' + \gamma T_1')\frac{dz}{d\xi_1} + \gamma T_1'\alpha\frac{d\xi_2}{d\xi_1} - \omega\beta\gamma T_1' \quad .. \quad (36)$$

which in view of (33), for $\omega = \omega^*$, gives

$$\omega\beta(\delta' - \gamma T_1'\delta' + \gamma T_1')\frac{dz}{d\xi_1} + \gamma T_1'\alpha\frac{d\xi_2}{d\xi_1} - \omega\beta\gamma T_1' > 0. \quad .. \quad .. \quad (37)$$

Since $d\xi_2 < 0$ and $\frac{d\omega}{d\xi_2} > 0$ for $\omega = \omega^*$, we get from (34)

$$\left[\gamma\omega\beta T_1'\frac{d\xi_1}{d\xi_2} - \omega\beta(\delta' - \gamma T_1'\delta' + \gamma T_1')\frac{dz}{d\xi_2} - \gamma T_1'\alpha\right]$$

$$> \left\{\xi_1(\gamma-\omega)T_1' + \xi_2\left[(T_2' - \omega\gamma T_1')\frac{d\xi_1}{d\xi_2} + \omega(\delta' - \gamma T_1'\delta' + \gamma T_1')\frac{dz}{d\xi_2}\right] - \omega z T_1'\right\}. \quad (38)$$

Since the R.H.S. is positive by (35),

$$\therefore \quad \gamma\omega\beta T_1'\frac{d\xi_1}{d\xi_2} - \omega\beta(\delta' - \gamma T_1'\delta' + \gamma T_1')\frac{dz}{d\xi_2} - \gamma T_1'\alpha > 0, \quad \text{for } \omega = \omega^* \quad (39)$$

which after some simplification becomes the same as (37).

Using (28) and (30) in (37), we get

$$\omega^* \frac{\beta}{\alpha} > \frac{\gamma\sqrt{T_1'^*}\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}}{(a'-b'f^*-c'f^{*2})-\gamma\sqrt{T_1'^*}\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right]} \quad \ldots \quad (40)$$

if $(a'-b'f^*-c'f'^{*2})-\gamma\sqrt{T_1'^*}\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right]$ be positive.

Now

$$a'-b'f^*-c'f^{*2} < a' \qquad \text{for } b' > 0,$$

and

$$a'-b'f^*-c'f^{*2} < a'-b' \quad \text{for } b' \leqslant 0.$$

Since c is always positive,

$$a'-b'f^*-c'f^{*2} < d', \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

where

$$\left.\begin{aligned} d' &= a' && \text{for } b' > 0 \\ &= a'-b' && \text{for } b' \leqslant 0 \end{aligned}\right\}. \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

Hence from the above inequality (40), we get

$$\omega^* \frac{\beta}{\alpha} > \frac{\gamma\sqrt{T_1'^*}\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}}{d'-\gamma\sqrt{T_1'^*}\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right]} \quad \ldots \quad \ldots \quad (43)$$

if

$$d'-\gamma\sqrt{T_1'^*}\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right] > 0. \quad \ldots \quad \ldots \quad (44)$$

Introducing

$$V_2/V_1 = K$$

we get

$$\beta = K(1+\alpha). \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

Taking $\omega^* = 0{\cdot}555$ and using (45) in (43), we get

$$K > \frac{\alpha\gamma\sqrt{T_1'^*}\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}}{(1+\alpha)\omega^*\left[d'-\gamma\sqrt{T_1'^*}\left[\phi_1+\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{\left(1-\omega^{*\frac{\gamma-1}{\gamma}}\right)}\right]\right]}$$

$$= \frac{1{\cdot}8018\alpha\gamma\sqrt{T_1'^*}\phi_2/(1+\alpha)\left[(d'-\gamma\sqrt{T_1'^*}\phi_1)\,4{\cdot}8054-\gamma\sqrt{T_1'^*}\phi_2\right]}{(1+\alpha)\left[(d'-\gamma\sqrt{T_1'^*}\phi_1)\;4{\cdot}8054-\gamma\sqrt{T_1'^*}\phi_2\right]}$$

$$= K_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (46)$$

giving the lower limit for the ratio of chamber volumes which must be satisfied in a RCL high-low pressure gun.

Particular Cases:

Case I. RCL high-low pressure gun using charges having quadratic formfunction: If we put $a = 1$, $b = -\theta$ and $c = 0$ we get

$$z = (1-f)(1+\theta f)$$

and

$$d' = 1-\theta \quad \text{for } b' > 0, \text{ i.e. for } \theta < 0,$$
$$= 1+\theta \quad \text{for } b' \leqslant 0, \text{ i.e. for } \theta > 0,$$

and (46) gives the lower limit for K.

Case II. High-low pressure gun: If we put $S_1 = 0$, i.e. $\phi_1 = 0$, the RCL high-low pressure gun becomes a high-low pressure gun and we obtain

$$K > \frac{1{\cdot}8018\gamma\alpha\phi_2\sqrt{T_1'^*}}{(1+\alpha)[4{\cdot}8054d' - \gamma\sqrt{T_1'^*}\phi_2]}. \quad \ldots \quad (47)$$

Case III. High-low pressure gun using charges having quadratic form-function: If we put $\phi_1 = 0$, $a' = 1-\theta$, $b' = -2\theta$ and $c' = 0$, the inequality (46) corresponds to that obtained by Ray (1962).

Estimation of z^*

From (17) we get

$$\xi_1 = \frac{\frac{T_2'}{T_1'}\xi_2\alpha + \omega z\beta + \omega z\xi_2}{\omega\beta - \left(\frac{T_2'}{T_1'} - \omega\right)\xi_2}. \quad \ldots \quad (48)$$

Since $\xi_1 > 0$ and for $\omega = \omega^*$, $\frac{T_2'}{T_1'} > \omega$,

$$\therefore \quad -\xi_2 < \frac{\omega z\beta}{\frac{T_2'}{T_1'}\alpha + \omega z} \quad \text{for } \omega = \omega^*. \quad \ldots \quad (49)$$

Now (36) can be written as

$$\xi_1(\gamma-\omega)\frac{d\xi_2}{d\xi_1} - \xi_2\left\{\frac{T_2'}{T_1'} - \omega\gamma + \omega\left(\frac{\delta'}{T_1'} - \gamma\delta' + \gamma\right)\frac{dz}{d\xi_1}\right\} - \omega z\frac{d\xi_2}{d\xi_1}$$
$$< \omega\beta\left(\frac{\delta'}{T_1'} - \gamma\delta' + \gamma\right)\frac{dz}{d\xi_1} + \gamma\alpha\frac{d\xi_2}{d\xi_1} - \omega\beta\gamma. \quad \ldots \quad (50)$$

Using (48) in (50), for $\omega = \omega^*$, we get

$$\left[-(\gamma-\omega)\left(\frac{\frac{T_2'}{T_1'}\xi_2\alpha+\omega z\beta+\omega z\xi_2}{\omega\beta-\left(\frac{T_2'}{T_1'}-\omega\right)\xi_2}\right)\frac{d\xi_2}{d\xi_1}-\xi_2\left\{\frac{T_2'}{T_1'}-\omega\gamma+\omega\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{dz}{d\xi_1}\right\}\right.$$

$$\left.-\omega z\frac{d\xi_2}{d\xi_1}\right] < \omega\beta\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{dz}{d\xi_1}+\gamma\alpha\frac{d\xi_2}{d\xi_1}-\omega\beta\gamma \quad \ldots \quad \ldots \quad \ldots \quad (51)$$

which after some simplification reduces to

$$-\xi_2\left[\left(\frac{T_2'}{T_1'}-\omega\gamma\right)\omega\beta+\left(\frac{T_2'}{T_1'}-\omega\right)\omega\beta\gamma-\left(\frac{T_2'}{T_1'}-\omega\gamma\right)\left(\frac{T_2'}{T_1'}-\omega\right)\xi_2\right.$$

$$\left.+\omega^2\beta\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{dz}{d\xi_1}-\omega\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\left(\frac{T_2'}{T_1'}-\omega\right)\xi_2\frac{dz}{d\xi_1}\right]$$

$$<\left[\left\{\omega\beta\gamma+\omega\left(\gamma-\frac{T_2'}{T_1'}\right)\xi_2\right\}(z+\alpha)\frac{d\xi_2}{d\xi_1}+\omega^2\beta^2\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{dz}{d\xi_1}\right.$$

$$\left.-\omega\beta\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\left(\frac{T_2'}{T_1'}-\omega\right)\xi_2\frac{dz}{d\xi_1}-\omega^2\beta^2\gamma\right]. \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

In (52), L.H.S. > 0, R.H.S. must be > 0.
Hence, for $\omega = \omega^*$,

$$\omega^2\beta^2\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{dz}{d\xi_1}-\omega\beta\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\left(\frac{T_2'}{T_1'}-\omega\right)\xi_2\frac{dz}{d\xi_1}$$

$$-\omega^2\beta^2\gamma+\omega\left\{\beta\gamma+\left(\gamma-\frac{T_2'}{T_1'}\right)\xi_2\right\}(z+\alpha)\frac{d\xi_2}{d\xi_1}>0$$

or

$$\left[-\omega\beta\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\left(\frac{T_2'}{T_1'}-\omega\right)\xi_2\frac{dz}{d\xi_1}-\omega(z+\alpha)\left(\gamma-\frac{T_2'}{T_1'}\right)\left(-\frac{d\xi_2}{d\xi_1}\right)\xi_2\right.$$

$$\left.+\omega^2\beta^2\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{dz}{d\xi_1}\right]>\omega^2\beta^2\gamma-\omega\beta\gamma(z+\alpha)\frac{d\xi_2}{d\xi_1}, \quad \ldots \quad \ldots \quad (53)$$

which on using (49) and simplifying becomes, for $\omega = \omega^*$,

$$\left[\omega\beta\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{T_2'}{T_1'}(z+\alpha)\frac{dz}{d\xi_1}+\frac{T_2'}{T_1'}(z\omega+\gamma\alpha)(z+\alpha)\frac{d\xi_2}{d\xi_1}\right]$$

$$>\omega\beta\gamma\left(\frac{T_2'}{T_1'}\alpha+\omega z\right). \quad \ldots \quad (54)$$

Now from (28), (30) and (54), for $\omega = \omega^*$, we have

$$\left[\omega\beta\left(\frac{\delta'}{T_1'}-\gamma\delta'+\gamma\right)\frac{T_2'}{T_1'}(z+\alpha)\left\{\frac{(a'-b'f-c'f^2)\sqrt{T_1'}}{\delta'\left(\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{1-\omega^{\frac{\gamma-1}{\gamma}}}\right)-\sqrt{T_1'}(\delta'-1)(a'-b'f-c'f^2)}\right\}\right.$$

$$\left.-\frac{T_2'}{T_1'}(z\omega+\gamma\alpha)(z+\alpha)\left\{\frac{\delta'\phi_2\omega^{\frac{1}{\gamma}}\sqrt{1-\omega^{\frac{\gamma-1}{\gamma}}}}{\delta'\left(\phi_1+\phi_2\omega^{\frac{1}{\gamma}}\sqrt{1-\omega^{\frac{\gamma-1}{\gamma}}}\right)-\sqrt{T_1'}(\delta'-1)(a'-b'f-c'f^2)}\right\}\right]$$

$$>\omega\beta\gamma\left(\frac{T_2'}{T_1'}\alpha+\omega z\right). \quad \ldots \quad (55)$$

Introducing

$$A = \delta'\phi_2\omega^{*\frac{1}{\gamma}}\sqrt{1-\omega^{*\frac{\gamma-1}{\gamma}}} \quad \ldots \quad (56)$$

and

$$B = \delta'\phi_1 \quad \ldots \quad (57)$$

in (55), we obtain after some simplification

$$\left[\omega^*\beta\sqrt{T_1'^*}\left[\left(\frac{\delta'}{T_1'^*}-\gamma\delta'+\gamma\right)\left(\frac{T_2'}{T_1'}\right)^*(z^*+\alpha)+\gamma\left\{\left(\frac{T_2'}{T_1'}\right)^*\alpha+\omega^*z^*\right\}\sqrt{T'^*}(\delta'-1)\right]\right.$$

$$\left.\times(a'-b'f^*-c'f^{*2})\right] > \omega^*\beta\gamma\left[\left(\frac{T_2'}{T_1'}\right)^*\alpha+\omega^*z^*\right](A+B)$$

$$+\left(\frac{T_2'}{T_1'}\right)^*(\omega^*z^*+\gamma\alpha)(z^*+\alpha)A,$$

where $f = f^*$ for $\omega = \omega^*$.
Since $a'-b'f^*-c'f^{*2} < d'$, we get

$$\left[\omega^*\beta\sqrt{T_1'^*}\left[\left(\frac{\delta'}{T_1'^*}-\gamma\delta'+\gamma\right)\left(\frac{T_2'}{T_1'}\right)^*(z^*+\alpha)+\gamma\left\{\left(\frac{T_2'}{T_1'}\right)^*\alpha+\omega^*z^*\right\}\sqrt{T_1'^*}(\delta'-1)\right]\right]d'$$

$$>\omega^*\beta\gamma\left[\left(\frac{T_2'}{T_1'}\right)^*\alpha+\omega^*z^*\right](A+B)+\left(\frac{T_2'}{T_1'}\right)^*(\omega^*z^*+\gamma\alpha)(z^*+\alpha)A. \quad \ldots \quad (58)$$

Putting $\left(\frac{T_2'}{T_1'}\right)^* = \frac{2}{\gamma+1}$ in (58) and rearranging, we get

$$2A\sqrt{T_1'^*}\,\omega^* z^{*2} + \Big[\omega^{*2}\beta\gamma(\gamma+1)\Big\{(A+B)\sqrt{T_1'^*} - T_1'^*(\delta'-1)d'\Big\} + 2A\sqrt{T_1'^*}\,\alpha(\gamma+\omega^*)$$
$$-2\omega^*\beta(\delta'-\gamma\delta' T_1'^* + \gamma T_1'^*)d'\Big]z^* < 2\omega^*\beta\alpha\Big\{\delta' d' - \gamma\sqrt{T_1'^*}(A+B)\Big\} - 2A\gamma\alpha^2\sqrt{T_1'^*}. \quad \text{.. (59)}$$

(59) can be written as

$$z^* + Mz^* < N, \quad \text{.. .. .. .. (60)}$$

where

$$2A\sqrt{T_1'^*}\,\omega^* M = \omega^{*2}\beta\gamma(\gamma+1)\Big\{(A+B)\sqrt{T_1'^*} - T_1'^*(\delta'-1)d'\Big\} + 2A\sqrt{T_1'^*}\,\alpha(\gamma+\omega^*)$$
$$-2\omega^*\beta(\delta'-\gamma\delta' T_1'^* + \gamma T_1'^*)d' \quad \text{.. .. .. (61)}$$

and

$$2A\sqrt{T_1'^*}\,\omega^* N = 2\omega^*\beta\alpha\Big\{\delta' d' - \gamma\sqrt{T_1'^*}(A+B)\Big\} - 2A\gamma\alpha^2\sqrt{T_1'^*} > 0. \quad \text{.. .. (62)}$$

Hence from (60) we find that

$$z^* < \tfrac{1}{2}\left[\sqrt{M^2+4N} - M\right] \quad \text{.. .. .. .. (63)}$$

giving an upper limit for the fraction of the charge to be burnt before the steady state of flow is reached.

Condition for z^* to be less than z_s

From (59) we find that if

$$2\omega^*\beta\alpha\Big\{\delta' d' - \gamma\sqrt{T_1'^*}(A+B)\Big\} - 2A\gamma\alpha^2\sqrt{T_1'^*}$$
$$< 2A\sqrt{T_1'^*}\,\omega^* z_s^2 + \Big[\omega^{*2}\beta\gamma\Big\{(A+B)\sqrt{T_1'^*} - T_1'^*(\delta'-1)d'\Big\}$$
$$+2A\sqrt{T_1'^*}\,\alpha(\gamma+\omega^*) - 2\omega^*\beta(\delta'-\gamma\delta' T_1'^* + \gamma T_1'^*)d'\Big]z_s \quad \text{.. .. (64)}$$

then

$$2A\sqrt{T_1'^*}\,\omega^* z^{*2} + \Big[\omega^{*2}\beta\gamma(\gamma+1)\Big\{(A+B)\sqrt{T_1'^*} - T_1'^*(\delta'-1)d'\Big\}$$
$$+2A\sqrt{T_1'^*}\,\alpha(\gamma+\omega^*) - 2\omega^*\beta(\delta'-\gamma\delta' T_1'^* + \gamma T_1'^*)d'\Big]z^*$$
$$< 2A\sqrt{T_1'^*}\,\omega^* z_s^2 + \Big[\omega^{*2}\beta\gamma(\gamma+1)\Big\{(A+B)\sqrt{T_1'^*} - T_1'^*(\delta'-1)d'\Big\}$$
$$+2A\sqrt{T_1'^*}\,\alpha(\gamma+\omega^*) - 2\omega^*\beta(\delta'-\gamma\delta' T_1'^* + \gamma T_1'^*)d'\Big]z_s.$$

The coefficients of z^{*2} and z_s^2 are positive and the coefficients of z^* and z_s will be positive if

$$\Big[\omega^{*2}\beta\gamma(\gamma+1)\Big\{(A+B)\sqrt{T_1^{'*}}-T_1^{'*}(\delta'-1)d'\Big\}+2A\sqrt{T_1^{'*}}\alpha(\gamma+\omega^*)$$
$$-2\omega^*\beta(\delta'-\gamma\delta' T_1^{'*}+\gamma T_1^{'*})d'\Big] > 0 \quad \ldots \quad (65)$$

and then we have

$$z^* < z_s.$$

Hence from (64) we get

$$\omega^*\beta\Big[2\alpha\Big\{\delta' d'-\gamma\sqrt{T_1^{'*}}(A+B)\Big\}+z_s\Big[2(\delta'-\gamma\delta' T_1^{'*}+\gamma T_1^{'*})d'$$
$$+\omega^*\gamma(\gamma+1)\Big\{(\delta'-1)T_1^{'*}d'-\sqrt{T_1^{'*}}(A+B)\Big\}\Big]\Big]$$
$$< 2A\sqrt{T_1^{'*}}(\gamma\alpha+\omega^* z_s)(\alpha+z_s) \quad \ldots \quad (66)$$

or

$$K < \frac{2A\sqrt{T_1^{'*}}(\gamma\alpha+\omega^* z_s)(\alpha+z_s)}{\omega^*(1+\alpha)\Big[2\alpha\Big\{\delta' d'-\gamma\sqrt{T_1^{'*}}(A+B)\Big\}+z_s\Big[2(\delta'-\gamma\delta' T_1^{'*}+\gamma T_1^{'*})d' +\omega^*\gamma(\gamma+1)\Big\{(\delta'-1)T_1^{'*}d'-\sqrt{T_1^{'*}}(A+B)\Big\}\Big]\Big]} = K_2. \quad \ldots \quad (67)$$

Hence from (46) and (67) we get

$$K_1 < K < K_2.$$

Thus for the steady state of flow to be reached before the shot starts, the upper and lower limits for the ratio of chamber volumes have been obtained.

Evaluation of K_1 and K_2

Taking $\delta' = 1$ we find here that $\phi_1\sqrt{T_1^{'*}}$ and $\phi_2\sqrt{T_1^{'*}}$ appear as parameters. Tables I, II, III and IV give the values of K_1 and K_2 for high-low pressure

Table I

$\delta' = 1$, $a = 1$, $b = 0$, $c = 0$, $\phi_1\sqrt{T_1'} = 0$, $\phi_2\sqrt{T_1'} = 1$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·0576	0·0703	0·831	0·0962	0·1091
0·2	0·1056	0·1173	0·1288	0·1408	0·1527
0·3	0·1462	0·1569	0·1678	0·1788	0·1896
0·4	0·1809	0·1906	0·2014	0·2110	0·2212
0·5	0·2112	0·2204	0·2304	0·2393	0·2487
0·6	0·2376	0·2461	0·2551	0·2651	0·2708
0·7	0·2608	0·2689	0·2774	0·2857	0·2936
1·0	0·3165	0·3236	0·3287	0·3377	0·3448

TABLE II

$\delta' = 1,\ a = 1,\ b = 0,\ c = 0,\ \phi_1 \sqrt{T_1'^*} = 0,\ \phi_2 \sqrt{T_1'^*} = 1{\cdot}5$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·1047	0·1185	0·1349	0·1525	0·1705
0·2	0·1920	0·2304	0·2172	0·2321	0·2475
0·3	0·2658	0·2761	0·2880	0·3008	0·3143
0·4	0·3291	0·3384	0·3489	0·3603	0·3725
0·5	0·3840	0·3924	0·4020	0·4123	0·4232
0·6	0·4320	0·4399	0·4491	0·4580	0·4678
0·7	0·4744	0·4816	0·4897	0·4984	0·5074
1·0	0·5760	0·5821	0·5887	0·5957	0·6029

TABLE III

$\delta' = 1,\ a = 0{\cdot}8216,\ b = 0{\cdot}0824,\ \phi_1 \sqrt{T_1'^*} = 0,\ \phi_2 \sqrt{T_1'^*} = 1$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·0661	0·0807	0·0956	0·1003	0·1254
0·2	0·1212	0·1349	0·1482	0·1618	0·1755
0·3	0·1679	0·1803	0·1928	0·2054	0·2179
0·4	0·2079	0·2190	0·2314	0·2424	0·2543
0·5	0·2425	0·2533	0·2641	0·2750	0·2858
0·6	0·2623	0·2828	0·3000	0·3032	0·3112
0·7	0·2995	0·3090	0·3187	0·3283	0·3374
1·0	0·3619	0·3718	0·3777	0·3881	0·3962

TABLE IV

$\delta' = 1,\ a = 0{\cdot}8216,\ b = 0{\cdot}0824,\ \phi_1 \sqrt{T_1'^*} = 0,\ \phi_2 \sqrt{T_1'^*} = 1{\cdot}5$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·1243	0·1387	0·1569	0·1768	0·1972
0·2	0·2279	0·2392	0·2543	0·2707	0·2880
0·3	0·3154	0·3257	0·3383	0·3521	0·3670
0·4	0·3906	0·3996	0·4106	0·4228	0·4369
0·5	0·4557	0·4639	0·4737	0·4846	0·4962
0·6	0·5127	0·5203	0·5290	0·5389	0·5494
θ·7	0·5629	0·5699	0·5780	0·5870	0·5965
1·0	0·6836	0·6893	0·0958	0·7030	0·7105

guns (where $\phi_1 = 0$), using tubular and multitubular powders, with $\phi_2\sqrt{T_1'^*} = 1$ and $\phi_2\sqrt{T_1'^*} = 1{\cdot}5$. It is interesting to note from Table I that the values of K_2 obtained in our case are much nearer to K_1 than those obtained by Ray (1962). In Tables V and VI, the values of K_1 and K_2 have been calculated for a RCL high-low pressure gun, with $\phi_1\sqrt{T_1'^*} = 0{\cdot}15$ and $\phi_2\sqrt{T_1'^*} = 1$, using tubular and multitubular propellants.

TABLE V

$\delta' = 1,\ a = 1,\ b = 0,\ c = 0,\ \phi_1\sqrt{T_1'^*} = 0{\cdot}15,\ \phi_2\sqrt{T_1'^*} = 1$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·0771	0·0852	0·0965	0·1088	0·1212
0·2	0·1413	0·1481	0·1567	0·1666	0.1772
0·3	0·1957	0·2012	0.2087	0·2171	0·2261
0·4	0·2423	0·2473	0·2536	0·2610	0·2688
0·5	0·2826	0·2873	0·2930	0·2993	0·3062
0·6	0·3180	0·3222	0·3273	0·3330	0·3368
0·7	0·3491	0·3531	0·3578	0·3629	0·3681
1·0	0·4239	0·4271	0·4309	0·4350	0·4393

TABLE VI

$\delta' = 1,\ a = 0{\cdot}8216,\ b = 0{\cdot}0824,\ \phi_1\sqrt{T_1'^*} = 0{\cdot}15,\ \phi_2\sqrt{T_1'^*} = 1$

α	K_1	K_2			
		$z_s = 0{\cdot}05$	$z_s = 0{\cdot}1$	$z_s = 0{\cdot}15$	$z_s = 0{\cdot}2$
0·1	0·0933	0·1012	0·1136	0·1276	0·1417
0·2	0·1711	0·1771	0·1860	0·1968	0·2987
0·3	0·2368	0·2415	0·2489	0·2578	0·2676
0·4	0·2932	0·2973	0·3033	0·3109	0·3191
0·5	0·3421	0·3458	0·3510	0·3573	0·3645
0·6	0·3848	0·3887	0·3927	0·3982	0·4017
0·7	0·4226	0·4255	0·4297	0·4346	0·4396
1·0	0·5131	0·5154	0·5185	0·5223	0·5264

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