EFFECTS OF CONDUCTING WALLS ON HEAT TRANSFER IN MHD COUETTE FLOW

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The paper presents the effects of conducting walls on heat transfer in fully developed MHD Couette flow under transversely applied magnetic field. The temperature is assumed to vary linearly along the walls, and expressions for temperature, Nusselt number and mean mixed temperature are obtained in closed form. Numerical calculations are presented under the condition of $\Phi_1 < \Phi_2$, where Φ_1 and Φ_2 are the wall electrical conductance ratios for the upper and the lower walls respectively. It is observed that for constant M, P_rRS and Φ_1, an increase in Φ_2 leads to an increase in the fluid temperature, whereas for constant M, P_rRS and Φ_2, an increase in Φ_1 leads to a decrease in the fluid temperature. Under various conditions, temperature profiles are shown graphically and the values of the Nusselt number and mean mixed temperature are presented in the form of a table followed by a discussion.

Notations

A	.. Temperature coefficient
c_p	.. Specific heat at constant pressure
E	.. Eckert number $(U^2/c_p\theta_1)$
H_0	.. Applied magnetic field
H	.. Dimensionless magnetic field (H_x/H_0)
$\vec{J}$	.. Current density vector
K	.. Thermal conductivity
$2L$	.. Separation between two walls
M	.. Hartmann number $\mu_c H_0 L(\sigma/\mu)^{\frac{1}{2}}$
Nu	.. Nusselt number $-\dfrac{1}{\theta(0)}\left(\dfrac{d\theta}{dy}\right)_{y=1}$
P_r	.. Prandtl number $\mu c_p/K$
R	.. Reynolds number $\rho UL/\mu$
R_m	.. Magnetic Reynolds number $4\pi\mu_c\sigma LU$
S	.. Dimensionless number A_1L/θ_1
T	.. Temperature
u	.. Dimensionless velocity u_1/U
U	.. Velocity of the walls

x, y .. Dimensionless coordinates $\left(\frac{x_1}{L}, \frac{y_1}{L}\right)$
μ .. Coefficient of viscosity
μ_c .. Magnetic permeability
σ .. Electrical conductivity of the fluid
η .. Magnetic permeability
$\sigma_{w,1}$ .. Electrical conductivity of the upper wall
$\sigma_{w,2}$ .. Electrical conductivity of the lower wall
ϕ_1 .. Electrical conductance ratio for upper wall $(\sigma_{w,1}l_1/\sigma_f L)$
ϕ_2 .. Electrical conductance ratio for lower wall $(\sigma_{w,2}l_2/\sigma_f L)$
A .. As defined by eqn. (8*a*)
θ_1 .. Characteristic temperature
θ .. Dimensionless temperature $\bar{\theta}/\theta_1$
l .. Thickness of the wall

Suffixes

f .. Fluid
w .. Confining walls
1 .. Upper wall
2 .. Lower wall

1. Introduction

Recently, an analysis was presented by Yen and Chang (1964) of the MHD Couette flow between two parallel walls, of different thicknesses and electrical conductivities, for fully developed laminar flow of an incompressible, viscous, electrically conducting fluid, under the action of transversely applied magnetic field. It was observed that variation of individual wall electrical conductances affects the flow field, and hence an MHD Couette flow with unequal wall electrical conductance ratios is quite different from the classical one and from the one with equal wall electrical conductance ratios. Chang and Yen (1962) also studied the influence of wall conductance on the flow of the conducting fluid between two stationary parallel walls, and Yen (1963) extended it to the study of heat transfer. It was observed in both these papers that the velocity field and temperature field are not influenced by the individual wall conductances, but are influenced by the sum of the wall electrical conductances.

The object of this paper is to study the heat transfer problem of the fluid in plane Couette flow, between two parallel and electrically conducting walls, under transverse magnetic field. Heat flux at the walls is not taken into consideration. The walls are assumed to be infinite in extent both in x_1 and z_1 directions and of unequal thicknesses and electrical conductivities, with

$\Phi_1 < \Phi_2$. But the wall temperature is assumed to vary linearly in the direction of flow.

With these assumptions in § 2, the problem is solved mathematically for temperature, Nusselt number and mean mixed temperature. In § 3, the temperature profiles are shown graphically, followed by a brief discussion. The numerical values for Nusselt number and mean mixed temperature are entered in Table I. A brief discussion follows this table.

In a subsequent note, the details will be presented for the case $\Phi_1 > \Phi_2$, for which the numerical calculations are being carried out.

2. Mathematical Analysis

As the walls are assumed to be infinite in extent, in both x_1- and in z_1-directions, the physical variables are functions of y_1 only. The x_1-axis is chosen along the centre line of the channel and y_1-axis is chosen normal to it. The flow is assumed to be steady and hence $\frac{\partial}{\partial t} = 0$. Hence the governing equations are

$$\mu \frac{d^2 u_1}{dy_1^2} + \frac{\mu_c H_0}{4\pi} \frac{dH_x}{dy_1} = 0, \qquad (1)$$

$$H_0 \frac{du_1}{dy_1} + \eta \frac{d^2 H_x}{dy_1^2} = 0, \qquad (2)$$

where

$$\eta = 1/4\pi\mu_c\sigma.$$

Introducing the following dimensionless quantities:

$$y = y_1/L, \quad H = H_x/H_0, \quad u = u_1/u \qquad (3)$$

in eqns. (1) and (2) we get

$$R_m \frac{d^2 u}{dy^2} + M^2 \frac{dH}{dy} = 0, \qquad (4)$$

$$R_m \frac{du}{dy} + \frac{d^2 H}{dy^2} = 0, \qquad (5)$$

where

$$M = \mu_c H_0 L (\sigma/\mu)^{\frac{1}{2}},$$

$$R_m = 4\pi\mu_c\sigma L U.$$

Here the Gaussian system of units is used. The boundary conditions on the velocity field are

$$u = \pm 1 \quad \text{at } y = \pm 1 \qquad (6)$$

and those at the conducting walls are the following (Shercliff 1956):

$$\left. \begin{aligned} \frac{dH}{dy} + \frac{1}{\Phi_1} H = 0 \quad \text{at } y = +1 \\ \frac{dH}{dy} - \frac{1}{\Phi_2} H = 0 \quad \text{at } y = -1 \end{aligned} \right\} \qquad (7)$$

where Φ_1, Φ_2 are as defined in the notation, and the subscripts 1 and 2 in (7) stand for upper and lower walls respectively.

The solutions of eqns. (4) and (5) in virtue of conditions (6) and (7) are

$$u = \frac{\sinh My}{\sinh M} + A(\cosh M - \cosh My), \qquad (8)$$

where

$$A = \frac{M(\Phi_1 - \Phi_2)}{\sinh M[M(\Phi_1+\Phi_2)\coth M + 2]}, \qquad (8a)$$

$$H = R_m\left(\frac{A \sinh My}{M} - \frac{\cosh My}{M \sinh M} - \frac{D_1}{M^2}\right), \qquad (9)$$

where

$$D_1 = M[A \sinh M(\Phi_1 M \coth M + 1) - (\Phi_1 M + \coth M)]$$

in terms of Φ_1, or

$$D_1 = -M[A \sinh M(\Phi_2 M \coth M + 1) + (\Phi_2 M + \coth M)]$$

in terms of Φ_2.

The current density is then given in non-dimensional form as

$$J = \frac{-1}{R_m}\left(\frac{dH}{dy}\right), \qquad (10)$$

where

$$J = j_z/\sigma B_0 U.$$

Hence, from (9) and (13) we get

$$J = -\left(A \cosh My - \frac{\sinh My}{\sinh M}\right). \qquad (11)$$

We now take the energy equation in the case of linearly varying wall temperature as

$$\rho c_p u_1 \frac{\partial T}{\partial x_1} = K\frac{\partial^2 T}{\partial y_1^2} + \mu\left(\frac{\partial u_1}{\partial y_1}\right)^2 + \frac{j_z^2}{\sigma}, \qquad (12)$$

and assume for T, as in (Siegel 1958)

$$T = A_1 x_1 + \bar{\theta}(y_1). \qquad (13)$$

Introducing the following additional non-dimensional quantities:

$$\theta = \bar{\theta}/\theta_1,$$

$$P_r = \frac{\mu c_p}{K},$$

$$R = \frac{\rho U L}{\mu},$$

$$S = \frac{A_1 L}{\theta_1},$$

$$E = U^2/c_p\theta_1$$

in equation (12), we get in virtue of (13)

$$\frac{d^2\theta}{dy^2} = P_rRSu - P_rE\left[M^2J^2 + \left(\frac{du}{dy}\right)^2\right]. \qquad .. \qquad .. \quad (14)$$

Substituting for u and J from (8) and (11) respectively in (14) and simplifying, we get

$$\frac{d^2\theta}{dy^2} = P_rRS\left[\frac{\sinh My}{\sinh M} + A(\cosh M - \cosh My)\right]$$

$$-P_rEM^2\left[\left(A^2 + \frac{1}{\sinh^2 M}\right)\cosh 2My - \frac{2A\sinh 2My}{\sinh M}\right]. \quad .. \quad (15)$$

The boundary conditions on θ are now

$$\theta = 0 \quad \text{at } y = \pm 1. \qquad .. \qquad .. \qquad .. \quad (16)$$

The solution of eqn. (15) subject to condition (16) is

$$\theta = P_rRS\left[\frac{\sinh My}{M^2\sinh M} - \frac{y}{M^2} + A\left(\frac{(y^2-1)\cosh M}{2} + \frac{\cosh M - \cosh My}{M^2}\right)\right]$$

$$-P_rE\left[\left(A^2 + \frac{1}{\sinh^2 M}\right)\frac{\cosh 2My - \cosh 2M}{4} + \frac{A(y\sinh 2M - \sinh 2My)}{2\sinh M}\right].$$

$$.. \quad (17)$$

Now, in technological problems, rate of heat transfer is expressed in terms of Nusselt number which is defined as

$$Nu = -\frac{L}{\bar{\theta}(0)}\left(\frac{dT}{dy_1}\right)_{y_1 = L}$$

$$= -\frac{1}{\theta(0)}\left(\frac{d\theta}{dy}\right)_{y=1}. \qquad .. \qquad .. \qquad .. \quad (18)$$

Hence from (17) and (18) we get

$$Nu = \frac{P_rRS\left[\frac{\coth M}{M} - \frac{1}{M^2} + A\left(\cosh M - \frac{\sinh M}{M}\right)\right] - P_rE\left[\left(A^2 + \frac{1}{\sinh^2 M}\right)\frac{M\sinh 2M}{2} + \frac{A(\sinh 2M - 2M\cosh 2M)}{2\sinh M}\right]}{P_rRSA\left[\frac{2 + (M^2-2)\cosh M}{2M^2}\right] + P_rE\left[\left(A^2 + \frac{1}{\sinh^2 M}\right)\left(\frac{1 - \cosh 2M}{4}\right)\right]}.$$

$$.. \quad (19)$$

The mean mixed temperature is defined as

$$T_{tm} = \frac{\int_{-1}^{1} u\theta\, dy}{\int_{-1}^{1} u\, dy}. \qquad .. \qquad .. \qquad .. \qquad .. \quad (20)$$

Hence from (17) and (20) we get, after simplification,

$$T_{im} = \frac{\left\{ P_rRS\left[\frac{\sinh 2M - 2M}{2M^2 \sinh^2 M} - \frac{2}{M^2}(\coth M - 1) + \frac{2A^2 \cosh M}{3M}(M(3-M)\cosh M - 3\sinh M) + \frac{A^2}{2M^2}((4-M^2)\sinh 2M - 4M\cosh^2 M - 2M^3)\right] - P_rE\left[\frac{A(A^2\sinh^2 M+1)(8\sinh^2 M + 3\sinh M + 4\cosh M(2-3\cosh^2 M))}{12\sinh^2 M} + \frac{A(3\sinh 2M(M\cosh M - \sinh M) - 2M\sinh^3 M)}{3M\sinh^2 M}\right]\right\}}{2A(M\cosh M - \sinh M)} \quad \text{.. (21)}$$

3. Physical Discussion of the Problem

Insight into the effects of magnetic field and the conductivity of the walls on the heat transfer can be obtained by the study of temperature distribution and the variation in the values of the Nusselt number and mean mixed temperature. Hence the values of θ, Nu and T_{im} are calculated by taking $\Phi_1 = 0$, 0·2, 0·4, 0·6, $\Phi_2 = 0$, 2, 4, 6, $M = 2$, 4, 6, $P_rRS = 0$, 0·2, 0·4, 0·6, $P_rE = 0{\cdot}2$. For temperature distribution, these are shown in Figs. 1–5 under different conditions and the values of Nu, T_{im} are entered in Table I. These findings may be given fuller perspective by looking into the details from the figures and tables.

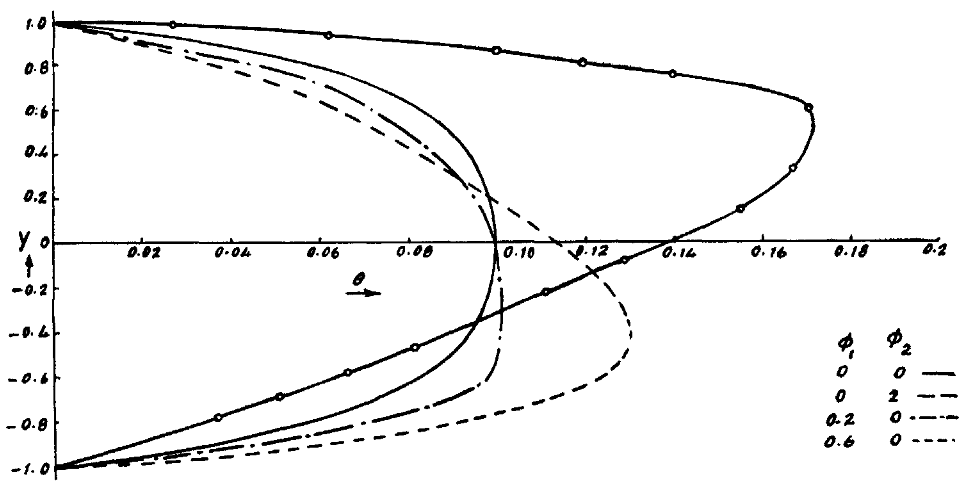

Fig. 1. Temperature profiles for $M = 2$, $P_rRS = 0$.

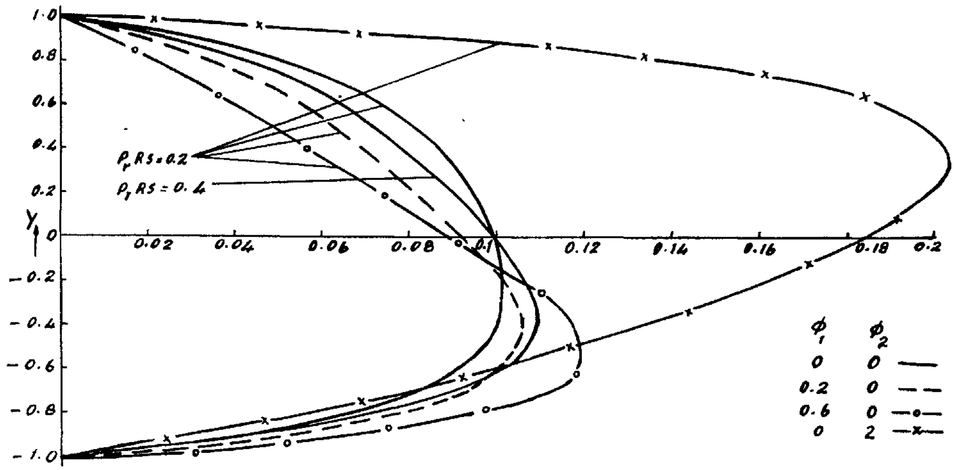

Fig. 2. Temperature profiles for $M = 2$.

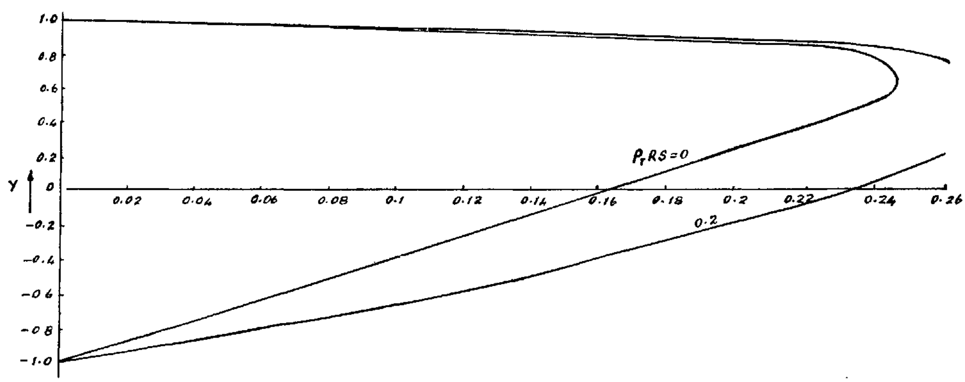

Fig. 3. Temperature profiles for $\Phi_1 = 0$, $\Phi_2 = 2$, $M = 4$, $P_rRS = 0$, 0·2.

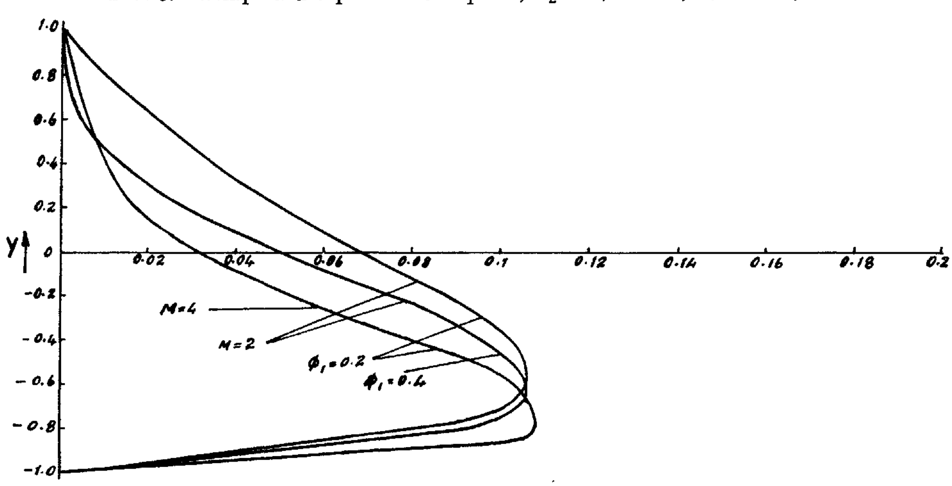

Fig. 4. Temperature profiles for $\Phi_2 = 0$, $P_rRS = 0{\cdot}6$, $\Phi_1 = 0{\cdot}2$, $0{\cdot}4$, $M = 2$, 4.

TABLE I

Values of Nusselt number and mean mixed temperature

			Nu		T_{im}	
Φ_2	Φ_1	M / P_rRS	2	4	2	4
0	0	0	0·3191	0·0107	–	–
	0·2		0·2485	0·0062	0·0406	0·0073
	0·4		0·2093	0·0045	0·0448	0·0110
	0·6		0·1858	0·0036	0·0493	0·0143
2	0		0·7696	0·0326	0·0722	0·0254
	0·2		0·6802	0·0280	0·0624	0·0191
	0·4		0·6073	0·0243	0·0552	0·0145
	0·6		0·5472	0·0214	0·0498	0·0113
4	0		0·8901	0·0359	0·0867	0·0303
	0·2		0·8244	0·0329	0·0787	0·0259
	0·4		0·7664	0·0303	0·0719	0·0222
	0·6		0·7150	0·0280	0·0662	0·0191
6	0		0·9448	0·0372	0·0936	0·0323
	0·2		0·8938	0·0350	0·0871	0·0290
	0·4		0·8471	0·0330	0·0814	0·0260
	0·6		0·8044	0·0312	0·0763	0·0234
0	0	0·2	0·2777	0·0102	–	–
	0·2		0·1951	0·0051	0·1305	−0·0084
	0·4		0·1467	0·0031	0·0707	−0·0201
	0·6		0·1160	0·0020	0·0470	−0·0258
2	0		0·7571	0·0337	0·1370	0·0874
	0·2		0·6657	0·0288	0·1071	0·0698
	0·4		0·5902	0·0249	0·0801	0·0553
	0·6		0·5270	0·0218	0·0546	0·0431
4	0		0·8784	0·0371	0·1743	0·0997
	0·2		0·8125	0·0340	0·1543	0·0885
	0·4		0·7540	0·0313	0·1359	0·0786
	0·6		0·7014	0·0288	0·1190	0·1698
6	0		0·9329	0·0385	0·1906	0·1045
	0·2		0·8821	0·0362	0·1754	0·0964
	0·4		0·8354	0·0341	0·1613	0·0890
	0·6		0·7924	0·0322	0·1480	0·0821
0	0	0·4	0·2364	0·0097	–	–
	0·2		0·1408	0·0040	0·2205	−0·0243
	0·4		0·0822	0·0017	0·0967	−0·0513
	0·6		0·0434	0·0004	0·0447	−0·0661
2	0		0·7454	0·0348	0·2017	0·1494
	0·2		0·6520	0·0296	0·1517	0·1205
	0·4		0·5738	0·0255	0·1050	0·0961
	0·6		0·5077	0·0222	0·0593	0·0750

TABLE I—(*concld.*)

			N_u		T_{im}	
Φ_2	Φ_1	P_rRS \ M	2	4	2	4
4	0		0·8676	0·0384	0·2619	0·1691
	0·2		0·8015	0·0351	0·2299	0·1512
	0·4		0·7421	0·0322	0·2000	0·1351
	0·6		0·6886	0·0296	0·1718	0·1205
6	0		0·9296	0·0398	0·2875	0·1768
	0·2		0·8713	0·0374	0·2637	0·1639
	0·4		0·8245	0·0352	0·2412	0·1519
	0·6		0·7811	0·0332	0·2198	0·1407
0	0	0·6	0·1951	0·0092	–	–
	0·2		0·0856	0·0030	0·3105	−0·0401
	0·4		0·0158	0·0003	0·1227	−0·0825
	0·6		−0·0319	−0·0011	0·0425	−0·1063
2	0		0·7343	0·0358	0·2664	0·2113
	0·2		0·6390	0·0305	0·1963	0·1712
	0·4		0·5582	0·0261	0·1300	0·1368
	0·6		0·4890	0·0226	0·0641	0·1068
4	0		0·8576	0·0396	0·3496	0·2385
	0·2		0·7911	0·0362	0·3055	0·2139
	0·4		0·7310	0·0331	0·2640	0·1916
	0·6		0·6765	0·0304	0·2247	0·1711
6	0		0·9118	0·0411	0·3844	0·2490
	0·2		0·8613	0·0386	0·3519	0·2314
	0·4		0·8143	0·0363	0·3210	0·2148
	0·5		0·7706	0·0342	0·2915	0·1994

Temperature profiles: One can observe from Figs. 1–5 that, in the case of conducting walls, the temperature profiles are deflected towards the wall of smaller electrical conductance ratio, though there is linear variation of temperature along the walls in the direction of flow. The temperature profiles are symmetrical about the centre line only when P_rRS, Φ_1, Φ_2 are all zeros. An increase in M and P_rRS leads to more deflection towards the wall of smaller electrical conductance ratio, which can be observed from Fig. 3.

From Fig. 5, one can conclude that for the same value of M, P_rRS and Φ_1 an increase in Φ_2 leads to an increase in the temperature of fluid, whereas an increase in Φ_1 leads to a decrease in fluid temperature when M, P_rRS and Φ_2 are maintained at constant value. Also, as M increases, θ increases when P_rRS, Φ_1, Φ_2 are constant.

Nusselt number: An increase in Φ_1 or M leads to a decrease in the value of Nu for the same value of Φ_2 and P_rRS. But, for the same value of Φ_1, M and P_rRS, an increase in Φ_2 leads to an increase in the value of Nu. Also, Nu decreases with the increase in P_rRS, when M, Φ_1, Φ_2 are maintained constant.

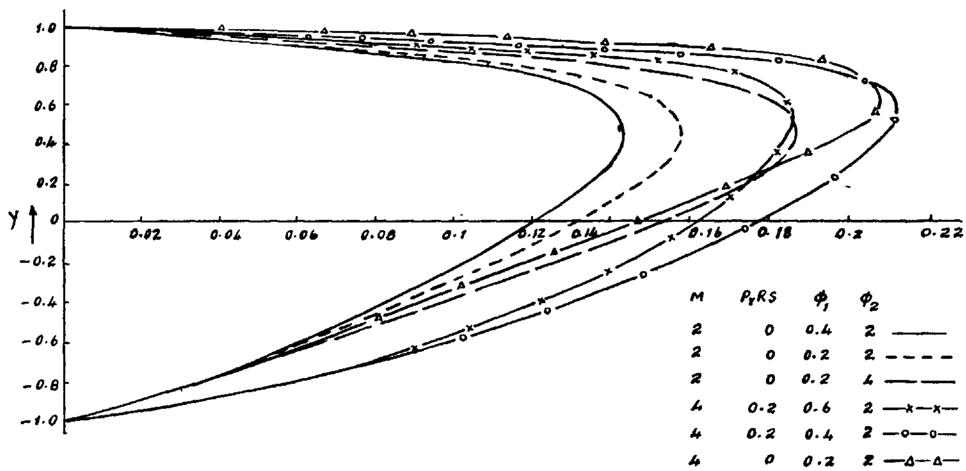

FIG. 5. Temperature profiles.

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