NOTE ON THE DISTRIBUTION OF SMALL PROPER MOTIONS IN 97 REGIONS COMMON TO POTSDAM PHOTOGRAPHISCHE HIMMELS KARTE AND OXFORD ASTROGRAPHIC CATALOGUES OF +32° and +33°

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Frequency distribution of small proper motions in eleven classes is discussed in the present paper.

Goyal (1958*a*, *b*, 1961-62, 1964, 1965), Goyal and Chaturvedi (1966) and Aravamudan (1956, 1959*a*, *b*, 1960, 1961, 1964, 1966) have derived the proper motions in Potsdam-Oxford section of plates in +32° and +33°. The distribution of large proper motions is discussed by Goyal and Sharma (1961), Shringi (1962-63), Goyal *et al.* (1961) have discussed the distribution of small proper motions in eleven classes in 87 regions. In the present paper the residuals in the sense x_0-x_p and y_0-y_p have been studied, where

$$x_p^1 = x_p+a\,.\,x_p+b\,.\,y_p+c$$

and

$$y_p^1 = y_p+d\,.\,x_p+e\,.\,y_p+f$$

further a, b, c, d, e, f are the plate constants and x_p, y_p are the Potsdam measures reduced to Oxford measures. The general procedure of deriving the plate constants is given in brief by Goyal (1961).

Table I gives the frequency distribution of the residuals Δx and Δy in each hour of R.A. in various class intervals. The class intervals were chosen in accordance with Struge's rule (Waugh 1943). In order to minimize the number of frequency curves, the data were combined according to the probable errors given in the last column. The p.e. and class intervals are given in the units of reseau interval. The unit being 0·001 r.i. The material was divided into four classes according to probable errors in R.A. $\leqslant 0{\cdot}00085$, $0{\cdot}00085 < \Delta x$ (p.e.) $\leqslant 0{\cdot}00095$, $0{\cdot}00095 < \Delta x$ (p.e.) $\leqslant 0{\cdot}00105$ and Δx (p.e.) $> 0{\cdot}00105$. The total frequency distributions so obtained are recorded in Table II. The first four columns give the frequency distribution for R.A. and the remaining four columns give the frequency data for the declination. The curves were drawn and smoothed. The smoothed values are recorded in parentheses in Table II and shall be represented by Y hereafter.

Table I

R.A.	& δ	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	p.e.
1^h	32°	3	3	10	18	21	27	21	15	9	6	4	±0·00103
		4	5	9	17	22	29	24	18	11	4	2	0·00100
2^h	32°	4	5	9	13	20	19	18	16	7	5	2	0·00109
		3	7	7	12	23	22	19	18	8	2	1	0·00100
3	32°	2	4	7	14	25	27	20	12	7	5	2	0·00098
		2	4	7	15	22	26	25	15	4	4	2	0·00089
4	32°	1	4	8	21	35	41	34	19	8	3	2	0·00080
		3	4	9	18	33	41	37	19	9	2	1	0·00084
5	32°	5	9	23	48	82	96	78	41	18	6	2	0·00081
		4	12	19	49	55	93	81	46	21	9	2	0·00088
6	32°	3	5	14	25	49	59	42	28	11	5	2	0·00078
		3	8	14	27	40	53	43	28	16	5	1	0·00087
7	32°	3	7	13	26	42	50	40	19	10	7	1	0·00088
		4	6	11	23	35	46	41	26	14	4	3	0·00091
8	32°	3	7	17	24	30	35	34	25	11	5	2	0·00102
		4	10	15	22	34	34	31	26	16	5	3	0·00103
1	33°	4	6	12	21	29	26	28	19	11	10	4	0·00110
		3	8	14	22	29	31	30	19	10	5	4	0·00102
2	33°	2	5	12	20	26	31	28	16	10	4	4	0·00097
		4	4	10	18	30	33	32	24	8	3	2	0·00092
*3	33°	4	8	17	30	46	62	44	30	17	8	5	0·00121
		5	8	19	35	49	64	50	28	10	5	3	0·00085
4	33°	3	7	15	27	43	51	40	26	12	5	2	0·00092
		5	5	12	26	46	49	44	25	12	4	4	0·00089
5	33°	4	10	26	58	94	115	93	46	24	9	3	0·00084
		7	12	24	52	72	114	88	56	24	9	6	0·00085
6	33°	3	4	15	32	56	86	55	27	12	5	2	0·00081
		2	8	13	30	52	64	53	36	14	4	5	0·00083
7	33°	3	7	13	27	44	47	41	24	12	3	2	0·00087
		4	7	14	22	42	44	43	27	13	5	1	0·00091
8	33°	3	4	10	15	16	18	15	13	6	2	2	0·00105
		3	3	8	13	17	20	16	11	8	4	2	0·00100

* A.M. in 3^h 31° +33° was very large so we gave it a weight 0·75 still the p.e. is very large.

If h is the modulus of precision and r the probable error, then we have $h \cdot r = 0{\cdot}4769$, the values of $1/h$ for all R.A. are 0·0017, 0·0019, 0·0020, 0·0023 and for declination 0·0018, 0·0019, 0·0020, 0·0020, then from a well-known formula (Smart 1938) we have

$$\bar{\epsilon} = -\frac{1}{2h^2}\frac{V'(x)}{V(x)} \quad \ldots \quad (1)$$

where $\bar{\epsilon}$ denotes average error for stars with the observed characteristic lying between X and $X+\Delta X$, $V(x)$ is the observed number of stars with the characteristic x and $V'(x)$ is the first derivative of $V(x)$. Equation (1) has been

TABLE II

Residuals	I	II	III	IV	V	VI	VII	VIII
−5	16 (8)	9 (10)	13 (13)	12 (12)	19 (20)	13 (10)	17 (8)	11 (8)
−4	32 (32)	21 (20)	23 (29)	19 (23)	44 (40)	18 (18)	26 (24)	23 (20)
−3	86 (82)	41 (38)	50 (49)	38 (38)	79 (75)	31 (34)	49 (49)	45 (40)
−2	184 (184)	80 (76)	90 (76)	64 (59)	174 (176)	71 (68)	85 (87)	69 (63)
−1	316 (320)	129 (125)	118 (112)	95 (89)	252 (262)	123 (117)	125 (125)	101 (100)
0	397 (380)	148 (150)	138 (130)	107 (110)	365 (355)	139 (132)	142 (140)	117 (109)
+1	305 (317)	121 (125)	118 (116)	90 (93)	302 (306)	132 (126)	128 (127)	99 (99)
+2	161 (164)	69 (69)	81 (85)	65 (66)	185 (185)	78 (92)	94 (95)	65 (63)
+3	74 (72)	34 (29)	43 (51)	35 (38)	84 (76)	39 (34)	47 (59)	28 (34)
+4	27 (29)	15 (9)	22 (23)	23 (20)	29 (35)	14 (17)	20 (26)	12 (15)
+5	8 (8)	5 (4)	16 (10)	11 (7)	15 (16)	6 (10)	13 (13)	8 (6)

extensively used by Dyson and Nassau (quoted by Smart 1938) for the correction of Parallaxes with the given mean probable errors. Goyal and Sharma (1961) have used this for correcting the large proper motions of Goyal, and Shringi (1962-63) for correcting the large proper motions of Aravamudan. In the present paper the authors have utilized it for correcting small proper motions in the zones $+32°$ and $+33°$. Equation (1) can be simplified to

$$\bar{\epsilon} = a \, . \, \frac{\Delta y}{y} \quad \ldots \quad (2)$$

where the values of a are −0·0014, −0·0018, −0·0020, −0·0026, −0·0016, −0·0018, −0·0020, −0·0020, and Δy is the difference of ordinates in the observed and smoothed curves. The corresponding values of $\bar{\epsilon}$ are entered in Table III.

Units for column I was 0·001 r.i. and 0·00001 r.i. for the rest. The frequency was recorded without decimals. A close look at the frequencies entered in Table II reveal that the residuals have a tendency to obey the Gaussian distribution (Richmond 1957). Representing the equations of the curves given by the data in Table II by $Y = mp$

where

$$m = N/\sigma \quad \ldots \quad (3)$$

$$p = (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \ldots \quad (4)$$

where the letters have their usual meaning. The values of N, σ, μ are given in Table IV for the eight curves.

TABLE III

Residuals	I	II	III	IV	V	VI	VII	VIII
−5	+70	−18	0	0	−8	+4	+106	+54
−4	0	+9	−5	−54	+15	0	+15	+26
−3	+7	+14	+4	0	+8	−17	0	+22
−2	0	+9	+31	+20	0	+7	−5	+9
−1	−2	+5	+10	+14	−6	+8	0	+2
0	+6	−2	+12	−7	+4	+9	+3	+13
+1	−5	−6	+3	−8	−2	+8	+1	0
+2	−3	0	−9	−4	0	−33	−4	+6
+3	+4	+3	−4	−22	+15	−27	−51	−12
+4	−10	+7	−9	−33	−33	−38	−60	−5
+5	+70	+4	+8	−94	−11	−12	0	+5

TABLE IV

Sl. No.	N	σ	μ
1	1,606	1·756	−0·0953
2	672	1·903	−0·1413
3	712	2·094	−0·0393
4	559	2·158	−0·0017
5	1,550	1·813	+0·0019
6	664	1·927	−0·0060
7	746	2·109	−0·0388
8	578	2·031	−0·2040

The goodness of the fit was tested by χ^2 test (Richmond 1957). The cumulative frequency distributions were computed.

A close examination of these cumulative frequency curves revealed that they represent straight lines between $x = -3$ and $x = +3$, a conclusion arrived at by Goyal *et al.* (1961). Eddington's relation given by Smart (1938) after some simplification can be written as:

$$U(x) = V(x) - q \,.\, b + r \,.\, d$$

where b and d are the second and fourth tabular differences respectively and

$$q = 12/(48h^2\alpha^2)$$

$$Y = (96h^4\alpha^4)^{-1}[2h^2\alpha^2+3]$$

where $\alpha = 0{\cdot}00375$ r.i. The true frequency function $U(x)$ can be easily calculated from the observed frequency function $V(x)$.

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