RADIATION FROM A CHARGED PARTICLE MOVING IN A PLASMA

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Interaction of a beam of charged particles with a plasma has been studied, taking into consideration the effect of beam temperature assuming progressive wave solution. The frequency and amplitude of the first few harmonics have been obtained for beam velocity $\bar{v} > v_\phi$ and $v_\phi \ll c$. The spectrum of radiation from a single charged particle moving in the progressive wave field excited by the beam has also been derived and some numerical results are given.

1. INTRODUCTION

When a neutral beam of charged particles moves through a plasma medium, longitudinal waves are generated; if the phase velocity v_ϕ of the longitudinal plasma waves is smaller than $\bar{v}$, the mean velocity of the beam, there is transference of energy from the beam to the waves. This problem has been discussed for $\bar{v}-v_\phi \ll v_\phi$ by Klimontovich and Silin (1961) and also for $\bar{v} \ll v_\phi$ by Pashitskii (1963) for low density beams. In the latter case the beam was taken as monoenergetic, but the method adopted may be extended to investigate more general cases as we have done here. Besides, under the influence of progressive longitudinal plasma waves, the motion of the individual charged particles becomes oscillatory and so the beam loses energy in the form of electromagnetic radiation. A similar situation is encountered sometimes in particle accelerators also. In the latter section, oscillations induced in plasma medium by an electron moving with harmonic velocity have been studied and the radiated energy has been calculated.

2. PROGRESSIVE WAVE SOLUTION

For a beam of electrons with mean velocity $\bar{v}$ and temperature T moving through a cold plasma, the usual hydrodynamic equations are written in the non-dimensional form (Pashitskii 1963):

$$\frac{\partial v_0}{\partial \tau}+v_0\frac{\partial v_0}{\partial \zeta} = \frac{\partial \phi_0}{\partial \zeta}, \qquad (1A)$$

$$\frac{\partial u_0}{\partial \tau}+(\beta_0+u_0)\frac{\partial u_0}{\partial \zeta}+\frac{\theta}{\alpha+n_{0b}}\cdot\frac{\partial n_{0b}}{\partial \zeta}=\frac{\partial \phi_0}{\partial \zeta}, \qquad \text{(1B)}$$

$$\frac{\partial n_{0e}}{\partial \tau}+v_0\frac{\partial n_{0e}}{\partial \zeta}+(1-\alpha+n_{0e})\frac{\partial v_0}{\partial \zeta}=0, \qquad \text{(1C)}$$

$$\frac{\partial n_{0b}}{\partial \tau}+(\beta_0+u_0)\frac{\partial n_{0b}}{\partial \zeta}+(\alpha+n_{0b})\frac{\partial u_0}{\partial \zeta}=0 \qquad \text{(1D)}$$

and the potential equation

$$\frac{\partial^2 \phi_0}{\partial \zeta^2}-\beta_\phi^2\frac{\partial^2 \phi_0}{\partial \tau^2}=n_{0e}+n_{0b}. \qquad (2)$$

The entire system of plasma+beam is assumed to be electrically neutral. Thus if N_e and N_b denote the equilibrium electron density for the plasma and the beam respectively, then

$$N_e+N_b=\text{const}=N_0. \qquad (3)$$

The dimensionless quantities in eqns. (1) and (2) are defined by the following relations:

$$\left.\begin{array}{l}\tau=\omega_0 t, \quad \zeta=\dfrac{\omega_0}{v_\phi}.z, \; v_0=\dfrac{v}{v_\phi}, \quad u_0=\dfrac{u}{v_\phi}, \; \beta_0=\dfrac{\bar{v}}{v_\phi} \\ n_{0e}=\dfrac{n_e}{N_0}, \; n_{0b}=\dfrac{n_b}{N_0}, \quad \phi_0=\dfrac{e\phi}{mv_\phi^2}, \; \beta_\phi=\dfrac{v_\phi}{c}, \quad \alpha=\dfrac{N_b}{N_0} \\ \theta=\dfrac{KT}{mv_\phi^2}, \; \omega_0^2=\dfrac{4\pi N_0 e^2}{m}\end{array}\right] \qquad (4)$$

Since we investigate the case $\beta_\phi \ll 1$, the vector potential A_0 which is lower than ϕ by an order of β_ϕ has been left out.

For progressive wave solutions $f(\zeta, \tau) \equiv f(\zeta-\tau) \equiv f(\xi)$ the set of eqns. (1) are readily integrated:

$$\left.\begin{array}{l}(1-v_0)^2=c_1+2\phi_0 \\ (1-\beta_0-u_0)^2-\theta\ln(u_0+\beta_0-1)^2=c_2+2\phi_0 \\ (1-\alpha+n_{0e})(1-v_0)=c_3 \\ (\alpha+n_{0b})(u_0+\beta_0-1)=c_4\end{array}\right] \qquad (5)$$

We now attempt to solve for

$$U=(u_0+\beta_0-1)^2=U_0+g(\xi), \; U_0=(\beta_0-1)^2 \qquad (6)$$

by eliminating the other unknowns from (2) and (5). The resulting equation being a rather complicated non-linear one, we introduce the following simplifying assumptions:

$$U_0 \gg 1 \qquad \text{(A)}$$

$$g(\xi) \approx 0(\epsilon), \; \epsilon \ll 1. \qquad \text{(B)}$$

Using (2), (5) and (6) and expanding in powers of g/U_0 whenever convenient we now obtain the equation for $g(\xi)$ in the following form to which Bogoliuboff's method of quasilinear approximation may be applied:

$$\frac{d^2g}{d\xi^2}+\frac{\theta}{U(U-\theta)}\left(\frac{dg}{d\xi}\right)^2=\frac{2U}{U-\theta}\left[-1+\frac{c_3}{1-v_0}+\frac{c_4}{\sqrt{U}}\right],$$

which is next written in the form

$$g''=G_0+G_1g+(G_2g^2+F_2g'^2)+(G_3g^3+F_3gg'^2)+\dots \qquad (7)$$

where primes denote differentiation,

$$G_0=\frac{2U_0}{U_0-\theta}\left[-1+\frac{c_3}{\sqrt{K_0}}+\frac{c_4}{\sqrt{U_0}}\right], \qquad (8A)$$

$$G_1=-\frac{2}{U_0-\theta}\left(1-\frac{U_0}{U_0-\theta}\right)-\frac{2c_3U_0}{\sqrt{K_0}(U_0-\theta)}\left(\frac{1}{2}\cdot\frac{K_1}{K_0}+\frac{\theta}{U_0(U_0-\theta)}\right)-\frac{c_4(U_0+\theta)}{(U_0-\theta)^2\sqrt{U_0}}, \qquad (8B)$$

$$G_2=\frac{2U_0}{U_0-\theta}\left[-\frac{c_3}{2\sqrt{K_0}}\cdot\frac{K_2}{K_0}+\frac{3}{8}\cdot\frac{c_3}{\sqrt{K_0}}\left(\frac{K_1}{K_0}\right)^2+\frac{3}{8}\cdot\frac{c_4}{\sqrt{U_0}}\cdot\frac{1}{U_0^2}\right.$$
$$+\frac{1}{(U_0-\theta)^2}\left(-1+\frac{c_3}{\sqrt{K_0}}+\frac{c_4}{\sqrt{U_0}}\right)\Bigg]+\frac{2}{U_0-\theta}\left[\frac{c_3}{\sqrt{K_0}}\left(-\frac{K_1}{2}-\frac{1}{U_0-\theta}\right)\right.$$
$$\left.-\frac{c_4}{\sqrt{U_0}}\left(\frac{1}{2U_0}+\frac{1}{U_0-\theta}\right)+\frac{1}{U_0-\theta}\right], \qquad (8C)$$

$$G_3=\frac{2U_0}{U_0-\theta}\left[\frac{c_3}{\sqrt{K_0}}\left(-\frac{1}{2}\cdot\frac{K_3}{K_0}+\frac{3}{4}\cdot\frac{K_1K_2}{K_0^2}-\frac{1}{(U_0-\theta)^3}\right)-\frac{c_4}{\sqrt{U_0}}\left(\frac{5}{16}\cdot\frac{1}{U_0^3}+\frac{1}{(U_0-\theta)^3}\right)\right.$$
$$+\frac{1}{(U_0-\theta)^3}+\frac{1}{U_0-\theta}\left\{\frac{c_3}{\sqrt{K_0}}\left(\frac{1}{2}\cdot\frac{K_2}{K_0}-\frac{3}{8}\cdot\frac{K_1^2}{K_0^2}-\frac{K_1}{2(U_0-\theta)}\right)\right.$$
$$\left.\left.+\frac{c_4}{\sqrt{U_0}}\left(-\frac{3}{8}\cdot\frac{1}{U_0^2}-\frac{1}{2U_0}\cdot\frac{1}{U_0-\theta}\right)\right\}\right]+\frac{2}{U_0-\theta}\left[\frac{c_3}{\sqrt{K_0}}\left(-\frac{1}{2}\cdot\frac{K_2}{K_0}+\frac{3}{8}\cdot\frac{K_1^2}{K_0^2}\right.\right.$$
$$\left.+\frac{1}{(U_0-\theta)^2}+\frac{K_1}{2K_0}\cdot\frac{1}{U_0-\theta}\right)-\frac{1}{(U_0-\theta)^2}+\frac{c_4}{\sqrt{U_0}}\left(\frac{3}{8}\cdot\frac{1}{U_0^2}+\frac{1}{(U_0-\theta)^2}\right.$$
$$\left.\left.+\frac{1}{2U_0}\cdot\frac{1}{U_0-\theta}\right)\right], \qquad (8D)$$

$$F_2=-\frac{\theta}{U_0-\theta},\quad F_3=\frac{\theta}{(U_0-\theta)^2}, \qquad (8E)$$

$$\left.\begin{aligned}K_0&=c_1-c_2+U_0-\theta\ln U_0,\quad K_1=1-\frac{\theta}{U_0}\\ K_2&=\frac{1}{2}\cdot\frac{\theta}{U_0^2},\quad K_3=-\frac{1}{3}\cdot\frac{\theta}{U_0^3}\end{aligned}\right\}. \qquad (9)$$

In view of the assumption (B) and the eqns. (8) we have

$$\frac{c_3}{\sqrt{K_0}}+\frac{c_4}{\sqrt{U_0}} \simeq 1+O(\epsilon^2). \qquad \qquad (10)$$

We shall now show that (10) is actually satisfied by averaging eqns. (5) over a whole period; as the average of perturbed quantities vanishes to order ϵ^2 we have

$$\left.\begin{aligned} c_1 &\simeq 1+O(\epsilon^2) \\ c_2 &\simeq U_0-\theta \ln U_0+O(\epsilon^2) \\ c_3 &\simeq 1-\alpha+O(\epsilon^2) \\ c_4 &\simeq \alpha+O(\epsilon^2) \end{aligned}\right] . \qquad \qquad (11)$$

Except the second, (11) are identical with those given by Pashitskii (1963). Eqns. (9) together with (11) now show that (10) is satisfied which justifies the quasilinear approximation even for not too small values of α, i.e. for beams with density comparable to plasma density.

If the amplitude and frequency of the periodic solution of (7) are written in the form

$$A = a_0+\epsilon a_2+\epsilon^2 a_2+\ldots$$

$$\psi = \Omega\xi, \ \Omega = \Omega_0+\epsilon\Omega_1+\epsilon^2\Omega_2+\ldots$$

and

$$g(\xi) = g_0+\epsilon g_1+\epsilon^2 g_2+\ldots$$

then we have by using (9) and (11)

$$a_0 \approx 0(\xi), \ \Omega_0 = 1-\alpha+\alpha/(U_0-\theta), \ g_0 = a_0 \cos\psi, \qquad \qquad (12A)$$

$$a_1 = 0, \ \Omega_1 = 0, \ g_1 = -\frac{1}{\Omega_0^2}G_0+\frac{a_0^2}{8\Omega_0^2}(G_2-\Omega_0^2F_2)\cos 2\Omega_0\xi, \qquad \qquad (12B)$$

$$a_2 = 0, \ \Omega_2 = \frac{a_0^2}{8\Omega_0}(3G_3+\Omega_0^2F_3)-\frac{a_0^2}{2\Omega_0^2}(G_2+\tfrac{2}{3}\Omega_0^2F_2)(G_2-\Omega_0^2F_2)-\frac{1}{\Omega_0}\cdot G_0G_2,$$

$$g_2 = -\frac{a_0^3}{8\Omega_0^2}\left[\tfrac{1}{4}(G_3-F_3)-\frac{1}{2}\cdot\frac{1}{\Omega_0^2}(G_2-\Omega_0^2F_2)(G_2-\tfrac{2}{3}\Omega_0^2F_2)\right]\cos 3\psi. \qquad \qquad (12C)$$

3. Evaluation of the Field

We shall now determine the electromagnetic field generated by the motion of a charged particle (prototype for the beam) under the influence of the progressive plasma wave discussed in the previous section. For small amplitude waves the velocity of the charged particle is approximately

$$\bar{v}+\bar{u}\cos\bar{\Omega}t,$$

$$\bar{u} = \frac{1}{2}\cdot\frac{a_0}{\beta_0-1}\cdot v_\phi, \ \bar{\Omega} = (\beta_0)\omega_0\Omega_0 \qquad \qquad (13)$$

so that the current density is given by

$$\vec{I}\left(\vec{r}, t\right) \equiv 0,\ 0,\ e(\bar{v}+\bar{u}\cos\Omega t)\delta(x)\delta(y)\delta(z-\bar{v}t-\bar{a}\sin\Omega t), \quad (14)$$

where

$$\bar{a} = \frac{\bar{u}}{\Omega}.$$

Due to interaction of the charged particle beam with the plasma the latter becomes anisotropic (Makhankov 1964; Neufeld and Wright 1961). However, if the density of the beam is sufficiently low, the plasma may be treated as isotropic. Further, for a sufficiently high frequency of oscillation the ion motion is also neglected. Under these conditions the plasma medium is described by the dielectric constant

$$\epsilon_0 = 1-\frac{\omega_0^2}{\omega^2}.$$

The Fourier transform of the electric field is now easily shown to be

$$\vec{E}\left(\vec{k}, \omega\right) = \frac{4\pi i\omega}{c^2\Delta}\bar{\bar{A}}\,.\,\vec{I}\left(\vec{k}, \omega\right), \quad (15)$$

where

$$I_x\left(\vec{k}, \omega\right) = I_y\left(\vec{k}, \omega\right) = 0, \quad (16A)$$

$$I_z\left(\vec{k}, \omega\right) = \frac{e}{(2\bar{u})^3}\sum_{n=-\infty}^{\infty} J_n(k_z\bar{a})\,.\left[\bar{v}+\frac{n\bar{u}}{k_z\bar{a}}\right].\,\delta(k_z\bar{v}+n\bar{\Omega}-\omega), \quad (16B)$$

$$A_{ij} = k_i k_j\left(k^2-\epsilon_0\,.\frac{\omega^2}{c^2}\right), \quad (17A)$$

$$A_{ii} = \left(k_i^2-\epsilon_0\,.\frac{\omega^2}{c^2}\right)\left(k^2-\epsilon_0\,.\frac{\omega^2}{c^2}\right), \quad (17B)$$

$$\Delta = \epsilon_0\,.\frac{\omega^2}{c^2}\,.\left(k^2-\epsilon_0\,.\frac{\omega^2}{c^2}\right). \quad (18)$$

Using cylindrical polar coordinates we obtain the field components from the above equations

$$E_x\left(\vec{r}, t\right) = \int_{-\infty}^{\infty} d\omega\,.\,e^{i\omega t}\,.\frac{4\pi i}{\omega\epsilon_0}\int d\vec{k}\,.\frac{k_\rho\cos\phi\,.\,I_z\left(\vec{k}, \omega\right)\exp\,.\,i[k_z\,.\,z+k_\rho\,.\,\rho\cos(\phi-\theta)]}{k_\rho^2+k_z^2-\epsilon_0\,.\,\omega^2/c^2}.$$

Integration over k_z and ϕ are easily performed; the integral over k_ρ is

$$\int_0^\infty \frac{k_\rho^2 J_1(k_\rho\,.\,\rho)}{k_\rho^2-b_n^2}\,.\,dk_\rho,$$

$$b_n^2 = \frac{\omega^2}{c^2}\,.\,\epsilon_0-\left(\frac{\omega-n\bar{\Omega}}{\bar{v}}\right)^2 > 0, \quad (19)$$

b_n^2 must be positive in order that there may be a non-evanescent field. In the far field approximation the contribution to the outgoing wave is

$$\sqrt{\frac{\pi}{2\rho}} \cdot e^{i(\pi/4 - b_n \cdot \rho)} \cdot b_n^{1/2}$$

so that we finally have

$$E_\rho(\vec{\mathbf{r}}, t) = \frac{1}{\sqrt{2\pi\rho}} \sum_{n=-\infty}^{\infty} \int d\omega\, e^{i\omega t} \cdot \frac{e}{\epsilon_0 \bar{v}} \cdot b_n^{1/2} \cdot F_n(\omega, \rho, z), \quad \text{(20A)}$$

where

$$F_n(\omega, \rho, z) = J_n\left(\frac{\omega - n\bar{\Omega}}{\bar{v}} \cdot \bar{a}\right) \exp i \left[\frac{\pi}{4} - \frac{\omega - n\bar{\Omega}}{\bar{v}} \cdot z - b_n \cdot \rho\right]. \quad \text{(21)}$$

Similarly the other components are obtained.

$$E_z(\vec{\mathbf{r}}, t) = -\frac{1}{\sqrt{2\pi\rho}} \cdot \sum_{n=-\infty}^{\infty} \int d\omega\, e^{i\omega t} \cdot \frac{e}{\epsilon_0} \cdot \frac{b_n^{3/2}}{\omega - n\bar{\Omega}} \cdot F_n(\omega, \rho, z), \quad \text{(20B)}$$

$$H_\phi(\vec{\mathbf{r}}, t) = \frac{1}{\sqrt{2\pi\rho}} \sum_{n=-\infty}^{\infty} \int d\omega\, e^{i\omega t} \cdot \frac{e}{c} \cdot \frac{\omega}{\omega - n\bar{\Omega}} \cdot b_n^{1/2} \cdot F_n(\omega, \rho, z), \quad \text{(20C)}$$

$$E_\phi = H_\rho = H_z = 0. \quad \text{(20D)}$$

The integration over ω in these equations may be done approximately (for $\bar{a}$ sufficiently small so that J_n is replaced by the first term in the expansion) by the method of steepest descent for large value of $t - z/\bar{v}$ (Jones 1964; Burshtein and Voskresenskii 1963). The nth term in the equation (20A) when written explicitly in terms of ω becomes

$$E_\rho^{(n)} = B \int_{-\infty}^{\infty} d\omega\, e^{R \cdot f(\omega)} \left[\frac{h(\omega)}{\omega - \omega_0} + \frac{h(\omega)}{\omega + \omega_0}\right],$$

where

$$R = i\rho/c = i\tau_0\gamma, \quad \tau_0 = t - (z/\bar{v}), \quad \gamma = \frac{\rho/c}{t - (z/\bar{v})}, \quad \text{(22A)}$$

$$f(\omega) = \frac{\omega}{\gamma} - \sqrt{(\omega - \omega_1)(\omega - \omega_2)}, \quad \text{(22B)}$$

ω_1 and ω_2 being $\dfrac{n\bar{\Omega} \pm \beta\sqrt{n^2\bar{\Omega}^2 - (1-\beta^2)\omega_0^2}}{1-\beta^2}$,

$$\beta = \frac{\bar{v}}{c}, \quad \text{(22C)}$$

$$h(\omega) \simeq \frac{\omega^2}{2\omega_0}\left[\frac{\omega^2}{c^2} \cdot \epsilon_0 - \left(\frac{\omega - n\bar{\Omega}}{\bar{v}}\right)^2\right]^{1/4} \cdot \frac{1}{2^n \cdot n!} \cdot \left(\frac{\omega - n\bar{\Omega}}{\bar{v}} \cdot \bar{a}\right)^n, \quad \text{(22D)}$$

$$B = \frac{e^{i\pi/4}}{\sqrt{2\pi\rho}} \cdot \frac{e}{\bar{v}} \cdot e^{in\bar{\Omega} z/\bar{v}}. \quad \text{(22E)}$$

Thus

$$E_\rho^{(n)} = 2\pi iBh(\omega_0)\exp i\tau_0\gamma\left[\frac{\omega_0}{\gamma}-\sqrt{(\omega_0-\omega_1)(\omega_0-\omega_2)}\right]$$

$$-2\pi iBh(\omega_{s_1})\exp\frac{i\tau_0}{2}[(\omega_1+\omega_2)+(\omega_1-\omega_2)\sqrt{1-\gamma^2}]+\tfrac{1}{2}\tau_0\gamma z_1^2$$

$$+\pi iBh(\omega_{s_1})\exp\left\{\frac{i\tau_0}{2}[(\omega_1+\omega_2)+(\omega_1-\omega_2)\sqrt{1-\gamma^2}]+\tfrac{1}{2}\tau_0\gamma z_1^2\right\}\operatorname{erf}c\left\{-\left(\frac{\tau_0\gamma}{2}\right)^{1/2}z_1\right\}$$

$$+iB\left\{\frac{\pi\gamma^2(\omega_1-\omega^2)}{\tau_0(1-\gamma^2)^{3/2}}\right\}^{1/2}\cdot\frac{h(\omega_{s_2})\exp i\left[\frac{\tau_0}{2}\{(\omega_1+\omega_2)-(\omega_1-\omega_2)\sqrt{1-\gamma^2}\}+\frac{\pi}{4}\right]}{\omega_{s_2}+\omega_0}$$

$$+iB\left\{\frac{\pi\gamma^2(\omega_1-\omega_2)}{\tau_0(1-\gamma^2)^{3/2}}\right\}^{1/2}\cdot\left[\frac{h(\omega_{s_1})\exp i\left[-\frac{\pi}{4}+\frac{\tau_0}{2}\{(\omega_1+\omega_2)+(\omega_1-\omega_2)\sqrt{1-\gamma^2}\}\right]}{\omega_{s_1}+\omega_0}\right.$$

$$\left.+\frac{h(\omega_{s_2})\exp i\left[\frac{\pi}{4}+\frac{\tau_0}{2}\{(\omega_1+\omega_2)-(\omega_1-\omega_2)\sqrt{1-\gamma^2}\}\right]}{\omega_{s_2}+\omega_0}\right], \quad \ldots \quad \ldots \quad (23)$$

where ω_{s_1} and ω_{s_2} are $\dfrac{\omega_1+\omega_2}{2}\pm\dfrac{\omega_1-\omega_2}{2\sqrt{1-\gamma^2}}$ and $z_1 = (\omega_0-\omega_{s_1})\{f''(\omega_{s_1})\}^{1/2}$.

Similarly the other two integrals may be obtained.

4. Radiation from the Charged Particle

The energy radiated by the electron across the surface of a cylinder (unit length) whose axis is the line of motion of the charged point is equal to

$$dW = 2\pi\rho\int_{-\infty}^{\infty} S_\rho\, dt, \quad \ldots \quad \ldots \quad \ldots \quad (24A)$$

S_ρ being the ρ-component of the Poynting vector. Using eqns. (20), we get the radiation loss per unit frequency interval as

$$dW = \frac{e^2}{c^2}\sum_{n=0}^{\infty} J_n^2\left(\frac{\omega-n\bar{\Omega}}{\bar{v}}\cdot\bar{a}\right)\cdot\omega\left[\frac{\omega^2}{(\omega-n\bar{\Omega})^2}-\frac{c^2}{\epsilon_0\bar{v}^2}\right]$$

$$= \sum_{n=0}^{\infty} I_n. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24B)$$

5. Results

For a charged particle belonging to the beam, $\bar{\Omega}$ is given by (13) and (12A). Since the amplitude of the progressive wave is not exactly determined, we only show the nature of the radiation spectrum in Figs. 1 and 2. Because of

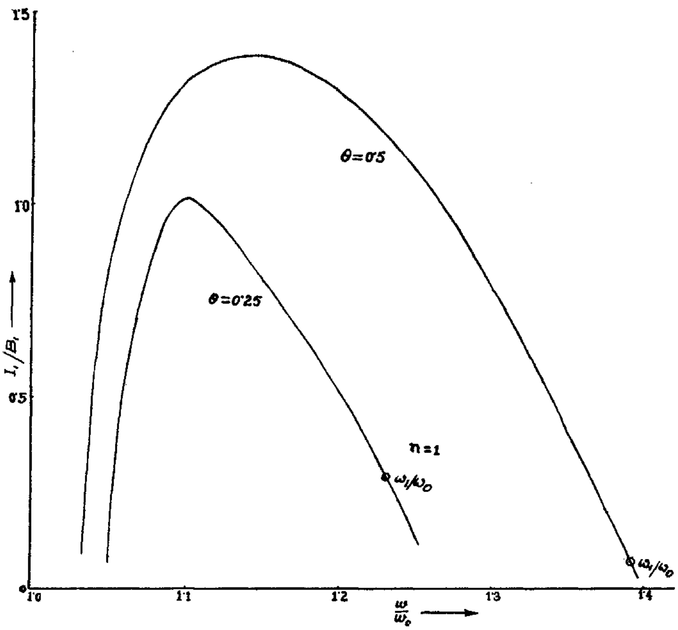
θ=0·5
θ=0·25
n=1
ω_1/ω_0
I_1/B_1
$\frac{\omega}{\omega_0}$
1·0
1·1
1·2
1·3
1·4
0·5
1·5

FIG. 1.

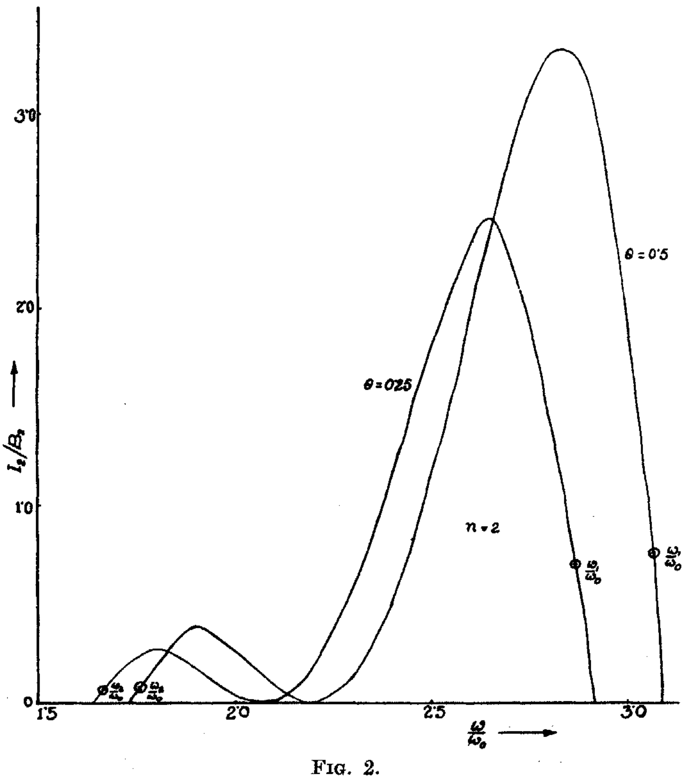
θ=0·5
θ=0·25
n=2
I_2/B_2
$\frac{\omega}{\omega_0}$
1·5
2·0
2·5
3·0
1·0

FIG. 2.

(19), the spectrum extends from ω_2 to ω_1 given by (22C). We have taken

$$\beta_0 = 2,\ \theta = 0{\cdot}5,\ 0{\cdot}25,\ \alpha = 0{\cdot}1,$$

$$\beta = 0{\cdot}3.$$

In the figures

$$B_1 = \frac{e^2}{4c^2}\left(\frac{\bar{a}}{\bar{v}}\right)^2 . \, \omega_0^3 \text{ and } B_2 = \frac{e^2}{64c^2}\left(\frac{\bar{a}}{\bar{v}}\right)^4 . \, \omega_0^5 .$$

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