HALL EFFECTS ON HEAT TRANSFER IN MHD CHANNEL FLOW UNDER CROSSED-FIELDS

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An investigation of the combined influence of tensor conductivity, magnetic field and electric field on heat transfer in a channel with segmented electrodes has been carried out. A solution for temperature and Nusselt number has been obtained in a closed form based upon an incompressible, two-dimensional flow with constant properties. Calculations presented here show the influence of $\omega_0\tau$ (product of electron cyclotron frequency and electron mean free time), M (Hartmann number), K (loading parameter) and $PrReS$ (product of Prandtl number, Reynolds number and a non-dimensional number) on the temperature field. It is found that an increase in $\omega_0\tau$ leads to an increase in fluid temperature and the Nusselt number, but an increase in $PrReS$ leads to a decrease in fluid temperature. In the case of the channel when working as a MHD generator, an increase in K leads to a decrease in the fluid temperature θ. Also Nusselt number increases with $\omega_0\tau$ as well as K.

Nomenclature

A = temperature gradient

a, b, c_1, c_2, c_3, c_4 = as defined in (2a)

$\bar{B}$ = magnetic induction vector $(B_{x_1}, B_{y_1}, B_{z_1})$

B_0 = applied magnetic field

c_p = fluid specific heat at constant pressure

E_x, E_y = electric field components

e_x = non-dimensional electric field component $(E_x/\bar{u}B_0)$

e = charge on electron

E = Eckert number $(\bar{u}/C_p\theta_1)$

$D_1, D_2 \ldots D_{21}$ = as defined in Appendix I

$\bar{J}$ = current density vector $(j_{x_1}, j_{y_1}, j_{z_1})$

J_x, J_y = non-dimensional current density $(j_{x_1}, j_{y_1}/\sigma_0\bar{u}B_0)$

K = loading parameter $(E_y/\bar{u}B_0)$

L = half width of the channel

M = Hartmann number $\left[B_0L\left(\frac{\sigma_0}{\mu}\right)^{\frac{1}{2}}\right]$

m_e = mass density of particles

Nu = Nusselt number

n_e = electron number density

P_x, P_y = pressure gradient components (P_{x_1}, $P_{y_1}/\rho\bar{u}^2$)
Pr = Prandtl number ($\mu c_p/\lambda$)
Re = Reynolds number ($\rho\bar{u}L/\mu$)
S = non-dimensional number (AL/θ_1)
T = fluid temperature
u, v = fluid velocity components
U, V = non-dimensional velocity components (u, $v/\bar{u}$)
$\bar{u}$ = mean fluid velocity
x, y, z = non-dimensional coordinates (x_1, y_1, z_1/L)
ρ = fluid density
λ = thermal diffusivity
σ_0 = electrical conductivity
μ = fluid viscosity
Φ = viscous dissipation function as defined in (11)
$\bar{\theta}$ = temperature as defined in (12)
θ_1 = reference temperature
ω_0 = electron cyclotron frequency (eB_0/m_e)
τ = electron mean free time ($\sigma_0 m_e/n_e e^2$)

1. Introduction

In recent years, MHD channel flows and their applications to propulsion systems and electric power generation are receiving a considerable amount of scientific attention. In the initial stages, the channel flows were described by assuming Ohm's law without Hall effects. But in the case of very strong magnetic fields, where the electron cyclotron frequency is much greater than the electron collision frequency, Hall effects become significant. Hence, the generalized Ohm's law must be used in solving problems on channel flows.

Recently, attempts have been made to describe the channel flows without neglecting Hall effects. Harris and Cobine (1961) described the advantages and disadvantages of three types of channel configurations depending upon the arrangement of the electrodes. One of the arrangements suggested is the segmentation of electrodes which prevents the flow of axial currents. Sherman and Sutton (1961) described the flow of conducting fluids moving in a channel with the segmented electrodes. In their treatment of the problem, the generalized Ohm's law, without ion-slip and pressure-diffusion effects, was assumed and the problem was solved with reference to the effects of Hall current on the velocity field, magnetic field, etc. But the fluid in such channels is always working at a very high temperature, and the effects of Hall current, magnetic field and electric field on the temperature field have not received attention as yet.

Hence the object of this paper is to describe the combined effects of Hall currents, magnetic field, electric field on the heat transfer in the channel flow of viscous, incompressible, electrically conducting fluids. The expressions for velocity and current density, as given by Sherman and Sutton (1961), are used to derive the expression for temperature distribution within the fluid and the Nusselt number under the assumption that the temperature along the walls varies linearly in the direction of flow.

2. Temperature Distribution

In this paper, the following assumptions are made:

1. The plates of a rectangular channel are extending to infinity in the x- and y-directions. The opposite faces of the channel are electrical insulators while the other two at $y_1 = \pm\infty$ are electrodes which are segmented infinitely in order to allow an axial electric field to develop so that no net current will flow upstream or downstream.

2. The physical properties of fluid are assumed to be constant.

3. As shown in Fig. 1, the fluid flows from left to right in the x_1-direction between two plates $z_1 = L$ and $z_1 = -L$ with modified Hartmann's velocity profile (Sherman and Sutton 1961) under the influence of a uniform magnetic field imposed in the z_1-direction.

Under these assumptions, the expressions for U, V, J_x, J_y as given by Sherman and Sutton (1961) are as follows:

$$U = -c_1 \sinh az \sin bz + c_2 \cosh az \cos bz + c_3 \qquad (1)$$

$$V = c_1 \cosh az \cos bz + c_2 \sinh az \sin bz + c_4 \qquad (2)$$

where

$$\left.\begin{aligned}
c_1 &= \frac{c_3 \sinh a \sin b - c_4 \cosh a \cos b}{\sinh^2 a + \cos^2 b} \\
c_2 &= \frac{-c_4 \sinh a \sin b - c_3 \cosh a \cos b}{\sinh^2 a + \cos^2 b} \\
c_3 &= -\omega_0\tau\left[\frac{ReP_y}{M^2}\right] - \left[\frac{ReP_x}{M^2}\right] + K \\
c_4 &= -\left[\frac{ReP_y}{M^2}\right] + \omega_0\tau\left[\frac{ReP_x}{M^2}\right] - e_x \\
a &= \left[\frac{M^2}{2(1+\omega_0^2\tau^2)}\left\{(1+\omega_0^2\tau^2)^{\frac{1}{2}}+1\right\}\right]^{\frac{1}{2}} \\
b &= \left[\frac{M^2}{2(1+\omega_0^2\tau^2)}\left\{(1+\omega_0^2\tau^2)^{\frac{1}{2}}-1\right\}\right]^{\frac{1}{2}}
\end{aligned}\right\} \qquad (2a)$$

$$\frac{ReP_y}{M^2} = P_y^* = \frac{B_1B_3 + B_2B_4}{B_1^2 + B_2^2} \qquad (3)$$

$$\frac{ReP_x}{M^2} = P_x^* = \frac{B_2B_3 - B_1B_4}{B_1^2 + B_2^2} \qquad (4)$$

where

$$\left.\begin{aligned}
B_1 &= \frac{(b+a\omega_0\tau)\sin b\cos b+(a-b\omega_0\tau)\sinh a\cosh a}{M^2(1+\omega_0^2\tau^2)^{-\frac{1}{2}}[\sinh^2 a+\cos^2 b]}-1\\
B_2 &= \frac{(a-b\omega_0\tau)\sin b\cos b-(b+a\omega_0\tau)\sinh a\cosh a}{M^2(1+\omega_0^2\tau^2)^{-\frac{1}{2}}[\sinh^2 a+\cos^2 b]}+\omega_0\tau\\
B_3 &= KB_2+\omega_0\tau\left[\frac{a\cosh a\sinh a+b\sin b\cos b}{M^2(1+\omega_0^2\tau^2)^{-\frac{1}{2}}(\sinh^2 a+\cos^2 b)}-1\right]\\
B_4 &= -KB_1+\omega_0\tau\left[\frac{a\sin b\cos b-b\sinh a\cosh a}{M^2(1+\omega_0^2\tau^2)^{-\frac{1}{2}}(\sinh^2 a+\cos^2 b)}-1\right]\\
e_x &= -\omega_0\tau(1-K)
\end{aligned}\right\} \quad \cdots \quad (5)$$

and

$$J_x = \frac{e_x+V-\omega_0\tau(K-U)}{1+\omega_0^2\tau^2} \quad \cdots \quad (6)$$

$$J_y = \frac{K-U+\omega_0\tau(e_x+V)}{1+\omega_0^2\tau^2} \quad \cdots \quad (7)$$

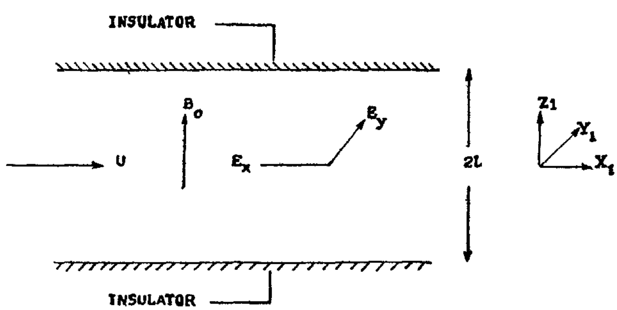

FIG. 1. Channel configuration.

and the non-dimensional parameters are defined as follows:

$$\left.\begin{aligned}
U &= u/\bar{u}, \quad V = v/\bar{u}, \quad z = z_1/L\\
x &= x_1/L, \quad y = y_1/L, \quad e_x = \frac{E_x}{\bar{u}B_0}\\
K &= \frac{E_y}{\bar{u}B_0}, \quad P = \frac{P}{\rho\bar{u}^2}, \quad \bar{u} = \frac{1}{2L}\int_{-L}^{L} u\,dz_1
\end{aligned}\right\} \quad \cdots \quad (8)$$

$$M^2 = \frac{\sigma_0 B_0^2 L^2}{\mu}, \quad Re = \frac{\rho\bar{u}L}{\mu},$$

$$J_x = \frac{j_{x_1}}{\sigma_0\bar{u}B_0}, \quad J_y = \frac{j_{y_1}}{\sigma_0\bar{u}B_0}.$$

In order to derive e_x and P^*_x, P^*_y, it is assumed that

$$\int_{-L}^{L} j_{x_1}\, dz_1 = 0, \quad \int_{-L}^{L} v\, dz_1 = 0. \qquad (9)$$

The channel configuration depends upon the value of K, called the loading parameter. It is as follows:

(1) $K = 0$ (short-circuited)
(2) $K < 1{\cdot}0$ (MHD generator)
(3) $K = 1{\cdot}0$ (MHD flowmeter)
(4) $K > 1{\cdot}0$ (MHD accelerator)

2.1. *Linear Variation of Temperature along the Plates*

The energy equation is given by

$$\rho c_p u \frac{\partial T}{\partial x_1} = \lambda \frac{\partial^2 T}{\partial z_1^2} + \frac{\bar{J}^2}{\sigma_0} + \mu \Phi \qquad (10)$$

where

$$\bar{J}^2 = j_{x_1}^2 + j_{y_1}^2 \quad \text{and} \quad \Phi = \left(\frac{\partial u}{\partial z_1}\right)^2 + \left(\frac{\partial v}{\partial z_1}\right)^2. \qquad (11)$$

For linear variation of temperature along the plates, we assume for temperature

$$T = Ax_1 + \bar{\theta}(z_1). \qquad (12)$$

Substituting (11) and (12) in (10), we get

$$\rho c_p u A = \lambda \frac{d^2\bar{\theta}}{dz_1^2} + \frac{1}{\sigma_0}\left(j_{x_1}^2 + j_{y_1}^2\right) + \mu\left[\left(\frac{du}{dz_1}\right)^2 + \left(\frac{\partial v}{dz_1}\right)^2\right]. \qquad (13)$$

Introducing the following non-dimensional quantities in eqn. (12):

$$Pr = \frac{\mu c_p}{\lambda} \text{ (Prandtl number)}$$

$$E = \frac{\bar{u}^2}{c_p \theta_1} \text{ (Eckert number)}$$

$$S = \frac{AL}{\theta_1} \text{ (a non-dimensional number)}$$

$$\theta = \frac{\bar{\theta}}{\theta_1} \text{ (non-dimensional temperature)}$$

in addition to those in (8) we have, on rearrangement,

$$\frac{d^2\theta}{dz^2} + PrE\left[M^2(J_x^2 + J_y^2) + \left(\frac{dU}{dz}\right)^2 + \left(\frac{dV}{dz}\right)^2\right] - PrRSU = 0 \qquad (14)$$

with the boundary conditions consistent with eqn. (12) being

$$\theta = 0 \quad \text{at } z = \pm 1 \qquad (15)$$

The choice $\theta = 0$ rather than $\theta =$ constant is purely for convenience. Physically, it means that the temperature of the fluid and the plates is the same.

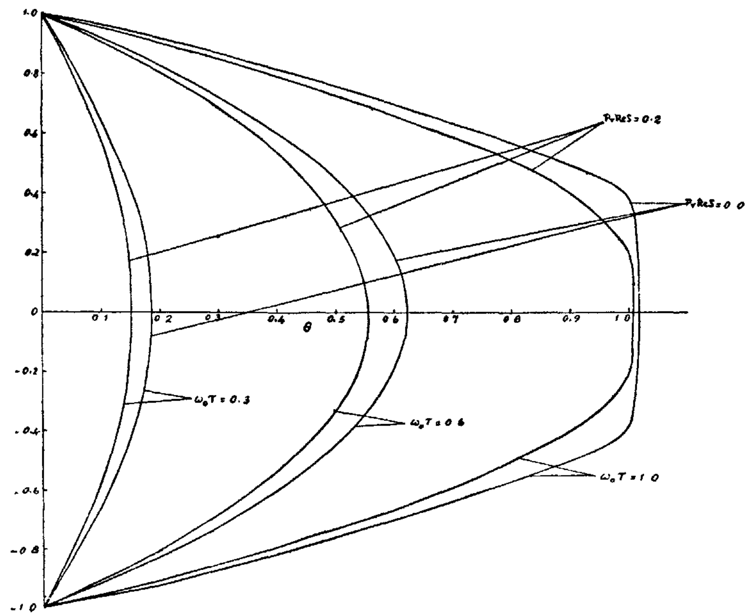

FIG. 2. Temperature profiles ($M = 10$; $K = 0$; $\omega\tau = 0{\cdot}3, 0{\cdot}6, 1$; $PrReS = 0, 0{\cdot}2$).

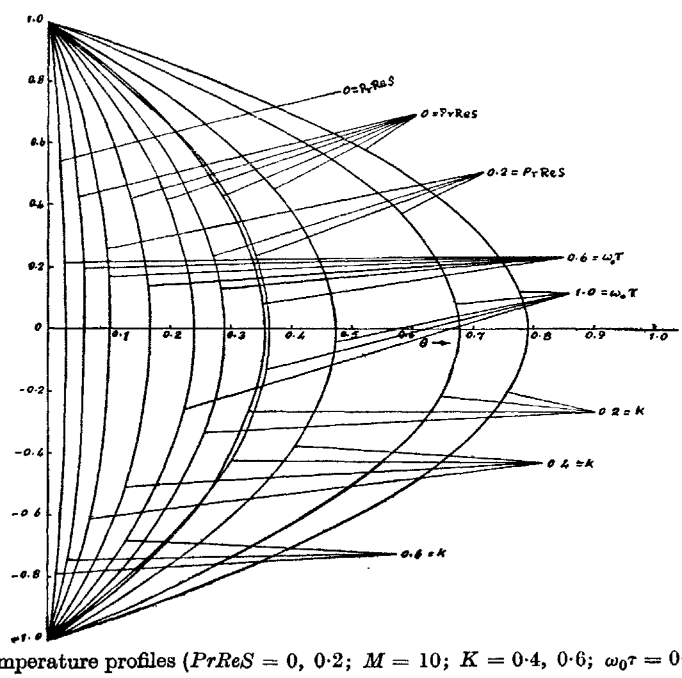

FIG. 3. Temperature profiles ($PrReS = 0, 0{\cdot}2$; $M = 10$; $K = 0{\cdot}4, 0{\cdot}6$; $\omega_0\tau = 0{\cdot}6, 1{\cdot}0$).

Substituting for U, V, J_x, J_y from eqns. (1), (2), (6) and (7) respectively in eqns. (14), simplifying and rearranging, we get

$$\frac{d^2\theta}{dz^2}+D_{14}\cos 2az+D_{15}\cos 2bz+D_{10}\sinh az\sin bz$$
$$+D_{11}\cosh az\cos bz+D_{12}=0 \quad \ldots \quad (16)$$

where D_1, $D_2 \ldots D_{15}$ are as defined in Appendix I.

The solution of eqn. (16), subject to the boundary condition (15), is given by

$$\theta = D_{18}(\cosh 2a-\cosh 2az)-D_{19}(\cos 2b-\cos 2bz)$$
$$+D_{20}(\sinh a\sin b-\sinh az\sin bz)$$
$$+D_{21}(\cosh a\cos b-\cosh az\cos bz)+\frac{D_{12}}{2}(1-z^2) \quad \ldots \quad (17)$$

where D_{16} $D_{17} \ldots D_{21}$ are as defined in Appendix I.

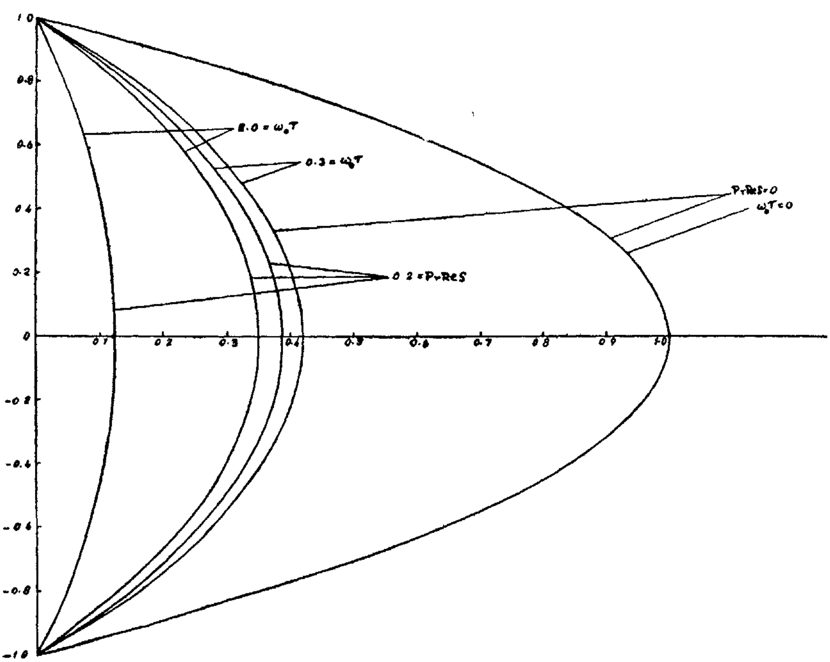

FIG. 4. Temperature profiles ($M = 10$; $K = 1{\cdot}0$; $PrReS = 0$, $0{\cdot}2$; $\omega_0\tau = 0{\cdot}3$, $0{\cdot}6$, 1).

Now, in the case of ionized gases, the Prandtl number Pr is $O(1)$ and, for the validity of incompressibility, the Eckert number must be $\ll 1$. Hence the value of PrE is chosen as 0·02. With this value of PrE, the temperature profiles are shown graphically in Figs. 2–5 for $M = 10$, 15, $PrRS = 0$, 0·1, 0·2 and $K = 0$, 0·2, 0·4, 0·6, 1, 2, 3.

In the technological fields, the rate of heat transfer is expressed in terms of Nusselt number, which is defined as

$$Nu = -\frac{L}{\theta(0)}\left(\frac{dT}{dz_1}\right)_{z_1 = L}$$

$$= -\frac{1}{\theta(0)}\left(\frac{d\theta}{dz}\right)_{z = 1}. \qquad \ldots \quad \ldots \quad \ldots \quad (18)$$

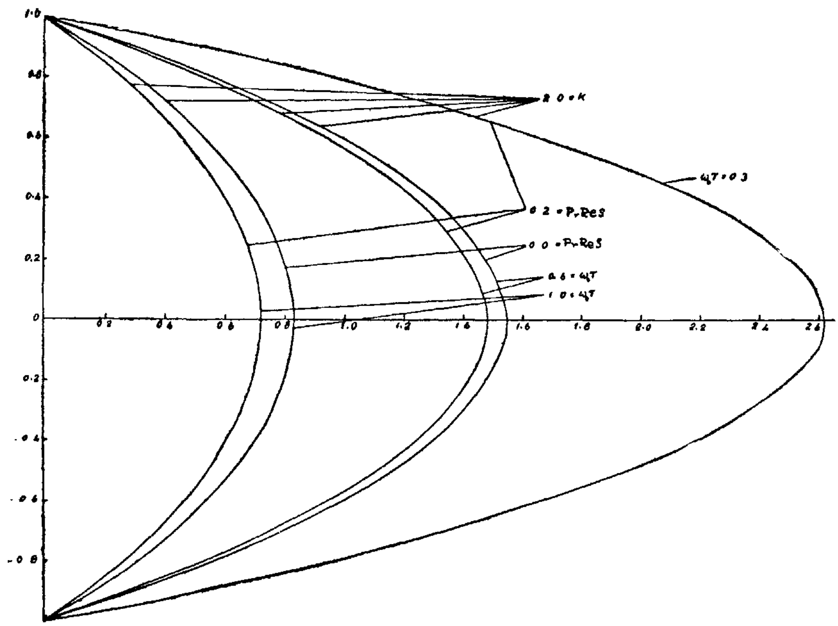

FIG. 5. Temperature profiles ($M = 10$; $K = 2$; $\omega_0\tau = 0{\cdot}3,\ 0{\cdot}6,\ 1{\cdot}0$; $PrReS = 0,\ 0{\cdot}2$).

Hence from (17) and (18), we get

$$Nu = -\frac{[(D_{21}b - D_{20}a)\cosh a \sin b - (D_{21}a + D_{20}b)\,.\sinh a \times \cos b - 2aD_{18}\sinh 2a - 2b\,.\,D_{19}\sin 2b - D_{12}]}{\left[D_{18}(\cosh 2a - 1) - D_{19}(\cos 2b - 1) + D_{20}\sinh a \sin b + D_{21}(\cosh a \cos b - 1) + \frac{D_{12}}{2}\right]}. \qquad \ldots \quad (19)$$

The Nusselt number Nu is plotted against $\omega_0\tau$ in Fig. 6.

3. Conclusions

Case I : $K = 0$ (short-circuited)

(1) From Fig. 1, it can be seen that an increase in the value of the Hall parameter $\omega_0\tau$ leads to an increase in the fluid temperature.

(2) Also, for the same value of $\omega_0\tau$, an increase in *PrRS* leads to a decrease in the fluid temperature.

Case II: $0 < K < 1$ (MHD generator)

(1) The influence of the parameters *PrReS* and $\omega_0\tau$ on the fluid temperature is the same as in case I.

(2) An increase in K leads to a decrease in the fluid temperature when M, *PrReS* and $\omega_0\tau$ are constant.

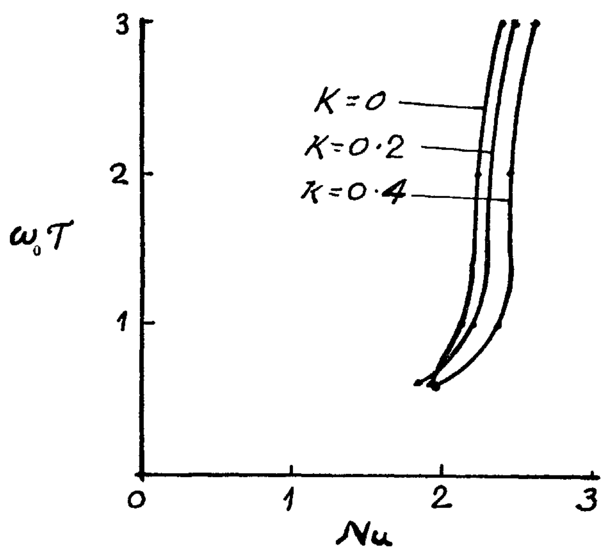

FIG. 6. Values of Nusselt number ($PrReS = 0{\cdot}1$, $M = 15$).

Case III: $K \geqslant 1$ (MHD accelerator, flowmeter)

It is the same as in case I. Here Nu increases with $\omega_0\tau$ as well as K.

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Appendix I

$$D_1 = c_1a + c_2b, \quad D_2 = c_2a - c_1b, \quad D_3 = D_1^2 + D_2^2$$

$$D_4 = \frac{PrEM^2}{1+\omega_0^2\tau^2}, \quad D_5 = c_1^2 + c_2^2, \quad D_6 = 2e_xc_1 - 2Kc_1 + 2c_4c_2 - 2c_1c_3$$

$$D_7 = 2e_xc_1 - 2Kc_2 + 2c_4c_1 + 2c_2c_3, \quad D_8 = e_x^2 + K^2 + 2e_xc_4 - 2Kc_3 + c_3^2 + c_4^2, \quad D_9 = D_4D_5$$

$$D_{10} = D_4D_6 + PrReSc_1, \quad D_{11} = D_4D_7 - PrReSc_2, \quad D_{12} = D_4D_8 - PrReSc_3$$

$$D_{13} = PrED_3, \quad D_{14} = \frac{D_9 + D_{13}}{2}, \quad D_{15} = \frac{D_9 - D_{13}}{2}$$

$$D_{16} = \frac{aD_{10} + bD_{11}}{a^2+b^2}, \quad D_{17} = \frac{aD_{11} - bD_{10}}{a^2+b^2}, \quad D_{18} = D_{14}/4a^2$$

$$D_{19} = D_{15}/4b^2, \quad D_{20} = \frac{aD_{16} + bD_{17}}{a^2+b^2}, \quad D_{21} = \frac{aD_{17} - bD_{16}}{a^2+b^2}$$