ON HEAT TRANSFER IN CROSSED-FIELDS MHD CHANNEL FLOW BETWEEN CONDUCTING WALLS

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A study of heat transfer aspect of fully developed flow in MHD channel under crossed-fields is presented. Closed form solutions for the temperature θ, Nusselt number Nu and the mean mixed temperature T_{im} are obtained under the following two conditions:

(i) Walls at different temperatures.

(ii) Linear variation of temperature along the walls.

Numerical values for θ, Nu, T_{im} are carried out for different values of M (the Hartmann number), e (the loading parameter), $\phi_u+\phi_l$ (sum of the electrical conductance ratios for the upper and lower walls respectively) and P_rRA (product of Prandtl number, Reynolds number and a non-dimensional number defined in the text). Some of the important effects of these parameters on the temperature field are brought out during the course of discussion.

1. INTRODUCTION

The phenomenon of heat transfer is encountered almost in all branches of technology. The MHD aspect of heat transfer in channel flow has been discussed by many research workers. Siegel (1958), Alpher (1961), Gershuni and Zukhovitskii (1958), Regirer (1960) and Yen (1963) have treated the heat transfer problem in case of fully developed flow, whereas Nigam and Singh (1960) considered the heat transfer at the entrance region of the channel. The effect of magnetic field on heat transfer in a channel flow between conducting and non-conducting walls has been considered in all these papers. MHD channel flows have a number of important applications such as MHD power generator, electromagnetic flow meter and electromagnetic accelerators. The last device has been used extensively in connection with nuclear power reactor to pump liquid sodium as a coolant. The channel works as a generator or an accelerator, depending upon the value of the loading parameter $e = E/UB$. The power output in the system is given by

$$P = \bar{J}\,.\,\bar{E} = JE$$

in a crossed-field generator or accelerator.

Since $J = \sigma uB(1-e)$, we have from above

$$P = \sigma u^2B^2e(1-e).$$

If $e < 1$, P is positive which means that the system delivers power to the load and this is consistent with the fact that the Lorentz force tends to retard the flow. Conversely, if $e > 1$, power must be supplied to the system, and the Lorentz force tends to accelerate the fluid, and the device may be called an accelerator. For $e = 1$, the power is neither supplied nor extracted from the system, and hence under this condition, the device is called the flow meter. In all these devices, heat is generated through viscous dissipation, joule heating and sometimes heat flux at the walls. So to get a good account of heat transfer problem in such practically used devices, in addition to magnetic field action, the action of electric field through loading parameter must also be considered.

Recently, such an attempt was made by Erickson *et al.* (1964) to study the effects of magnetic field, electric field and viscous dissipation on heat transfer in the entrance region of the channel with non-conducting walls. Heat flux at the channel walls was also not considered by Erickson *et al.* But under working conditions, the non-conducting walls of the channel do become conducting and hence the conductivity of the walls must also be considered, in order to obtain the full account of the heat transfer aspect.

Hence, the object of this paper is to study the effects of magnetic field, electric field and the electrical conductivity of the walls on the heat transfer in a fully developed channel flow, in the absence of externally imposed heat flux at the channel walls. To account for the electrical conductivity of the walls, the boundary conditions are proposed by Shercliff (1956) by assuming the continuity of the tangential electric field across the boundary between the fluid and the wall, which in the non-dimensional form are

$$\frac{dH}{dy} - \frac{1}{\phi_u} H = 0 \text{ at } y = 1$$

$$\frac{dH}{dy} - \frac{1}{\phi_l} H = 0 \text{ at } y = -1$$

where the electrical conductance ratios ϕ_u, ϕ_l, for the upper and lower walls respectively are defined as

$$\phi_u = \frac{\sigma_{w,u} \, . \, l_u}{\sigma_f L}, \; \phi_l = \frac{\sigma_{w,l} \, . \, l_l}{\sigma_f \, . \, L}$$

with σ_w as the electrical conductivity of the wall, the suffixes u or l denoting the upper or lower wall respectively, and l denotes the thickness of the wall. When the walls are non-conducting, $\sigma_w = 0$ and hence the boundary conditions in (1) reduce to $H = 0$ at $y = \pm 1$. But as pointed out by Sutton and Sherman (1965), this will only be the correct boundary condition in the event that the net current flow through the channel is zero, which occurs only for the open-circuit condition, regardless of the amount of current flowing in the channel.

Recently, it has been observed by Chang and Yen (1962) and Yen (1963) that, in case of channel with conducting walls, the flow field and the temperature field are affected not by the individual electrical conductance ratio of the wall, but by the sum of the electrical conductance ratios of both the walls. Hence, the particular solution obtained in case of non-conducting walls, by putting $\phi_u+\phi_l = 0$, corresponds to open-circuited case, i.e. for $e \neq 0$. Hence while comparing the two cases, viz. $\phi_u+\phi_l = 0$ and $\phi_u+\phi_l \neq 0$, the short-circuited case should be avoided, when the effects of loading parameter e on the temperature field are to be discussed.

In §2, the problem is suitably posed, and expression for velocity is derived. Then the energy equation is integrated with the following two assumptions:

(1) The two walls at different temperatures.

(2) Linear variation of temperature along the walls.

Temperature profiles are shown graphically under different conditions. Expressions for Nusselt number and mean mixed temperature are derived and their numerical values are entered in the tables. In §3, comparative discussion is presented in case of channel with conducting and non-conducting walls.

2. Mathematical Analysis

The steady, laminar flow of a viscous, incompressible, electrically conducting fluid between two infinite and conducting parallel walls in the x_1- and z_1-directions is assumed here. The separation between two parallel walls is assumed to be $2L$. The fluid is supposed to flow in the direction of x_1-axis, where x_1-axis is taken along the centre line of the channel. A uniform magnetic field is assumed to be applied transversely to the direction of flow and parallel to y_1-axis. The magnetic permeability μ_c and the electrical conductivity σ_f of the fluid medium are assumed to be constant scalar quantities. In the steady flow problem considered here, the displacement current vanishes identically.

Under these assumptions, the governing equations in non-dimensional form are

$$R_m \frac{d^2u}{dy^2}+M^2\frac{dH}{dy} = -P \qquad \text{(1)}$$

$$R_m \frac{du}{dy}+\frac{d^2H}{dy^2} = 0 \qquad \text{(2)}$$

where

$$\left.\begin{aligned} &x = x_1/L, \quad y = y_1/L, \quad H = H_x/H_o \\ &u = u_1/U, \quad p = p_1/\rho U^2, \quad P_1 = \frac{\partial P}{\partial x} \\ &P = -P_1 R R_m, \quad R = \frac{\rho UL}{\mu}, \\ &M = \mu_c H_o L\left(\frac{\sigma_f}{\mu}\right)^{\frac{1}{2}}, \quad \text{Hartmann number} \\ &R_m = 4\pi\mu_c\sigma_f LU, \quad \text{Magnetic Reynolds number.} \end{aligned}\right\} \qquad \text{(3)}$$

Here Gaussian system of units is used.

The non-dimensional form of boundary conditions are

$$\left.\begin{aligned} &u = 0 \text{ at } y = \pm 1 \\ &\frac{dH}{dy}+\frac{1}{\phi_u}H = 0 \text{ at } y = 1 \\ &\frac{dH}{dy}-\frac{1}{\phi_l}H = 0 \text{ at } y = -1 \end{aligned}\right\} \quad \ldots \quad (4)$$

where

$$\phi_u = \sigma_w l_u/\sigma_f L$$
$$\phi_l = \sigma_w l_l/\sigma_f L.$$

Solving eqns. (1) and (2), under the boundary conditions, we have the expression for the velocity as

$$u = \frac{\Phi P}{R_m}[\cosh M - \cosh My] \quad \ldots \quad (5)$$

where

$$\Phi = \frac{\phi_u+\phi_l+2}{M[(\phi_u+\phi_l)M \cosh M+2 \sinh M]}.$$

Integration of Energy Equation

Case I—Two walls at constant temperatures.

The energy equation in usual notation is

$$K\frac{d^2T}{dy_1^2}+\mu\left(\frac{du_1}{dy_1}\right)^2+\frac{j^2}{\sigma_f} = 0. \quad \ldots \quad (6)$$

By Ohm's law,

$$j = \sigma_f(E_0+\mu_c U_1 H_0). \quad \ldots \quad (7)$$

If we assume

$$E_0 = -eUB_0 \quad \ldots \quad (8)$$
$$B_0 = \mu_c H_0$$

then eqn. (6), by virtue of relations (7), (8) and (3) becomes

$$\frac{d^2\theta_I}{dy^2}+P_rE\left[\left(\frac{du}{dy}\right)^2+M^2(u-e)^2\right] = 0 \quad \ldots \quad (9)$$

where

$$\theta_I = \frac{T-T_2}{T_1-T_2},\ P_r = \frac{\mu c_p}{K},\ E = \frac{U^2}{c_p(T_1-T_2)}.$$

Here c_p, K, P_r, E have their usual meaning and T_1, T_2 are the temperatures of the upper and lower walls respectively. Then the boundary conditions are

$$\left.\begin{aligned} &\theta_I = 1 \text{ at } y = 1 \\ &\theta_I = 0 \text{ at } y = -1 \end{aligned}\right\}. \quad \ldots \quad (10)$$

Substituting for u from (5) in (9) and simplifying, we get

$$\frac{d^2\theta_I}{dy^2} = -P_rE[B_1+B_2 \cosh My+A_1^2M^2 \cosh 2My] \quad \ldots \quad (11)$$

where

$$A_1 = \frac{\Phi P}{R_m}$$

$$B_1 = M^2 A_1^2 \cosh^2 M + M^2 e^2 - 2eA_1 M^2 \cosh M$$

$$B_2 = 2(eA_1 M^2 - M^2 A_1^2 \cosh M).$$

The solution of eqn. (11), subject to conditions (10), is given as

$$\theta_I = \frac{y+1}{2} + P_r E\left[\frac{B_1(1-Y^2)}{2} + \frac{B_2}{M^2}(\cosh M - \cosh My) + \frac{A_1^2}{4}(\cosh 2M - \cosh 2My)\right]. \quad \ldots \quad (12)$$

The Nusselt number is defined as

$$Nu_1 = \frac{L}{T_1 - T_2}\left(\frac{dT}{dy_1}\right)_{y_1 = -L} = \left(\frac{d\theta_I}{dy}\right)_{y=-1}. \quad \ldots \quad (13)$$

Hence, from eqns. (12) and (13), we have

$$Nu_1 = P_r E\left(B_1 + \frac{B_2}{M}\cosh M + \frac{A_1^2 M}{2}\sinh 2M\right) + \tfrac{1}{2}. \quad \ldots \quad (14)$$

The mean mixed temperature is given by

$$T_{im} = \frac{\int_{-1}^{1} u\theta dy}{\int_{-1}^{1} u dy}. \quad \ldots \quad (15)$$

Hence, substituting for u and θ_I respectively from eqns. (5) and (12) in eqn. (15) and simplifying, we get

$$T_{im_1} = \frac{\left[M\cosh M - \sinh M + P_r E\left\{\frac{2B_1}{3M^2}(\sinh M + M(M^2-1)\cosh M) + \frac{B_2}{M}\left((2+\cosh 2M - \frac{3\sinh 2M}{2M}\right) + \frac{A_1^2}{16}(8M\cosh 2M\cosh M - 4\cosh M\sinh 2M - \sinh 4M + 1)\right\}\right]}{2(M\cosh M - \sinh M)}. \quad \ldots \quad (16)$$

The values of θ_I are plotted against y for different values of M, e and $\phi_u + \phi_l$ in the Figs. 1, 2 and 3.

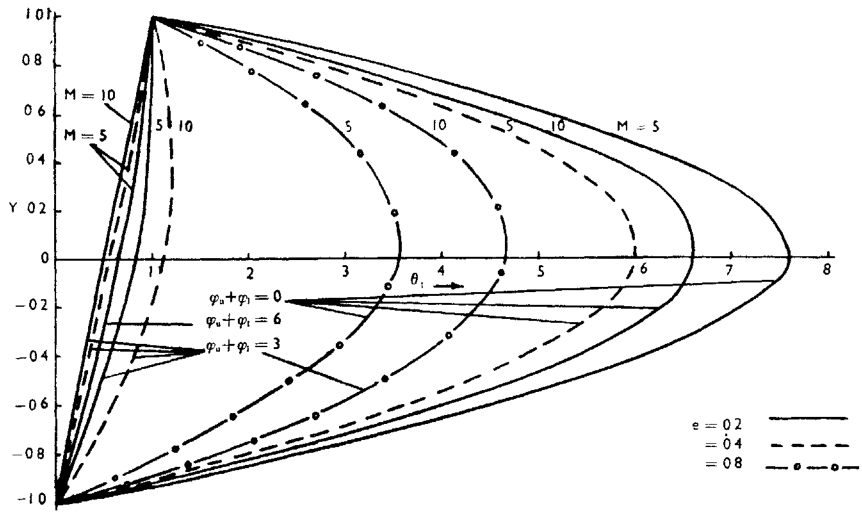

FIG. 1. Temperature profiles ($M = 5, 10;\ e = 0{\cdot}2, 0{\cdot}4, 0{\cdot}8;\ \phi_u+\phi_l = 0, 3, 6$).

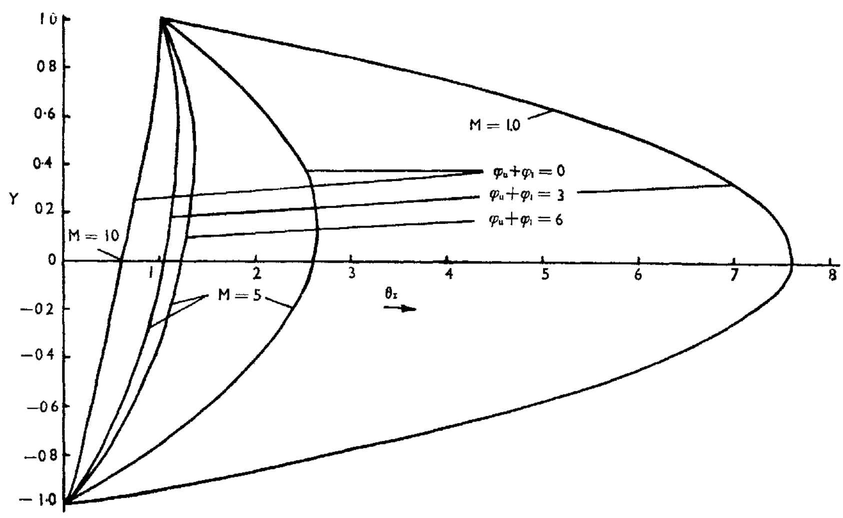

FIG. 2. Temperature profiles ($M = 5, 10;\ e = 1;\ \phi_u+\phi_l = 0, 3, 6$).

The values of Nu_1 and T_{im_1} are recorded in Table I.

Case II—Linear variation of temperature along the walls.

The energy equation in this case is

$$\rho c_p u_1 \frac{dT}{dx_1} = K\frac{d^2T}{dy_1^2}+\mu\left(\frac{du_1}{dy_1}\right)^2+\frac{j^2}{\sigma_f}. \qquad (17)$$

For varying temperature, we assume following Siegel (1958),

$$T = \tau x_1+\theta(y_1). \qquad (18)$$

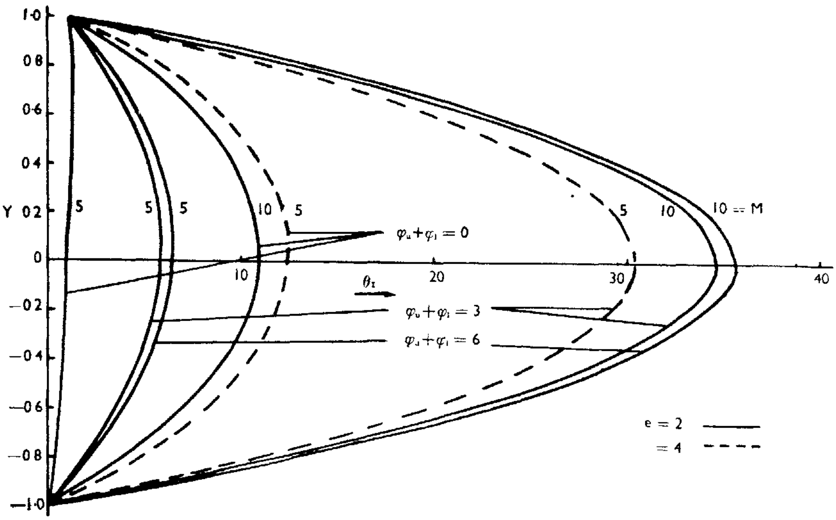

FIG. 3. Temperature profiles ($M = 5, 10$; $e = 2, 4$; $\phi_u+\phi_l = 0, 3, 6$).

TABLE I

Values of $\mathrm{Nu_I}$ *and* $\mathrm{Tim_1}$

M	$\phi_u+\phi_l$ \ e	0·2	0·4	0·6	0·8	1	2	4	6
					Nu_1				
5	0	13·5029	10·9025	8·7021	6·9018	5·5014	4·5000	32·5000	100
	3	1·1429	0·8017	0·8605	1·3193	2·1781	12·4721	63·0600	153
	6	0·9001	0·7000	0·9000	1·5000	2·5000	13·4998	65·5000	156
10	0	12·1000	7·3000	4·1000	2·5000	2·5000	26·5000	194	522
	3	0·6144	1·8894	4·7644	9·2394	15·3145	69·6895	297	686
	6	0·6706	2·1416	5·2125	9·8835	16·1545	71·5094	301	692
15	0	9·7666	3·9666	1·7666	3·1666	8·1666	87·1667	514	1302
	3	1·3197	5·5282	13·3368	24·7453	39·7538	168·7000	696	1584
	6	1·4672	5·8933	13·9194	25·5455	40·7716	170·9000	700	1590
					T_{im_1}				
5	0	6·2241	4·9926	3·9226	3·0138	2·2664	0·9492	10·4146	36·0129
	3	0·7628	0·5715	0·5415	0·6728	0·9655	4·8488	24·7150	60·7142
	6	0·6570	0·5307	0·5657	0·7620	1·1197	5·3280	25·8442	62·4934
10	0	5·0014	3·0215	1·6289	0·8236	0·6055	8·3240	67·8056	186
	3	0·5192	0·9531	1·9742	3·5826	5·7783	25·5656	109	251
	6	0·5421	1·0539	2·1529	3·8392	6·1127	26·2894	110	253
15	0	3·9115	1·6155	0·6000	0·8648	2·4000	29·3418	179	457
	3	0·7707	2·2427	4·9951	9·0278	14·3410	60·1125	247	563
	6	0·8263	2·3802	5·2145	9·3292	14·7243	60·9054	249	565

Substituting for T from (18) in (17), we get by virtue of the relations (3), (7) and (8),

$$P_r RAu = \frac{d^2\theta_{II}}{dy^2} + P_r E\left[\left(\frac{du}{dy}\right)^2 + M^2(u-e)^2\right] \quad \ldots \quad (19)$$

where

$$A = \frac{\tau L}{\theta_1}, \ \theta_{II} = \bar{\theta}\,\theta_1$$

where θ_1 is a characteristic temperature.

Substituting for u from (5) in (19) and simplifying, we get

$$\frac{d^2\theta_{II}}{dy^2} = A_2 - A_3 \cosh My - A_4 \cosh 2\,My \quad \ldots \quad (20)$$

where

$$A_2 = P_r \,.\, R \,.\, A \,.\, A_1 \cosh M - P_r E M^2 (A_1^2 \cosh^2 M + e^2 - 2eA_1 \cosh M)$$

$$A_3 = P_r \,.\, RA \,.\, A_1 + M^2 \,.\, P_r \,.\, E(2eA_1 - 2A_1^2 \cosh M)$$

$$A_4 = P_r \,.\, E \,.\, M^2 \,.\, A_1^2 \,.$$

The boundary conditions on θ_{II} following Siegel (1958) are now assumed as

$$\theta_{II} = 0 \text{ at } y = \pm 1 \quad \ldots \quad (21)$$

(the temperature of the fluid and the wall is assumed as the same).

Then the solution of eqn. (20), subject to the conditions (21), is given by

$$\theta_{II} = \frac{A_2}{2}(y-1) + \frac{A_3}{M^2}(\cosh M - \cosh My) + \frac{A_4}{4M^2}(\cosh 2M - \cosh 2My). \quad \ldots \quad (22)$$

The Nusselt number is defined as

$$Nu_2 = -\frac{L}{\bar{\theta}(0)}\left(\frac{dT}{dy_1}\right)_{y_1 = L} = -\frac{1}{\theta(0)}\left(\frac{d\theta}{dy}\right)_{y=1}. \quad \ldots \quad (23)$$

Hence, from eqns. (23) and (22), we get

$$Nu_2 = \frac{2M(2A_2M - 2A_3 \sinh M - A_4 \sinh 2M)}{2A_2M^2 - 4A_3[\cosh M - 1] - A_4[\cosh 2M - 1]}. \quad \ldots \quad (24)$$

The mean mixed temperature is obtained from eqns. (15) and (22) as

$$T_{tm_2} = \frac{\begin{array}{c}[8A_2(M(3-M^2)\cosh M - 3\sinh M) + 6A_3(4\cosh^2 M - 3\sinh 2M + 2M) \\ + A_4(6M(2\cosh M - \sinh M)\cosh 2M - 3\cosh M \,.\, \sinh 2M \\ + \sinh 3M + 3\sinh M)]\end{array}}{24M^2(M\cosh M - \sinh M)}. \quad (25)$$

The values of θ_{II} are plotted against y for different values of M, e, $P_r \,.\, RA$ and $\phi_u + \phi_l$ in Figs. 4–7.

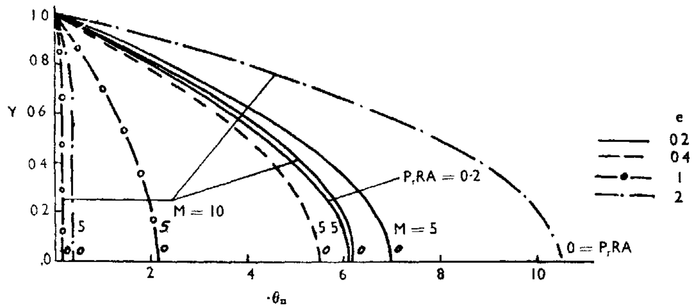

FIG. 4. Temperature profiles ($\phi_u+\phi_l = 0$; $M = 5, 10$; $e = 0{\cdot}2, 0{\cdot}4, 1, 2$; $P_rRA = 0, 0{\cdot}2$)

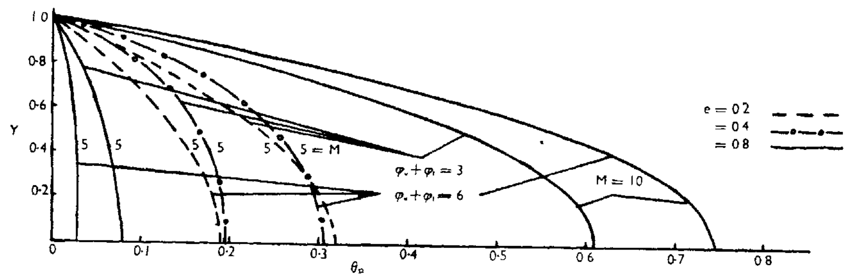

FIG. 5. Temperature profiles ($P_rRA = 0$; $e = 0{\cdot}2, 0{\cdot}4, 0{\cdot}8$; $M = 5, 10$; $\phi_u+\phi_l = 3, 6$).

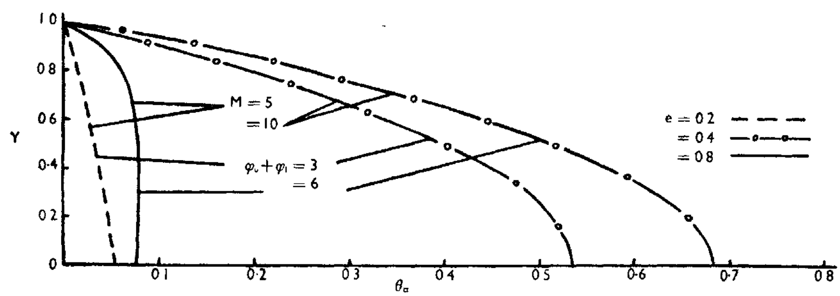

FIG. 6. Temperature profiles ($P_rRA = 0{\cdot}2$; $e = 0{\cdot}2, 0{\cdot}4, 0{\cdot}8$; $M = 5, 10$; $\phi_u+\phi_l = 3, 6$).

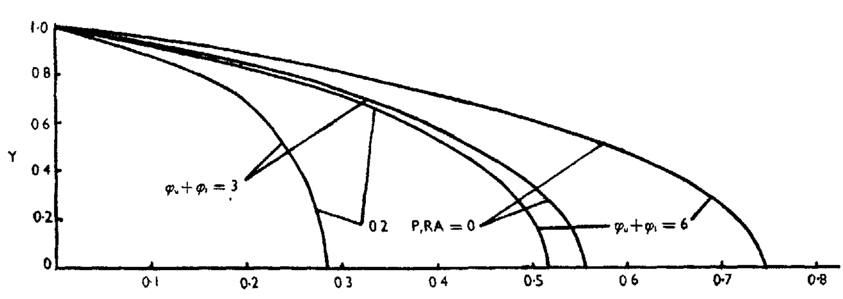

FIG. 7. Temperature profiles ($\phi_u+\phi_l = 3, 6$; $e = 1$; $M = 5$; $P_rRA = 0, 0{\cdot}2$).

The values of Nu_2 and T_{tm_2} are given in Tables II and III respectively.

TABLE II

Values of Nu_2

M	$P_r RA$	$\phi_u+\phi_l$ / e	0·2	0·4	0·6	0·8	1	2	4	6
5	0	0	1·8363	1·8782	1·9547	2·0963	2·3687	10·0027	2·6715	2·2958
		3	2·0010	3·7959	9·5700	4·1832	3·0288	2·2398	2·0904	2·0554
		6	2·0964	6·6077	5·7363	3·2341	2·6714	2·1863	2·0739	2·0459
	0·2	0	1·8512	1·9063	2·0159	2·2515	2·8574	−4·6027	2·7493	2·3079
		3	3·4195	0·8822	0·4719	−4·6461	4·2648	2·2667	2·0936	2·0566
		6	0·0000	1·0000	0·0000	7·6025	3·0864	2·2044	2·0764	2·0469
	0·4	0	1·8712	1·9485	2·1250	2·6426	6·6917	−0·5541	2·8414	2·3205
		3	1·3526	1·3831	1·1518	3·5233	6·0290	2·2965	2·0969	2·0577
		6	1·4829	1·3945	1·0236	−1·3208	4·1648	2·2240	2·0790	2·0478
10	0	0	1·8770	1·9653	2·3376	4·7618	2·0000	2·4762	2·1248	2·0706
		3	4·7068	2·2710	2·1329	2·0873	2·0049	2·0283	2·0132	2·0086
		6	3·0642	2·1891	2·1000	2·0676	2·0510	2·0228	2·0108	2·0071
	0·2	0	1·8804	1·9865	2·5714	−15·7151	−2·8205	2·5075	2·1264	2·0710
		3	0·5009	2·3331	2·1447	2·0920	2·0673	2·0287	2·0133	2·0087
		6	−7·2371	2·2216	2·1076	2·0709	2·0528	2·0232	2·0109	2·0071
	0·4	0	1·8846	2·0161	3·2142	−0·3571	−0·2272	2·5420	2·1279	2·0715
		3	1·2949	2·4160	2·1575	2·0968	2·0699	2·0291	2·0134	2·0087
		6	0·8705	2·2606	2·1157	2·0742	2·0546	2·0235	2·0110	2·0071
15	0	0	1·9225	2·2033	10·0000	5·5550	2·9113	2·1452	2·0510	2·0307
		3	2·1627	2·0553	2·0331	2·0236	2·0183	2·0086	2·0042	2·0027
		6	2·1141	2·0426	2·0260	2·0187	2·0146	2·0070	2·0034	2·0022
	0·2	0	1·9254	2·2884	−3·1636	13·6634	3·0588	2·1473	2·0512	2·0308
		3	2·1912	2·0578	2·0339	2·0240	2·0185	2·0087	2·0042	2·0027
		6	2·1296	2·0443	2·0267	2·0191	2·0148	2·0070	2·0034	2·0022
	0·6	0	1·9287	2·4350	−0·0416	−7·8688	3·2557	2·1495	2·0514	2·0309
		3	2·2263	2·0604	2·0348	2·0244	2·0188	2·0087	2·0042	2·0027
		6	2·1472	2·0461	2·0273	2·0194	2·0150	2·0070	2·0034	2·0022

3. NUMERICAL CALCULATIONS AND DISCUSSION

In order to study the effects of different fields represented by their respective parameters, numerical calculations for θ, Nu and T_{tm} are carried out for $P/Rm = 10$, $P_r R = 5$, $P_r E = 0{\cdot}2$, $P_r RA = 0$, $0{\cdot}2$, $0{\cdot}4$, $\phi_u + \phi_l = 0$, 3, 6; $M = 5$, 10, 15, and $e = 0{\cdot}2$, $0{\cdot}4$, $0{\cdot}6$, $0{\cdot}8$, 1, 2, 4, 6.

TABLE III

Values of Tim_2

M	$P_r RA$	$\phi_u+\phi_l$ \ e	0·2	0·4	0·6	0·8	1	2	4	6
5	0	0	6·7663	5·4655	4·3154	3·3160	2·4672	0·4834	7·8154	30·2137
		3	3·3708	0·1140	0·0417	0·1200	0·3490	3·7538	21·8630	55·0389
		6	2·0719	0·0515	0·0465	0·1922	0·4885	4·2301	23·0130	56·8622
	0·2	0	6·0029	4·7021	3·5521	2·5526	1·7039	−0·2799	7·0520	29·4503
		3	0·1125	−0·1104	−0·1827	0·1044	0·1245	3·5239	21·6387	54·8144
		6	0·0163	−0·1392	−0·1442	0·0013	0·2977	4·0393	22·8222	56·6714
	0·4	0	5·2395	3·9388	2·7887	2·7893	0·9405	−1·0433	6·2886	28·6870
		3	−0·1119	−0·3349	−0·4072	−0·3289	−0·0999	3·3048	21·4142	54·5890
		6	−0·1744	−0·3301	−0·3351	−0·1894	0·1068	3·8484	22·6314	56·4800
10	0	0	4·7397	2·7172	1·2713	0·4020	0·1092	7·2944	64·90	180
		3	0·0163	0·4300	1·4203	2·9872	5·1307	24·4970	106·47	246
		6	0·0379	0·5302	1·5991	3·2446	5·4667	25·2260	107·98	248
	0·2	0	4·3788	2·3563	0·9104	0·0410	−0·2516	6·9335	65·54	179
		3	−0·0400	0·3736	1·3639	2·9308	5·0744	24·4400	106·00	246
		6	−0·0086	0·4836	1·5525	3·1980	5·4201	25·1790	107 00	248
	0·4	0	4·0179	1·9954	0·5494	−0·3198	−0·6125	6·5725	64·18	179
		3	−0·0964	0·3172	1·3075	2·8744	5·0180	24·3844	106·00	246
		6	−0·0552	0·4370	1·5060	3·1515	5·3736	25·1330	107·00	248
15	0	0	3·5079	1·1782	0·1182	0·3279	1·8074	28·2550	176	451
		3	0·2648	1·7189	4·4427	8·4363	13·6996	59·0600	345	557
		6	0·3204	1·8567	4·6628	8·7386	14·0842	59·8500	246	560
	0·2	0	3·2727	0·9430	−0·1169	0·0927	1·5721	28·0150	276	451
		3	0·2398	1·6939	4·4177	8·4113	13·6746	59·0360	245	557
		6	0·2999	1·8363	4·6424	8·7182	14·0637	59·8370	246	560
	0·4	0	3·0375	0·7077	−0·3522	−0·1424	1·3369	27·7790	276	451
		3	0·2147	1·6689	4·3927	8·3862	13·6490	59·0110	245	557
		6	0·2794	1·8158	4·6219	8·6977	14·0430	59·8100	246	560

Now when $e < 1{\cdot}0$, the channel flow corresponds to a MHD generator. The efficiency of a MHD generator may be defined as the ratio of the electrical power to the flow power, which is identical to the value of the electric field factor e. But as pointed out by Moffatt (1963), the value of e for the maximum power is 0·5. In general, a reasonable compromise is, therefore, required between the conflicting requirements for maximum efficiency and the maximum power; $e = 0{\cdot}8$ is the generally accepted value. Hence some values are chosen which are greater than 0·5. The Prandtl number P_r for electrically conducting fluids can at most be of order unity and, for the validity of the assumption of incompressibility, the value of the Eckert number E_c must also be very small. Hence the value of P_rE is chosen as 0·2. The case corresponding to the value $\phi_u+\phi_l = 0$ is that of a channel with non-conducting walls.

Temperature Profiles

The results for θ_{I} (case I) are shown graphically in Figs. 1–3. In Fig. 1 temperature profiles in case of MHD generator are shown. We have the following observations:

1. In case of non-conducting and conducting walls, for $e = 0{\cdot}2$, an increase in Hartmann number M leads to a decrease in θ_{I}, but for $e = 0{\cdot}8$ an increase in M leads to an increase in θ_{I}.
2. For $e = 0{\cdot}2$, $M = 5$, an increase in $\phi_u+\phi_l$ leads to a decrease in θ_{I}.
3. For $M = 5$, $\phi_u+\phi_l = 0$ or 3, an increase in e leads to a decrease in θ_{I}.

In Fig. 2 temperature profiles in case of MHD flowmeter are shown. We have the following observations:

1. In case of non-conducting walls, an increase in M leads to a decrease in θ_{I} whereas, in case of conducting walls, an increase in M leads to an increase in θ_{I}.
2. For the same M, an increase in $\phi_u+\phi_l$ leads to an increase in θ_{I}.

In Fig. 3 temperature profiles in case of MHD accelerator are shown. We have the following observations:

1. In case of conducting as well as non-conducting walls, an increase in M leads to an increase in θ_{I}.
2. For the same M, an increase in $\phi_u+\phi_l$ leads to an increase in θ_{I}.
3. For $M = 5$, $\phi_u+\phi_l = 0$ or 3, an increase in e leads to an increase in θ_{I}.

Nusselt Number

Case I—1. In case of MHD generator, for low value of M (say) $= 5$, an increase in e leads to an increase in Nu_1 when the walls are non-conducting but, in case of conducting walls, there is first a decrease and then an increase in Nu_1, as e increases, but at high M, say $M = 15$, there is a decrease in Nu_1 first, and then the trend is towards an increase in case of non-conducting walls, whereas for conducting walls, an increase in e leads to an increase in Nu_1.

2. In MHD accelerator, an increase in e leads to an increase in Nu_1, both in case of conducting as well as non-conducting walls.

3. In case of generator, an increase in M leads to a decrease in Nu_1, in case of non-conducting walls, but in case of conducting walls, it decreases first and then the trend is towards the increase, for small values of e.

For $e = 0{\cdot}8$ and $\phi_u+\phi_l = 0$, as M increases, there is first decrease in Nu_1, and then the trend is towards increase. But in case of conducting walls, an increase in M or $\phi_u+\phi_l$ leads to an increase in Nu_1.

4. In case of accelerator, an increase in M or $\phi_u+\phi_l$ leads to an increase in Nu_1.

Case II—1. In case of a generator, for small values of M, and $P_r RA = 0$, $\phi_u+\phi_l = 0$, an increase in e leads to an increase in Nu_2. But at high values, Nu_2 first increases and then the trend is towards the decrease.

2. In case of an accelerator, an increase in e leads to a decrease in Nu_2.

3. In general, an increase in $\phi_u+\phi_l$ leads to a decrease in Nu_2.

4. Also, an increase in $P_r RA$ leads to an increase in Nu_2.

Mean Mixed Temperature

Case I—Generator: 1. For low values of M, an increase in e leads to a decrease in T_{im_1} in case of non-conducting walls, but at large M, it first decreases and then the trend is towards the increase. But in case of conducting walls, it increases with e.

2. For $e = 0{\cdot}2$, $0{\cdot}4$ and $\phi_u+\phi_l = 0$, as M increases, T_{im_1} decreases. But for $e = 0{\cdot}8$, it first decreases and then increases. For $\phi_u+\phi_l = 3$, for $e = 0{\cdot}2$, it decreases first and then increases. But for $e > 0{\cdot}2$, T_{im_1} increases with M.

Accelerator: 3. An increase in e or M or $\phi_u+\phi_l$ leads to an increase in T_{im_1}.

Case II—Generator: 1. In case of non-conducting walls, T_{im_2} decreases as e increases for small M but, for large values of M, it decreases first and then the trend is towards the increase.

2. For $M = 10$, 15, an increase in $\phi_u+\phi_l$ leads to an increase in T_{im_2}.

Accelerator: 1. T_{im_2} increases with M, e, $\phi_u+\phi_l$ or $P_r \,.\, RA$.

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