ON HEAT TRANSFER IN MHD CHANNEL FLOW IN SLIP-FLOW REGIME

by V. M. SOUNDALGEKAR, *Department of Mathematics, Indian Institute of Technology, Powai, Bombay 76*

(Communicated by V. R. Thiruvenkatachar, F.N.I.)

(*Received 12 December 1967*)

An analysis of heat transfer in a MHD channel flow of a viscous, incompressible rarefied gas is presented here under slip-flow and temperature-jump boundary conditions. It is observed that an increase in K (loading parameter) leads to a decrease in Nusselt number for all λ, Γ (slip-parameter, temperature-jump coefficient). Also, with the increase in λ and Γ, there is a decrease in the Nusselt number.

1. INTRODUCTION

The flow of fluids in channels is of importance in technological fields. Such a study has been undertaken by a number of researchers in the case of flow field and temperature field. They have also considered the effects of transversely applied magnetic field on the channel flows. One such attempt was made by Soundalgekar (1969) where most of the references of earlier work were cited. In all these problems, the fluid was assumed to be of normal density, i.e. density changes due to an increase in temperature or a decrease in pressure were not taken into account.

But the channel flows of ionized gases, which are electrically conducting, are receiving considerable importance in recent years, due to their practical applications in MHD generators, accelerators, etc. The working of the channel under different conditions, as a MHD generator or an accelerator, has been described in detail by Soundalgekar (1969) in the case of continuum flows. Now the channel flow of an electrically conducting, viscous, incompressible and low-density gases has been recently described by Inman (1965*a*) in the case of fully developed flows. Inman (1965*b*) also studied the heat transfer aspect of the channel flows of low-density gases without the magnetic field. But the heat transfer aspect of these electrically conducting low-density gases under magnetic field has not received any attention.

Hence, as a step towards a better understanding of the heat transfer aspect of the low-density gases under transversely applied magnetic field, an analysis is performed for forced convection heat transfer in MHD channel flows. In section 2, the problem is posed and the expressions for velocity profiles derived by Inman (1965*a*) are assumed. Then the energy equation is integrated under the assumption of the two walls of the channel to be at different temperatures. The expression for Nusselt number is derived in terms of mean mixed temperature and other parameters.

2. The Mathematical Analysis

Here the steady, fully developed flow of the low-density gases, which are viscous, incompressible and electrically conducting, is considered under the action of the transversely applied magnetic field. The x-axis is taken at the centre-line of the channel in the direction of the flow and the z-axis chosen normal to it. A uniform magnetic field is assumed to be applied transversely, i.e. parallel to z-axis. The magnetic permeability μ_e and the electrical conductivity σ_f of the ionized gases are assumed to be constant scalar quantities.

In the steady flow problem considered here, the displacement current vanishes identically. Under these assumptions, the expression for the non-dimensional velocity under the first-order velocity slip boundary conditions as derived by Inman (1965a) is

$$u = \left(1-\frac{\alpha \cosh M\eta}{\cosh M}\right)\Big/A \qquad (1)$$

where

$$A = 1-\frac{\alpha \tanh M}{M}, \quad \alpha = \frac{1}{1+\lambda M \tanh M}$$

$$\lambda = \xi_u/L, \text{ the rarefaction parameter}$$

$$u = u/\bar{u}, \quad \eta = z/L.$$

The slip coefficient ξ_u is given by the expression (Inman 1965a)

$$\xi_u = \left[\frac{2-\beta}{\beta}\right]-\bar{l}$$

where $\bar{l}$ is the mean free path, given by (Inman 1965a)

$$\bar{l} = [(\sqrt{\pi/8})/0{\cdot}499]\mu(\sqrt{RT/p})$$

and β is termed Maxwell's reflection coefficient, R the gas constant, $\bar{u}$ the mean gas velocity, M the Hartmann number and L the half width of the channel.

The energy equation for the fully developed flow is

$$k\frac{d^2t}{dz^2}+\mu\left(\frac{du}{dz}\right)^2+\frac{j^2}{\sigma_j} = 0 \qquad (2)$$

where t is the temperature of the fluid. Here k, μ and j have their usual meaning.

5B

The boundary conditions are

$$\left.\begin{aligned} t_{g,1}-t_{w,1} &= -\xi_t\left(\frac{dt_{g,1}}{dz}\right)_{z=L} \\ t_{g,2}-t_{w,2} &= \xi_t\left(\frac{dt_{g,2}}{dz}\right)_{z=-L} \end{aligned}\right\} \qquad \ldots \quad \ldots \quad \ldots \quad (3)$$

where

$$\xi_t = \frac{2-a}{a}\,\frac{2\lambda}{\lambda+1}\,\frac{1}{Pr}$$

is the temperature-jump coefficient. Here a is an accommodation coefficient, λ the ratio of the specific heats and Pr the Prandtl number. Eqns. (2) and (3) may be expressed in terms of non-dimensional quantities as

$$\frac{d^2T}{d\eta^2}+PrE\left[\left(\frac{du}{d\eta}\right)^2+M^2J^2\right] = 0 \qquad \ldots \quad \ldots \quad \ldots \quad (4)$$

$$\left.\begin{aligned} T(1) &= T_{g,1} = 1-2\Gamma\left(\frac{dT}{d\eta}\right)_{\eta=1} \\ T(-1) &= T_{g,2} = -\left[1-2\Gamma\left(\frac{dT}{d\eta}\right)_{\eta=-1}\right] \end{aligned}\right\} \qquad \ldots \quad \ldots \quad (5)$$

where

$$T = \frac{t-t_{w,m}}{t_{w,1}-t_{w,m}}, \quad T_{g,1} = \frac{t_{g,1}-t_{w,m}}{t_{w,1}-t_{w,m}}$$

$$T_{g,2} = \frac{t_{g,2}-t_{w,m}}{t_{w,1}-t_{w,m}}, \quad Pr = \frac{\mu C_p}{k}$$

$$E = \frac{\bar{u}^2}{C_p(t_{w,1}-t_{w,m})} \text{ (Eckert number)}, \quad J = j/\sigma_f B_0\bar{u}, \quad \Gamma = \xi_t/2L.$$

Also, from Ohm's law, we have

$$J = K-U \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

where $K = E_0/\bar{u}B_0$ is the loading parameter.
Substituting for u and J from eqns. (1) and (6) respectively in eqn. (4), we get, after doing some algebra,

$$\frac{d^2T}{d\eta^2}+PrE[A_1+A_2\cosh M\eta+A_3\cosh 2M\eta] = 0 \qquad \ldots \quad \ldots \quad (7)$$

where

$$A_1 = M^2K^2-\frac{2KM}{A}+\frac{M^2}{A^2}$$

$$A_2 = \frac{2M^2K\alpha}{A\cosh M}-\frac{2\alpha M^2}{A^2\cosh M}$$

$$A_3 = \frac{\alpha^2M^2}{A^2\cosh^2 M}.$$

The solution of eqn. (7) subject to the boundary conditions (5) is

$$T = \frac{\eta}{1+2\Gamma}+PrE\left[\frac{A_1}{2}(1-\eta^2)+\frac{A_2}{M^2}(\cosh M-\cosh M\eta)\right.$$
$$+\frac{A_3}{4M^2}(\cosh 2M-\cosh 2M\eta)$$
$$\left.+2\Gamma\left(A_1+\frac{A_2\sinh M}{M}+\frac{A_3\sinh 2M}{2M}\right)\right]. \quad \ldots \quad \ldots \quad (8)$$

The temperature profiles, calculated from eqn. (8), are shown in Figs. 1–3. When the wall temperatures are specified, the wall heat-flux variations

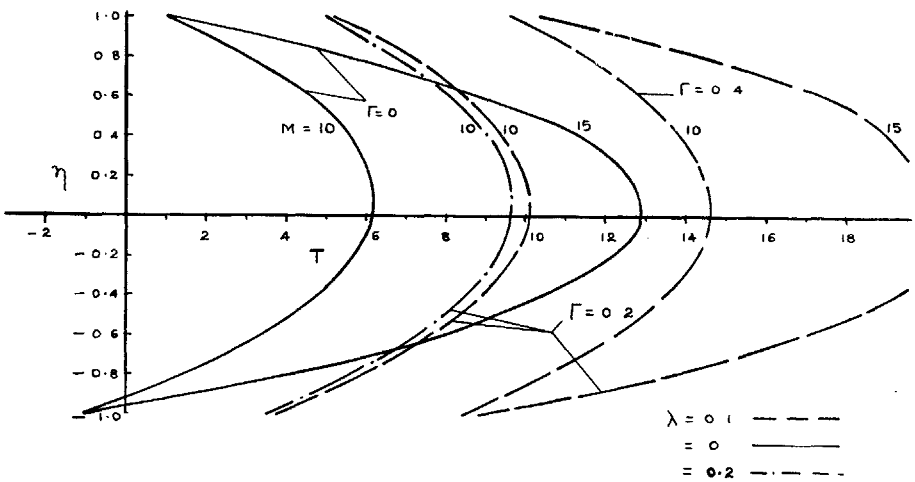

FIG. 1. Temperature profiles ($K = 0$, $\lambda = 0$, 0·1, 0·2; $\Gamma = 0$, 0·2, 0·4).

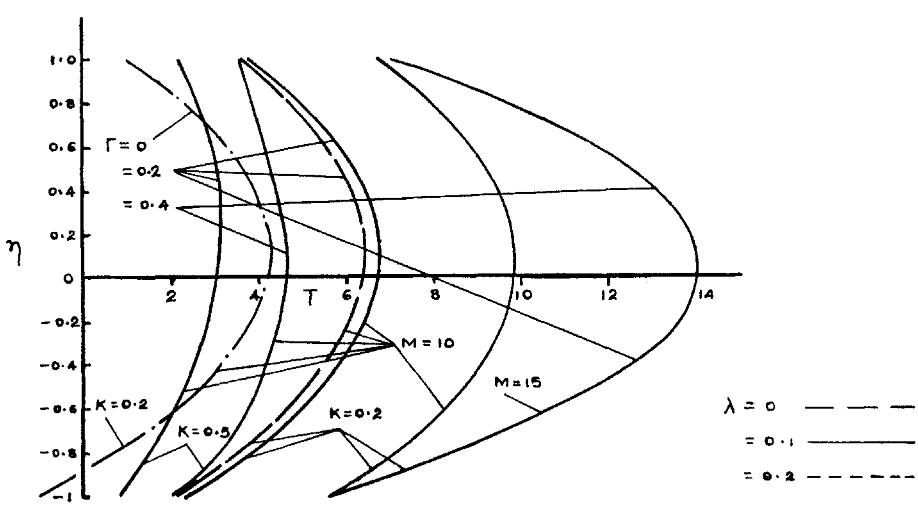

FIG. 2. Temperature profiles ($M = 10$, 15; $K = 0$·2, 0·5).

along the channel length required to maintain the wall temperatures constant are of practical importance. If a sign convention is adopted to give positive heat-transfer rates at both the walls, then the heat-transfer value from the upper wall to the gas is given by

$$q_{w,1} = k\left(\frac{\partial t}{\partial z}\right)_{z=L} = \frac{k}{L}(t_{w,1}-t_{w,m})\left(\frac{\partial T}{\partial \eta}\right)_{\eta=1} \quad \ldots \quad \ldots \quad (9)$$

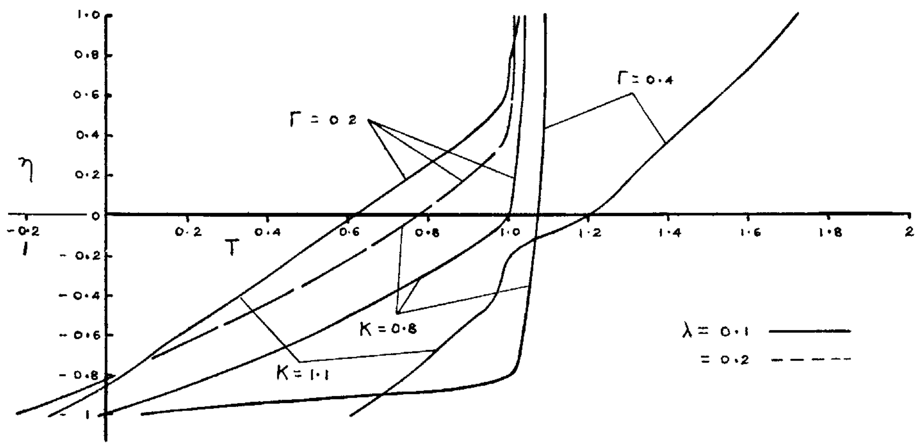

FIG. 3. Temperature profiles ($M = 10$, $K = 0{\cdot}8,\ 1{\cdot}1$; $\lambda = 0{\cdot}1,\ 0{\cdot}2$; $\Gamma = 0{\cdot}2,\ 0{\cdot}4$).

and from the gas to the lower wall by

$$q_{w,2} = \frac{k}{L}(t_{w,1}-t_{w,m})\left(\frac{\partial T}{\partial \eta}\right)_{\eta=-1}. \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

In continuum heat-transfer theory, it is customary to represent the heat-transfer results in terms of a heat-transfer coefficient $h = q_w/(t_w - t_b)$ and a Nusselt number $Nu = hD_T/k$, where D_T is the thermal diameter and defined by

$$D_T = 4\,\frac{\text{cross-sectional area}}{\text{heated perimeter}}\,.$$

For the flat duct, $D_t = 4L$. We extend these to low-density gas flows which is natural and appropriate.
Hence,

$$Nu_1 = \frac{q_{w,1}}{t_{w,1}-t_b}\cdot\frac{4L}{k} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

where t_b, the mean mixed temperature of the gas, is given by

$$t_b = \frac{\int_{-1}^{1} ut\,d\eta}{\int_{-1}^{1} u\,d\eta} = \frac{1}{2}\int_{-1}^{1} ut\,d\eta\,.$$

Hence, the Nusselt number Nu may be expressed alternatively as

$$Nu_1 = \frac{4}{1-T_b}\left(\frac{\partial T}{\partial \eta}\right)_{\eta=1} \quad \ldots \quad (12)$$

where

$$T_b = \frac{t_b - t_{w,m}}{t_{w,1} - t_{w,m}} = \frac{1}{2}\int^{1} uT\, d\eta . \quad \ldots \quad (13)$$

The Nusselt number at the lower wall may be written in the same manner as

$$Nu_2 = \frac{q_{w,2}}{t_b - t_{w,2}}\,\frac{4L}{k}$$

$$= \frac{4}{1+T_b}\left(\frac{dT}{d\eta}\right)_{\eta=-1} \quad \ldots \quad (14)$$

Substituting for T and u from eqns. (8) and (1) respectively in eqn. (13) and evaluating the integral, we obtain the mean mixed temperature as

$$T_b = \frac{1}{2A}\Bigg\{ PrE\Bigg[\frac{2A_1}{3} + \frac{2A_2}{M^3}(M \cosh M - \sinh M)$$

$$+ \frac{A_3}{4M^3}(2M \cosh 2M - \sinh 2M)$$

$$+ 4\Gamma\left(A_1 + \frac{A_2 \sinh M}{M} + \frac{A_3 \sinh 2M}{2M}\right)\Bigg]$$

$$- \frac{PrE\alpha}{\cosh M}\Bigg[\frac{2A_1}{M^3}(M \cosh M - \sinh M) + \frac{A_2}{2M^3}(\sinh 2M - 2M)$$

$$+ \frac{A_3}{12M^3}(6M \cosh 2M \sinh M - 3 \sinh M - \sinh 3M)$$

$$+ \frac{4\Gamma \sinh M}{M}\left(A_1 + \frac{A_2 \sinh M}{M} + \frac{A_3 \sinh 2M}{2M}\right)\Bigg] + \frac{2\alpha}{M^2(1+2\Gamma)}\Bigg\}. \quad \ldots \quad (15)$$

Again, substituting for T from (8) in (12) and (14), one can obtain

$$Nu_1 = \frac{4}{1-T_b}\left[\frac{1}{1+2\Gamma} - PrE\left\{A_1 + \frac{A_2 \sinh M}{M} + \frac{A_3}{2M}\sinh 2M\right\}\right] \quad \ldots \quad (16)$$

and

$$Nu_2 = \frac{4}{1+T_b}\left[\frac{1}{1+2\Gamma} + PrE\left\{A_1 + \frac{A_2 \sinh M}{M} + \frac{A_3}{2M}\sinh 2M\right\}\right] \ldots \quad (17)$$

The numerical values of T_b, Nu_2 are entered in Table I.

3. Conclusions

The numerical values of T, Nu_2 and T_b are calculated for $M = 10, 15$; $k = 0, 0{\cdot}2, 0{\cdot}5, 0{\cdot}8$ and $1{\cdot}1$; $\lambda = 0, 0{\cdot}1, 0{\cdot}2$ and $\Gamma = 0, 0{\cdot}2, 0{\cdot}4$ and $PrE = 0{\cdot}1$.

TABLE I

Values of T_b, Nu_2

M	K	λ	G	T_b	Nu_2
10	0	0	0	5·1448	9·4910
		0·1	0·2	4·2191	9·2518
			0·4	4·4457	8·7502
		0·2	0·2	4·0183	9·1940
			0·4	4·3114	8·5672
	0·2	0·0	0	3·7293	9·2869
		0·1	0·2	2·9235	8·6367
			0·4	3·1541	8·0044
		0·2	0·2	2·7057	8·5649
			0·4	2·9507	7·8728
	0·5	0	0	2·1466	9·0049
		0·1	0·2	1·5309	7·2251
			0·4	1·7991	6·3061
		0·2	0·2	1·2906	7·0456
			0·4	1·5051	6·1887
	0·8	0	0	1·2125	9·0036
		0·1	0·2	0·7992	5·4946
			0·4	1·1428	4·3173
		0·2	0·2	0·5403	5·0241
			0·4	0·7740	4·0449
	1·1	0	0	0·9271	9·7143
		0·1	0·2	0·7285	5·2525
			0·4	1·1855	3·6840
		0·2	0·2	0·4548	4·4947
			0·4	0·7572	3·3598

Mean mixed temperature—In normal density gas flows ($\lambda = 0 = \Gamma$), an increase in k leads to a decrease in the mean mixed temperature. It also decreases with increasing λ (the rarefaction parameter), but decreases with increasing Γ (the temperature jump coefficient).

Nusselt number—(a) Nu_2 in short-circuited case is greater than in open-circuited case for $K < 1$. (b) In open-circuit case ($0 < K < 1$), an increase in K leads to a decrease in Nusselt number for all λ and Γ. (c) For the same Γ, an increase in λ leads to a decrease in Nu_2. (d) Nu_2 also decreases as Γ increases.

REFERENCES

Inman, R. M. (1965*a*). Laminar slip-flow of an electrically conducting incompressible rarefied gas in a channel with a transverse magnetic field. *Appl. Sci. Res*, **11** B, 391–400.

———— (1965*b*). Heat transfer in thermal entrance region with laminar slip-flow between parallel plates at unequal temperatures. *NASA TN D*—2980.

Soundalgekar, V. M. (1969). On heat transfer in crossed-fields MHD channel flow between conducting walls. *Proc. natn. Inst. Sci. India*, A **35**, 329.