THERMAL STRESSES IN AN INFINITE THICK PLATE WITH A CIRCULAR CYLINDRICAL HOLE

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The thermo-elastic equations are solved involving non-dimensional parameters and the results are obtained in integral forms. The graphs show that the stresses are exponential in character and die out quickly.

1. Introduction

The problem of a cylindrical hole embedded in an otherwise infinite medium can be regarded as of special significance to give the results approximately valid for a thick hollow cylinder of finite height. The problem of the hollow cylinder in classical elasticity was first solved by Lamé (1852). He calculated the stress system submitted to uniform pressure on the inner and the outer surfaces.

In thermo-elasticity the problem of a circular hole within a cylinder was first solved in the steady state by Lorenz (1907) with the surfaces free from external load. Liu and Chang (1965) solved the dynamic problem of an infinitely long hollow cylinder subjected to an internal axisymmetric blast and a sudden change of temperature, using the method of treating the time-dependent boundary conditions developed by Mindlin and Goodman (1950).

In the present paper a similar problem of a cylindrical hole in a thick plate of infinite radius is solved. We employ the dynamic theory of thermo-elasticity in which the heat conduction equation is taken independent of elastic field. The plate is assumed to be unstressed and to be initially at temperature zero. Then suddenly a uniform and constant temperature is applied on the inner surface of the hole. The heat thus flows through the plate parallel to the planes. The problem, therefore, reduces to one of a one-dimensional type and we are to find the stresses thus developed in the system. The method of solution is based on the simple theory of operators whereby much labour is saved. The thermo-elastic equations are solved in terms of non-dimensional variables and the expressions for the stresses and the displacement are obtained in the integral forms. To show the nature of the radial stress, the graphs are plotted for the integrand versus the variable of integration at different points in the plate and at various times. The area under these curves gives the stress obtained. The stress components thus obtained are exponential in character having maximum value at some time and then dying out quickly.

A similar method for Cartesian system of coordinates was also developed by the author (*in press*) and attempts are being made to develop a theory for spherical polar coordinates also.

2. Governing Equations

The equation of motion in the cylindrical coordinate system (r, θ, z), when the displacement vector (u_r, u_θ, u_z) is uniaxial, is (Nowacki 1962)

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}-\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=m\frac{\partial T}{\partial r}, \qquad (2.1)$$

where

$m=\dfrac{3\lambda+2\mu}{\lambda+2\mu}\alpha_T$, $\dfrac{1}{c^2}=\dfrac{\rho}{\lambda+2\mu}$, λ and μ are the Lamé constants and α_T is the coefficient of thermal expansion. The heat conduction equation, when the temperature field is given as $T=T(r, t)$, is (Nowacki 1962)

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial T}{\partial r}-\frac{1}{k}\frac{\partial T}{\partial t}=0, \qquad (2.2)$$

where k is the thermal diffusivity.

The components of stress in this case will be

$$\sigma_{rr}=\lambda\left(u,_r+\frac{1}{r}u\right)+2\mu u,_r-(3\lambda+2\mu)\alpha_T T, \qquad (2.3)$$

$$\sigma_{\theta\theta}=\lambda\left(u,_r+\frac{1}{r}u\right)+2\mu\cdot\frac{u}{r}-(3\lambda+2\mu)\alpha_1 T, \qquad (2.4)$$

$$\sigma_{zz}=\lambda\left(u,_r+\frac{1}{r}u\right)-(3\lambda+2\mu)\alpha_T T, \qquad (2.5)$$

$$\sigma_{r\theta}=\sigma_{rz}=\sigma_{\theta z}=0.$$

Introducing the non-dimensional variables, given as

$$U=\frac{u}{mT_0 b},\ \rho=\frac{r}{b},\ \tau=\frac{kt}{b^2},\ \theta=\frac{T}{T_0}, \qquad (2.6)$$

where

$b=\dfrac{k}{c}$, the eqns. (2.1) and (2.2) give

$$\frac{\partial^2 U}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial U}{\partial \rho}-\frac{U}{\rho^2}-\frac{\partial^2 U}{\partial \tau^2}=\frac{\partial \theta}{\partial \rho}, \qquad (2.7)$$

and

$$\frac{\partial^2 \theta}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial \theta}{\partial \rho}-\frac{\partial \theta}{\partial \tau}=0. \qquad (2.8)$$

Putting $u = \phi_{,\rho}$ in (2.7) and integrating once with respect to ρ, we get

$$\frac{\partial^2\phi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\phi}{\partial\rho}-\frac{\partial^2\phi}{\partial\tau^2}=\theta, \qquad (2.9)$$

where ϕ is the non-dimensional displacement potential. The relations (2.3) to (2.5) also reduce to the forms

$$\sigma_{\rho\rho}=\frac{\sigma_{rr}}{mT_0(\lambda+2\mu)}=\frac{\partial^2\phi}{\partial\rho^2}+\frac{\nu}{1-\nu}\cdot\frac{1}{\rho}\frac{\partial\phi}{\partial\rho}-\theta, \qquad (2.10)$$

$$\sigma_{\phi\phi}=\frac{\sigma_{\theta\theta}}{mT_0(\lambda+2\mu)}=\frac{\nu}{1-\nu}\cdot\frac{\partial^2\phi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\phi}{\partial\rho}-\theta, \qquad (2.11)$$

and

$$\sigma_{\xi\xi}=\frac{\sigma_{zz}}{mT_0(\lambda+2\mu)}=\frac{\nu}{1-\nu}\left[\frac{\partial^2\phi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\phi}{\partial\rho}\right]-\theta, \qquad (2.12)$$

where ν is Poisson's ratio.

3. Method of Solution

Eliminating θ from eqns. (2.8) and (2.9), we get

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial^2}{\partial\tau^2}\right)\phi=0. \qquad (3.1)$$

The solution of this equation can be taken in the form

$$\phi=\phi_1+\phi_2, \qquad (3.2)$$

where ϕ_1 is the solution of the equation

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial^2}{\partial\tau^2}\right)\phi_1=0, \qquad (3.3)$$

and ϕ_2 is that of

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial}{\partial\tau}\right)\phi_2=0. \qquad (3.4)$$

Substituting (3.2) in (2.9) and using (3.3), we get

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial^2}{\partial\tau^2}\right)\phi_2=\theta. \qquad (3.5)$$

Subtracting it from (3.4), we have

$$\left(\frac{\partial^2}{\partial\tau^2}-\frac{\partial}{\partial\tau}\right)\phi_2=-\theta. \qquad (3.6)$$

The solution of eqn. (3.6) is

$$\phi_2=f_1(\rho)+e^{\tau}f_2(\rho)-\int e^{\tau}\left\{\int e^{-\tau}\theta\,d\tau\right\}d\tau, \qquad (3.7)$$

where $f_1(\rho)$ and $f_2(\rho)$ are the functions of ρ only and are such that (3.7) satisfies (3.4). Substituting (3.7) in eqn. (3.4), we get

$$f_1''(\rho)+\frac{1}{\rho}f_1'(\rho) = \chi(\rho), \quad \ldots \quad (3.8)$$

and

$$f_2''(\rho)+\frac{1}{\rho}f_2'(\rho)-f_2(\rho) = 0, \quad \ldots \quad (3.9)$$

where $\chi(\rho)$ is a known function of ρ and is given as

$$\chi(\rho) = \left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{\partial}{\partial\tau}\right)\int e^{\tau}\left\{\int e^{-\tau}\theta\, d\tau\right\}d\tau.$$

Thus we have by solving (3.8) and (3.9)

$$f_1(\rho) = A\log\rho+B+F(\rho), \quad \ldots \quad (3.10)$$

and

$$f_2(\rho) = CI_0(\rho)+DK_0(\rho), \quad \ldots \quad (3.11)$$

where

$$F(\rho) = \int\frac{1}{\rho}\left\{\int\rho\chi(\rho)d\rho\right\}d\rho.$$

The general solution of (3.3) can be taken in the form

$$\phi_1 = \int_{\delta}^{\infty} e^{-\tau\alpha}\,[N(\alpha)I_0(\alpha\rho)+M(\alpha)K_0(\alpha\rho)]\,d\alpha,\ \delta>0, \quad \ldots \quad (3.12)$$

where δ is small. Since the solution does not exist at $\alpha = 0$, the value at $\alpha = 0$ is excluded from the general solution (3.12). Thus the complete solution of the equation (3.1) is

$$\phi = \int_{\delta}^{\infty} e^{-\alpha\tau}[N(\alpha)I_0(\alpha\rho)+M(\alpha)K_0(\alpha\rho)]d\alpha+A\log\rho+B$$
$$+F(\rho)-\int e^{\tau}\left\{\int e^{-\tau}\theta\, d\tau\right\}d\tau, \quad \ldots \quad (3.13)$$

where the term $e^{\tau}f_2(\rho)$ is merged in the value of ϕ_1. The constants A and B and the functions $M(\alpha)$ and $N(\alpha)$ can be determined from the boundary conditions of the problem.

4. Boundary Conditions

The surface of the hole is assumed initially at zero temperature and then suddenly heated and maintained at constant temperature T_0 uniformly. The surface is, otherwise, assumed to be free from external traction. The boundary conditions are, therefore,

$$T = 0,\ t \leqslant 0, \text{ and } r \geqslant R, \quad \ldots \quad (4.1)$$
$$T = T_0,\ r = R, \text{ and } t > 0, \quad \ldots \quad (4.2)$$
$$T = 0 \text{ as } r \to \infty, \text{ for all } t > 0, \quad \ldots \quad (4.3)$$
$$\sigma_{rr} = 0 \text{ at } r = R \text{ for all } t > 0, \quad \ldots \quad (4.4)$$

and all the stresses should be zero at infinity.

In the form of non-dimensional variables, given as in (2.6), these boundary conditions reduce to

$$\theta = 0,\ \tau \leqslant 0, \qquad \text{(4.5)}$$

$$\theta = 1,\ \rho = a\left(=\frac{R}{b}\right),\ \tau > 0, \qquad \text{(4.6)}$$

$$\theta = 0 \text{ as } \rho \to \infty, \text{ for all } \tau > 0, \qquad \text{(4.7)}$$

and

$$\sigma_{\rho\rho} = 0 \text{ at } \rho = a, \qquad \text{(4.8)}$$

where R is the radius of the cavity and a is the non-dimensional radius.

5. Solution

For finite displacement and to satisfy the regularity conditions at infinity, we take $N(\alpha)$ as equal to zero in the equations (3.13). Hence ϕ becomes

$$\phi = \int_{\delta}^{\infty} e^{-\alpha\tau} M(\alpha) K_0(\alpha\rho) d\alpha + A \log \rho + B + F(\rho) - \int e^{\tau} \left\{ \int e^{-\tau}\theta\, d\tau \right\} d\tau. \qquad \text{(5.1)}$$

The solution of the heat conduction equation (2.8) with the boundary conditions (4.5) to (4.7) is given as (Carslaw and Jaegar 1947)

$$\theta = 1 + \frac{2}{\pi} \int_0^{\infty} e^{-x^2\tau} \frac{J_0(x\rho)Y_0(xa) - J_0(xa)Y_0(x\rho)}{J_0^2(xa) + Y_0^2(xa)} \cdot \frac{dx}{x}. \qquad \text{(5.2)}$$

Substituting the value of θ from (5.2) into (5.1) and simplifying, we get

$$\phi = \int_{\delta}^{\infty} M(\alpha) e^{-\alpha\tau} K_0(\alpha\rho) d\alpha + A \log \rho + B + T + F(\rho)$$
$$- \frac{2}{\pi} \int_{\delta^2}^{\infty} \frac{e^{-x^2\tau}}{x^3(x^2+1)} \cdot \frac{J_0(x\rho)Y_0(xa) - J_0(xa)Y_0(x\rho)}{J_0^2(xa) + Y_0^2(xa)} . dx, \qquad \text{(5.3)}$$

the integral from 0 to δ^2 can be neglected as the integrand $\to 0$, as $x \to 0$.

Substituting the value of θ in $\chi(\rho)$ and then in $F(\rho)$, we have

$$F(\rho) = \tfrac{1}{4}\rho^2. \qquad \text{(5.4)}$$

Replacing x^2 by α and substituting the value of the function $F(\rho)$ from (5.4) in (5.3), we get

$$\phi = \int_{\delta}^{\infty} M(\alpha) e^{-\alpha\tau} K_0(\alpha\rho) d\alpha + A \log \rho + B + \tfrac{1}{4}\rho^2 + T$$
$$- \frac{2}{\pi} \int_{\delta}^{\infty} \frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)} \frac{J_0(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha}) - J_0(a\sqrt{\alpha})Y_0(\rho\sqrt{\alpha})}{J_0^2(a\sqrt{\alpha}) + Y_0^2(a\sqrt{\alpha})} d\alpha. \qquad \text{(5.5)}$$

Differentiating eqn. (5.5) with respect to ρ, we get

$$\frac{\partial \phi}{\partial \rho} = \int_{\delta}^{\infty} \alpha M e^{-\alpha\tau} K_0'(\alpha\rho) d\alpha + \frac{A}{\rho} + \tfrac{1}{2}\rho$$

$$-\frac{2}{\pi}\int_{\delta}^{\infty} \frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)} \sqrt{\alpha}\, \frac{J_0'(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha}) - J_0(a\sqrt{\alpha})Y_0'(\rho\sqrt{\alpha})}{J_0^2(a\sqrt{\alpha}) + Y_0^2(a\sqrt{\alpha})} d\alpha, \quad (5.6a)$$

and

$$\frac{\partial^2 \phi}{\partial \rho^2} = \int_{\delta}^{\infty} \alpha^2 M e^{-\alpha\tau} K_0''(\alpha\rho) d\alpha - \frac{A}{\rho^2} + \tfrac{1}{2}$$

$$-\frac{2}{\pi}\int_{\delta}^{\infty} \frac{e^{-\alpha\tau}}{2\alpha(\alpha+1)} \cdot \frac{J_0''(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha}) - J_0(a\sqrt{\alpha})Y_0''(\rho\sqrt{\alpha})}{J_0^2(a\sqrt{\alpha}) + Y_0^2(a\sqrt{\alpha})} d\alpha. \quad \ldots (5.6b)$$

Now substituting from (5.6a, b) and (5.2) in eqn. (2.10), we have

$$\sigma_{\rho\rho} = \int_{\delta}^{\infty} \alpha M e^{-\alpha\tau} \left\{ \alpha K_0''(\alpha\rho) + \frac{\nu}{1-\nu} \cdot \frac{1}{\rho} K_0'(\alpha\rho) \right\} d\alpha - \tfrac{1}{2}\left(1 + \frac{2A}{\rho^2}\right) \cdot \frac{1-2\nu}{1-\nu}$$

$$-\frac{2}{\pi}\int_{\delta}^{\infty} \frac{e^{-\alpha\tau}}{2\alpha(\alpha+1)} \left[\left\{ J_0''(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha}) - J_0(a\sqrt{\alpha})Y_0''(\rho\sqrt{\alpha}) \right\} + \frac{\nu}{1-\nu} \cdot \frac{1}{\rho\sqrt{\alpha}} \right.$$

$$\times \left\{ J_0'(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha}) - J_0(a\sqrt{\alpha})Y_0'(\rho\sqrt{\alpha}) \right\} + (\alpha+1)\{ J_0(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha})$$

$$\left. - J_0(a\sqrt{\alpha})Y_0(\rho\sqrt{\alpha}) \} \right] \frac{d\alpha}{J_0^2(a\sqrt{\alpha}) + Y_0^2(a\sqrt{\alpha})}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots (5.7)$$

Using the boundary conditions (4.8), we have

$$0 = \int_{\delta}^{\infty} e^{-\alpha\tau} \left[M\alpha \left\{ \alpha K_0''(\alpha a) + \frac{\nu}{1-\nu} \cdot \frac{1}{a} K_0'(a\alpha) \right\} \right.$$

$$\left. - \frac{2}{\pi} \cdot \frac{1}{2\alpha(\alpha+1)} \cdot \frac{2}{\pi a^2 \alpha} \cdot \frac{1-2\nu}{1-\nu} \cdot \frac{1}{J_0^2(a\sqrt{\alpha}) + Y_0^2(a\sqrt{\alpha})} \right] d\alpha - \tfrac{1}{2} \cdot \frac{1-2\nu}{1-\nu}\left(1 + \frac{2A}{a^2}\right).$$

Thus we have

$$A = -\tfrac{1}{2}a^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots (5.8)$$

and

$$M(\alpha) = \frac{2}{\pi^2 a^2} \cdot \frac{1}{\alpha^3(\alpha+1)} \cdot \frac{1-2\nu}{1-\nu} \cdot \frac{1}{J_0^2(a\sqrt{\alpha}) + Y_0^2(a\sqrt{\alpha})}$$

$$\times \frac{1}{\alpha K_0''(a\alpha) + \frac{\nu}{1-\nu} \cdot \frac{1}{a} K_0'(\alpha a)}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots (5.9)$$

Hence we have

$$\sigma_{\rho\rho} = \frac{2}{\pi}\int_{\delta}^{\infty} Z d\alpha - \tfrac{1}{2}\left(1 - \frac{a^2}{\rho^2}\right) \cdot \frac{1-2\nu}{1-\nu}, \quad \ldots \quad \ldots \quad \ldots (5.10)$$

where

$$Z=\frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)}\left[\frac{2}{\pi a^2}\cdot\frac{1-2\nu}{1-\nu}\cdot\frac{\alpha k_0(\alpha\rho)+\frac{1}{\rho}\cdot\frac{1-2\nu}{1-\nu}\cdot k_1(\alpha\rho)}{\alpha k_0(a\alpha)+\frac{1}{a}\,\frac{1-2\nu}{1-\nu}\cdot k_1(\alpha a)}\right.$$

$$+\frac{1-2\nu}{1-\nu}\cdot\frac{\sqrt{\alpha}}{\rho}\{J_0(a\sqrt{\alpha})Y_1(\rho\sqrt{\alpha})-J_1(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha})\}$$

$$\left.-\alpha^2\{J_0(\rho\sqrt{a})Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})Y_0(\rho\sqrt{\alpha})\}\right]\frac{1}{J_0^2(a\sqrt{\alpha})+Y_0^2(a\sqrt{\alpha})}. \quad (5.11)$$

Substituting the values of $M(\alpha)$ and A in eqn. (5.6a), we get the displacement as

$$U(\rho,T)=\frac{\partial\phi}{\partial\rho}=\frac{2}{\pi}\int_\delta^\infty\frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)}\left[-\frac{2}{\pi a^2}\cdot\frac{1-2\nu}{1-\nu}\cdot\frac{K_1(a\alpha)}{\alpha K_0(\alpha a)+\frac{1-2\nu}{1-\nu}\cdot\frac{1}{a}K_1(a\alpha)}\right.$$

$$\left.+\sqrt{\alpha}\{J_1(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})Y_1(\rho\sqrt{\alpha})\}\right]\frac{d\alpha}{J_0^2(a\sqrt{\alpha})+Y_0^2(a\sqrt{\alpha})}$$

$$-\frac{1}{2\rho}(a^2-\rho^2). \quad (5.12)$$

From eqns. (2.4) and (2.5), substituting the values from (5.2), (5.6a, b), (5.8) and (5.9), we have

$$\sigma_{\phi\phi}=\frac{2}{\pi}\int_\delta^\infty\frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)}\left[\frac{2}{\pi a^2}\cdot\frac{1-2\nu}{1-\nu}\cdot\frac{\frac{\nu}{1-\nu}\cdot\alpha K_0(\alpha\rho)-\frac{1}{\rho}\cdot\frac{1-2\nu}{1-\nu}K_1(\alpha\rho)}{\alpha K_0(a\alpha)+\frac{1}{a}\cdot\frac{1-2\nu}{1-\nu}\cdot K_1(a\alpha)}\right.$$

$$+\frac{1-2\nu}{1-\nu}\cdot\frac{\sqrt{\alpha}}{\rho}\{J_1(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})Y_1(\rho\sqrt{\alpha})\}$$

$$\left.-\alpha\left(\alpha+\frac{1-2\nu}{1-\nu}\right)\{J_0(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})Y_0(\rho\sqrt{\alpha})\}\right]$$

$$\times\frac{d\alpha}{J_0^2(a\sqrt{\alpha})+Y_0^2(a\sqrt{\alpha})}\cdot-\frac{1-2\nu}{2(1-\nu)}\cdot\left(1-\frac{a^2}{\rho^2}\right), \quad (5.13)$$

and

$$\sigma_{\xi\xi}=\frac{2\nu}{\pi(1-\nu)}\int_\delta^\infty\frac{e^{-\alpha\tau}}{2\alpha^2(\alpha+1)}\left[\frac{2}{\pi a^2}\cdot\frac{1-2\nu}{1-\nu}\cdot\frac{\alpha K_0(\alpha\rho)}{\alpha K_0(\alpha a)+\frac{1}{a}\cdot\frac{1-2\nu}{1-\nu}K_1(a\alpha)}\right.$$

$$\left.-\alpha^2\{J_0(\rho\sqrt{\alpha})Y_0(a\sqrt{\alpha})-J_0(a\sqrt{\alpha})Y_0(\rho\sqrt{\alpha})\}\right]\frac{d\alpha}{J_0^2(a\sqrt{\alpha})+Y_0^2(a\sqrt{\alpha})}. \quad (5.14)$$

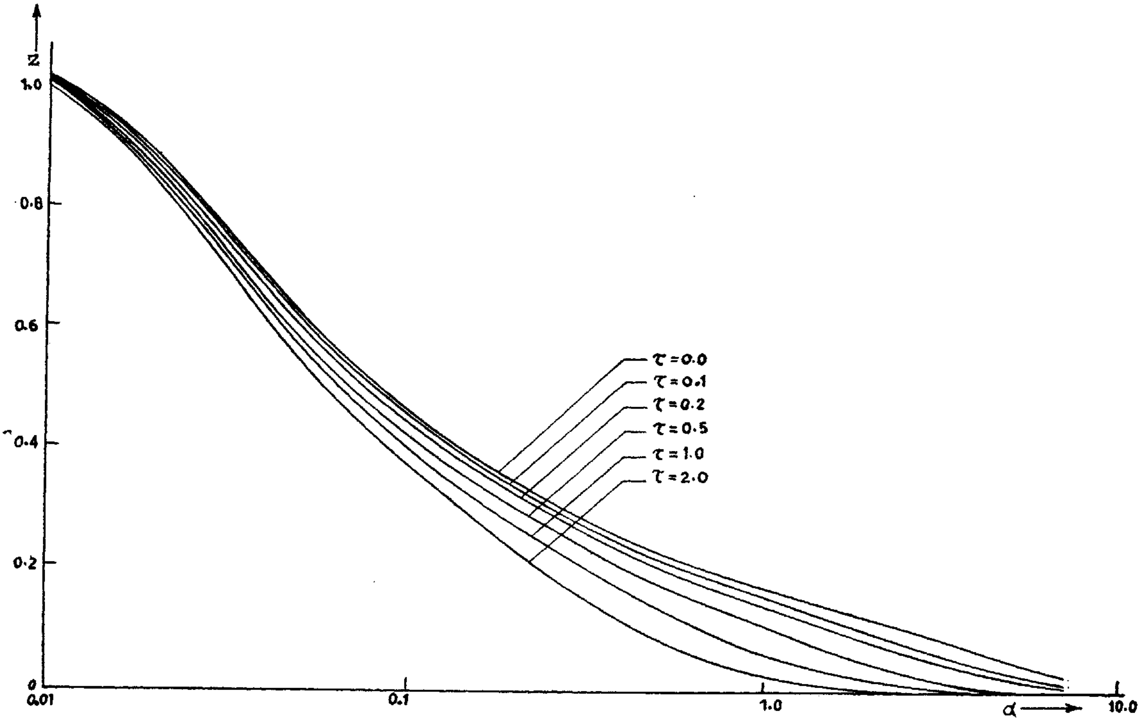

FIG. 1. Showing $\sigma_{\rho\rho}$ for $\rho = 2{\cdot}0$ and for various τ.

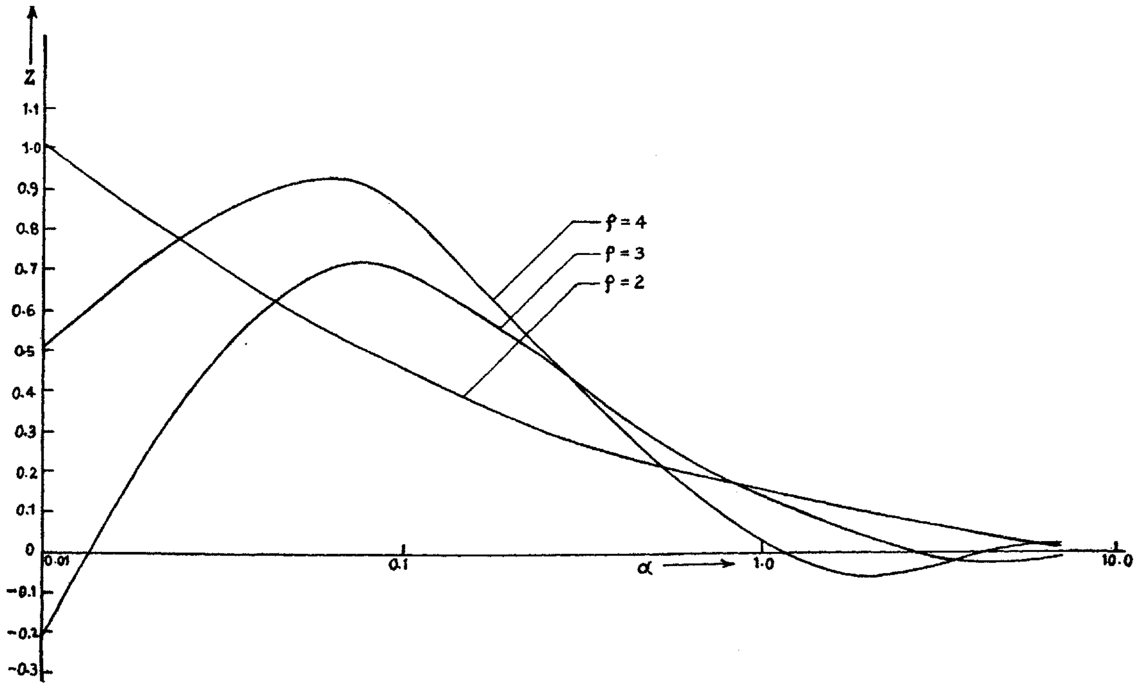

FIG. 2. Showing $\sigma_{\rho\rho}$ for $\tau = 0{\cdot}1$ and for various ρ.

6. Numerical Calculations

The graphs are drawn for the function Z for $\rho = 2$, $\tau = 0$, $0{\cdot}1$, $0{\cdot}2$, $0{\cdot}5$, 1 and 2 in Fig. 1 and in Fig. 2 for $\rho = 2$, 3, 4 and $\tau = 0{\cdot}1$. The value of Poisson's ratio ν is taken equal to $1/4$ in all calculations. From the graphs we find that the area enclosed by the curve in Fig. 1 decreases with time and tends to zero as τ becomes large.

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