MAGNETOHYDRODYNAMIC FLOW BETWEEN TWO PARALLEL PLATES, ONE IN UNIFORM MOTION AND THE OTHER AT REST WITH UNIFORM SUCTION AT THE STATIONARY PLATE

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Hydromagnetic Couette type flow with small uniform suction at the stationary plate has been considered in the present paper. The basic equations have been solved by reducing them to second and third order non-linear differential equations under appropriate boundary conditions. The longitudinal and transverse velocity profiles for $\lambda = 0{\cdot}1$, $R = 100$ and different values of M are drawn. It is noted that at a distance, depending on the Hartmann number, an adverse pressure gradient develops which causes a back flow. The pressure increase in the main flow direction decreases with the increase of Hartmann number. The coefficient of skin friction decreases with the increase of suction parameter or with the increase of Hartmann number.

INTRODUCTION

Hartmann (1937) has done pioneer work in the study of steady magnetohydrodynamic channel flow of a conducting fluid under a uniform magnetic field transverse to an electrically insulated channel wall. Cramer (1959) and Lilley (1959) have studied the problem of porous wall Couette type flow with superimposed axial pressure gradient. Mehta (1963) has studied the Couette type flow of an incompressible, viscous and infinitely electrically conducting fluid between two equally porous parallel planes under the assumption that the rate of suction at one wall is equal to the rate of injection of the fluid at the other wall.

Verma and Bansal (1966) have studied the Couette flow between two plates with small suction at the stationary plate. They have reported that due to suction at the stationary plate an adverse pressure gradient is developed which causes a back flow at large distance from the mouth of the channel. In this paper we have discussed the shear flow of a conducting fluid under a uniform magnetic field transverse to electrically insulated parallel plates with small uniform suction at the stationary plate. The problem is treated for small values of suction parameter, neglecting the induced magnetic field. It is seen that due to suction at the stationary plate an adverse pressure gradient is developed which causes a back flow at a distance from the mouth of the channel which will depend on the Hartmann number. The longitudinal and transverse velocity profiles, the axial pressure increase and the variation of coefficient of friction at the stationary plate are shown graphically.

1. Basic Equations

The equation of motion for laminar flow of an incompressible, electrically conducting fluid in the presence of a transverse magnetic field is, in the usual notation,

$$\rho \frac{dV}{dt} = -\text{grad } p + \mu \nabla^2 V + (J \times B) \quad \ldots \quad \ldots \quad (1.1a)$$

where J and B are given by Maxwell's equations and Ohm's law, namely

$$\left.\begin{aligned} \text{curl } H = 4\pi J, \quad \text{div } B = 0 \\ \text{curl } E = 0, \quad \text{div } E = 0 \\ J = \sigma[E + V \times B] \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (1.1b)$$

and

The equation of continuity is

$$\text{div } V = 0. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1.1c)$$

When a conducting fluid moves through the magnetic lines of force, B_0, the positive and negative charges are each accelerated in such a way that their average motion gives rise to an electric current $j = \sigma V \times B$, where $B = B_0 + b$. The quantity b is the magnetic induction resulting from the electric current j in the fluid. In this analysis b will be considered as a perturbation on the basic field strength and negligible in comparison with B_0, i.e. $B_0 \gg b$.

The fluid is assumed to be ionized and thereby an electrical conductor. However, within any small but finite volume the number of particles with positive and negative charges are nearly equal. The total excess charge density (θ) and imposed electric field intensity (E) are assumed to be zero (Rossow 1958).

Let us consider the shear flow of a conducting fluid between two parallel plates. The upper plate is moving with uniform velocity U and the suction at the stationary plate is uniform and small. Let y_0 be the distance between the two plates, u and v be the components of the velocity in the x and y directions which are along the stationary plate and perpendicular to it respectively. A constant magnetic field of strength H_0 is applied perpendicular to the plates and fixed relative to them. The following assumptions are made:

(a) Electrical conductivity σ_e of the fluid is sufficiently large so that the displacement current is neglected,

(b) no external electric field is applied,

(c) the secondary effects of magnetic induction are neglected.

With these assumptions, (1.1a)–(1.1c) are reduced simply to

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma_e B_0^2 u, \quad \ldots \quad (1.1)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \quad \ldots \quad (1.2)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \quad \ldots \quad (1.3)$$

where p is the pressure, ρ the density of the fluid, μ the coefficient of viscosity, B_0 ($=\mu_e H_0$) the electromagnetic induction, μ_e the magnetic permeability.

Since there is uniform suction, $\frac{\partial v}{\partial x} = 0$, therefore v is a function of y only. From (1.3), we have

$$\frac{\partial^2 u}{\partial x^2} = 0. \quad \ldots \quad (1.4)$$

From (1.1)–(1.3), we have

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}-\sigma_e B_0^2 u, \quad \ldots \quad (1.5)$$

$$\rho v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y}+\mu\frac{\partial^2 v}{\partial y^2} \quad \ldots \quad (1.6)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \quad \ldots \quad (1.7)$$

with the boundary conditions

$$\left.\begin{aligned} &u = 0,\ v = -v_0 \quad \text{at } y = 0 \\ \text{and} \quad &u = U,\ v = 0 \quad \text{at } y = y_0 \end{aligned}\right\} \quad \ldots \quad (1.8)$$

where v_0 is the velocity of suction.

Let us introduce the following dimensionless quantities:

$$\bar{x} = \frac{x}{y_0}, \quad \bar{y} = \frac{y}{y_0}, \quad \bar{u} = \frac{u}{U}, \quad \bar{v} = \frac{v}{-v_0}, \quad \bar{p} = \frac{p}{\rho U^2}.$$

M (Hartmann number) $= \mu_e H_0 y_0\left(\frac{\sigma_e}{\rho\nu}\right)^{\frac{1}{2}}$, R (Reynolds number) $= \frac{U y_0}{\nu}$,

λ (suction parameter) $= \frac{v_0 y_0}{\nu}$, where ν is the kinematic viscosity.

From (1.5)–(1.7), we have

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}-\frac{\lambda}{R}\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{1}{R}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}-\frac{M^2}{R}\bar{u}, \quad \ldots \quad (1.9)$$

$$\frac{\lambda^2}{R^2}\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{\partial \bar{p}}{\partial \bar{y}}-\frac{\lambda}{R^2}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \quad \ldots \quad (1.10)$$

and

$$\frac{\partial \bar{u}}{\partial \bar{x}} - \frac{\lambda}{R}\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad \ldots \quad (1.11)$$

with the boundary conditions

$$\left.\begin{aligned} \bar{u} = 0,\ \bar{v} = 1 \quad \text{at } \bar{y} = 0 \\ \text{and} \quad \bar{u} = 1,\ \bar{v} = 0 \quad \text{at } \bar{y} = 1 \end{aligned}\right\}. \quad \ldots \quad (1.12)$$

Let

$$\left.\begin{aligned} \bar{p}(\bar{x}, \bar{y}) &= p_0 + p_1(\bar{x}, \bar{y}) \\ \bar{u}(\bar{x}, \bar{y}) &= u_0 + u_1(\bar{x}, \bar{y}) \\ \bar{v}(\bar{y}) &= v_1(\bar{y}) \end{aligned}\right\} \quad \ldots \quad (1.13)$$

where p_1, u_1, v_1 are the perturbations caused by the suction and p_0, u_0 are the known quantities for the flow when there is no suction (i.e. plane Couette flow under transverse magnetic field) satisfying the equations

$$\frac{\partial p_0}{\partial \bar{x}} = 0,\ \frac{\partial p_0}{\partial \bar{y}} = 0;\ \frac{\partial u_0}{\partial \bar{x}} = 0 \text{ and } \frac{\partial^2 u_0}{\partial \bar{y}^2} - M^2 u_0 = 0.$$

Therefore, we have

$$\left.\begin{aligned} p_0 &= \text{constant} \\ \text{and} \quad u_0 &= \frac{\sinh M\bar{y}}{\sinh M} \end{aligned}\right\}. \quad \ldots \quad (1.14)$$

From (1.9)–(1.14), on dropping the bars, we have

$$\left(\frac{\sinh My}{\sinh M} + u_1\right)\frac{\partial u_1}{\partial x} - \frac{\lambda}{R}v_1\left(\frac{M\cosh My}{\sinh M} + \frac{\partial u_1}{\partial y}\right) = -\frac{\partial p_1}{\partial x} + \frac{1}{R}\frac{\partial^2 u_1}{\partial y^2} - \frac{M^2}{R}u_1, \quad (1.15)$$

$$\frac{\lambda^2}{R^2}v_1\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} - \frac{\lambda}{R^2}\frac{\partial^2 v_1}{\partial y^2} \quad \ldots \quad (1.16)$$

and

$$\frac{\partial u_1}{\partial x} - \frac{\lambda}{R}\frac{\partial v_1}{\partial y} = 0 \quad \ldots \quad (1.17)$$

with the boundary conditions

$$\left.\begin{aligned} u_1 = 0,\ v_1 = 1 \quad \text{at } y = 0 \\ \text{and} \quad u_1 = 0,\ v_1 = 0 \quad \text{at } y = 1 \end{aligned}\right\}. \quad \ldots \quad (1.18)$$

2. Method of Solution

Let

$$v_1 = f(y) \quad \ldots \quad (2.1)$$

therefore, from eqn. (1.17), we have

$$u_1 = \frac{\lambda}{R}xf'(y) + F(y) \quad \ldots \quad (2.2)$$

where $f(y)$ and $F(y)$ are to be determined.

Putting the values of u_1 and v_1 in (1.15) and (1.16), we get

$$\left[\frac{\sinh My}{\sinh M}+\frac{\lambda}{R}xf'+F\right]\frac{\lambda}{R}f'-\frac{\lambda}{R}f\left[\frac{M\cosh My}{\sinh M}+\frac{\lambda}{R}xf''+F'\right]$$

$$=-\frac{\partial p_1}{\partial x}+\frac{1}{R}\left[\frac{\lambda}{R}xf'''+F''\right]-\frac{M^2}{R}\left[\frac{\lambda}{R}xf'+F\right] \quad .. \quad (2.3)$$

and

$$\frac{\lambda^2}{R^2}ff'=-\frac{\partial p_1}{\partial y}-\frac{\lambda}{R^2}f''. \quad .. \quad (2.4)$$

In the absence of external pressure gradient, from (2.3) we have

$$F''-M^2F+\lambda\left[f\frac{M\cosh My}{\sinh M}+F'f-f'\frac{\sinh My}{\sinh M}-Ff'\right]=0. \quad .. \quad (2.5)$$

Differentiating (2.4) with respect to x, we have

$$\frac{\partial^2 p_1}{\partial x\,\partial y}=0. \quad .. \quad (2.6)$$

Now differentiating (2.3) with respect to y and using (2.5) and (2.6), we get

$$\frac{d}{dy}[f'''-M^2f'+\lambda\{ff''-f'^2\}]=0 \quad .. \quad (2.7)$$

or

$$f'''-M^2f'+\lambda\{ff''-f'^2\}=C \quad .. \quad (2.8)$$

where C is the constant of integration to be determined. Now we are to solve eqns. (2.5) and (2.8) to get f and F with the following boundary conditions:

and

$$\left.\begin{array}{l} f(0)=1,\quad f'(0)=0,\quad F(0)=0 \\ f(1)=0,\quad f'(1)=0,\quad F(1)=0 \end{array}\right\}. \quad .. \quad (2.9)$$

3. Solution for Small λ

The solution for eqn. (2.8) can be expressed for small values of λ by a power series developed near $\lambda=0$ as follows:

$$f=f_0+\lambda f_1+\ldots+\lambda^n f_n \quad .. \quad (3.1)$$

and

$$C=C_0+\lambda C_1+\ldots+\lambda^n C_n \quad .. \quad (3.2)$$

where f_n's and C_n's are taken independent of λ.

By substituting eqns. (3.1) and (3.2) in (2.8) and equating the coefficients of like powers of λ, we have

$$f_0'''-M^2f_0'=C_0 \quad .. \quad (3.3)$$

and

$$f_1'''-M^2f_1'+f_0f_0''-f_0'^2=C_1. \quad .. \quad (3.4)$$

The boundary conditions are

$$\left.\begin{aligned} &f_0(0) = 1,\ f_n(0) = 0 &&\text{for } n \geqslant 1 \\ &f_n(1) = 0 &&\text{for } n \geqslant 0 \\ &f_n'(0) = 0 \text{ and } f_n'(1) = 0 &&\text{for } n \geqslant 0 \end{aligned}\right\}. \qquad \ldots \quad \ldots \quad (3.5)$$

From (3.3) and (3.5), we have

$$f_0 = K_1 + K_2 y + K_3 \cosh My + K_4 \sinh My \qquad \ldots \quad \ldots \quad (3.6)$$

where

$$K_1 = \frac{C_0 M - M^3 \cosh M - C_0 \sinh M}{M^3(1-\cosh M)},$$

$$K_2 = -\frac{C_0}{M^2},$$

$$K_3 = \frac{M^3 - C_0 M + C_0 \sinh M}{M^3(1-\cosh M)},$$

$$K_4 = \frac{C_0}{M^3}$$

and

$$C_0 = \frac{M^3 \sinh M}{M \sinh M + 2(1-\cosh M)}.$$

From (3.4) – (3.6), we have

$$f_1 = \alpha_1 + \alpha_2 \cosh My + \alpha_3 \sinh My - \frac{\{C_1 + K_2^2 - M^2(K_3^2 - K_4^2)\}}{M^2} y$$

$$+ \frac{(7K_2K_4 - 2MK_1K_3)}{4M} y \cosh My + \frac{(7K_2K_3 - 2MK_1K_4)}{4M} y \sinh My$$

$$- \frac{K_2K_3}{4} y^2 \cosh My - \frac{K_2K_4}{4} y^2 \sinh My \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3.7)$$

where

$$\alpha_1 = \frac{\{C_1 + K_2^2 - M^2(K_3^2 - K_4^2)\}}{M^2(1-\cosh M)} - \frac{(7K_2K_4 - 2MK_1K_3 - MK_2K_3)}{4M(1-\cosh M)} \cosh M$$

$$- \frac{\sinh M}{4M^3(1-\cosh M)} \left[7K_2K_3M^2 - 2K_1K_4M^3 - K_2K_4M^3 - 7K_2K_4M\right.$$

$$\left. + 2M^2K_1K_3 + 4\{C_1 + K_2^2 - M^2(K_3^2 - K_4^2)\}\right],$$

$$\alpha_2 = -\alpha_1,$$

$$\alpha_3 = \frac{4\{C_1 + K_2^2 - M^2(K_3^2 - K_4^2)\} - 7MK_2K_4 + 2M^2K_1K_3}{4M^3}$$

and

$$C_1 = \frac{M \sinh M}{4[M \sinh M+2(1-\cosh M)]}\big[\cosh M(2K_2K_4M+2K_1K_4M-7K_2K_3)$$
$$+\sinh M(2K_1K_3M+2K_2K_3M-7K_2K_4)+\{7K_2K_3-2K_1K_4M+5K_2K_4M$$
$$-2K_1K_3M^2-K_2K_3M^2-4K_2^2+4M^2(K_3^2-K_4^2)\}\big]-\frac{(1-\cosh M)}{4[M \sinh M+2(1-\cosh M)]}$$
$$\times\big[8K_2^2-8M^2(K_3^2-K_4^2)+5K_2K_3M^2-2K_1K_4M^3-K_2K_4M^3\big].$$

Thus, for the first order perturbation, we have

$$f(y) = (K_1+\lambda\alpha_1)+\frac{[K_2M^2-\lambda C_1-\lambda K_2^2+\lambda M^2(K_3^2-K_4^2)]}{M^2}y+(K_3+\lambda\alpha_2)\cosh My$$
$$+(K_4+\lambda\alpha_3)\sinh My+\lambda\frac{(7K_2K_4-2K_1K_3M)}{4M}y\cosh My$$
$$+\lambda\frac{(7K_2K_3-2K_1K_4)}{4M}y\sinh My-\lambda\frac{K_2K_3}{4}y^2\cosh My-\lambda\frac{K_2K_4}{4}y^2\sinh My \quad .. \quad (3.8)$$

and

$$C = C_0+\lambda C_1. \quad .. \quad (3.9)$$

Now, the solution for eqn. (2.5) can be expressed for small values of λ by a power series developed near $\lambda = 0$ as follows:

$$F = F_0+\lambda F_1+\ldots+\lambda^n F_n. \quad .. \quad (3.10)$$

Putting the values of F and f from (3.1) and (3.10) in eqn. (2.5) and equating the coefficients of like powers of λ, we have

$$F_0''-M^2F_0 = 0 \quad .. \quad (3.11)$$

and

$$F_1''-M^2F_1+\frac{M\cosh My}{\sinh M}f_0-F_0'f_0-\frac{\sinh My}{\sinh M}f_0'-F_0f_0' = 0 \quad .. \quad (3.12)$$

with the boundary conditions

$$\left.\begin{aligned} Fn(0) &= 0 \quad \text{for } n \geqslant 0 \\ Fn(1) &= 0 \quad \text{for } n \geqslant 0 \end{aligned}\right\}. \quad .. \quad (3.13)$$

From (3.11) and (3.13), we have

$$F_0 = 0. \quad .. \quad (3.14)$$

From (3.6) and (3.12) – (3.14), we have

$$F_1 = A_1\cosh My+A_2\sinh My-\frac{K_1}{2\sinh M}y\sinh My+\frac{3K_2}{4M\sinh M}y\cosh My$$
$$-\frac{K_2}{4\sinh M}y^2\sinh My+\frac{K_3}{M\sinh M} \quad .. \quad (3.15)$$

where

$$A_1 = -\frac{K_3}{M \sinh M}$$

and

$$A_2 = \frac{(2K_1+K_2)}{4M \sinh M} - \frac{(3K_2-4K_3)}{4M \sinh^2 M} \cosh M - \frac{K_3}{M \sinh^2 M}.$$

Therefore, from (1.14), (2.2), (3.8) and (3.15), we have

$$u = \frac{\sinh My}{\sinh M} + \frac{\lambda}{R} x f'(y) + \lambda F_1(y) \quad \ldots \quad (3.16)$$

and

$$v = f(y) \quad \ldots \quad (3.17)$$

where $f(y)$ and $F_1(y)$ are given by (3.8) and (3.15).

From (2.3) and (2.4), the pressure distribution is

$$p(0, 0) - p(x, y) = \frac{\lambda}{R^2} f' - \frac{\lambda C}{2R^2} x^2 \quad \ldots \quad (3.18)$$

where C is given by (3.9).

The pressure increase in x-direction is

$$p(x, y) - p(0, y) = \frac{\lambda C}{2R^2} x^2. \quad \ldots \quad (3.19)$$

The shearing stress at the lower plate is

$$\tau_0 = \frac{\mu U}{y_0} \left(\frac{\partial u}{\partial y}\right)_{y=0}.$$

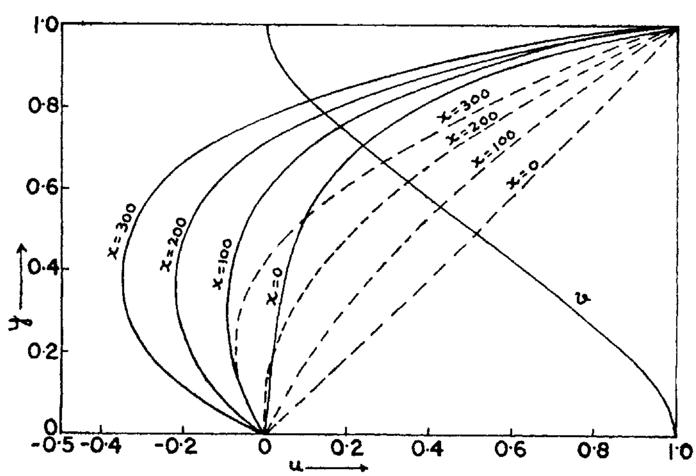

FIG. 1. Longitudinal and transverse velocity profiles plotted against y ($\lambda = 0{\cdot}1$, $R = 100$; $M = 5$ ——— and $M = 0$ - - - -).

And the coefficient of skin friction is given by

$$C_f = \frac{2\tau_0}{\rho U^2} = \frac{2}{R}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= \frac{2}{R}\left[\frac{M}{\sinh M} + \frac{3K_2\lambda}{4M \sinh M} + A_2 M\lambda + \frac{K_3 M^2 \lambda}{R} x\right]. \qquad .. (3.20)$$

4. Numerical Discussion

The longitudinal and transverse velocity profiles for $\lambda = 0{\cdot}1$, $R = 100$, $M = 5$ at various cross-sections of the channel are shown in Fig. 1 and they are compared with the case without magnetic field, i.e. $M = 0$. It is noted that due to a small suction at the stationary plate a flow from large values of x towards the mouth of the channel is developed near the stationary plate (a similar phenomenon is reported by Verma and Bansal for the case of non-conducting fluids). In the present case, due to the presence of magnetic field, this back flow occurs right up to the small distances from the mouth of the channel. If the Hartmann number is small, viscosity dominates and the velocity profile is similar to the case of non-conducting fluid. As the Hartmann number increases, the region of back flow is extended from the stationary plate towards the upper plate and it is shown in Fig. 2 for various values of

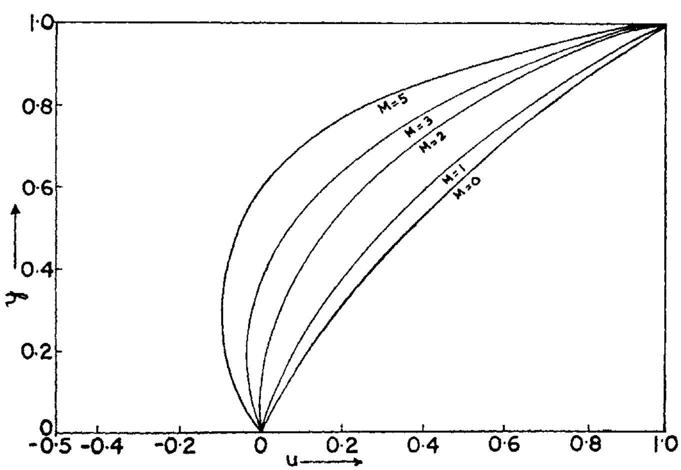

Fig. 2. Longitudinal velocity profile plotted against y ($\lambda = 0{\cdot}1$, $R = 100$ and $x = 100$).

Hartmann number and $\lambda = 0{\cdot}1$, $R = 100$ and $x = 100$. This is due to the fact that viscosity becomes unimportant as the Hartmann number increases except near the upper plate.

The pressure distribution is shown in Fig. 3 for $\lambda = 0{\cdot}1$, $R = 100$ and $M = 0$, 1. The pressure in the main flow direction is parabolic and it is found that this pressure increase decreases with the increase of Hartmann number.

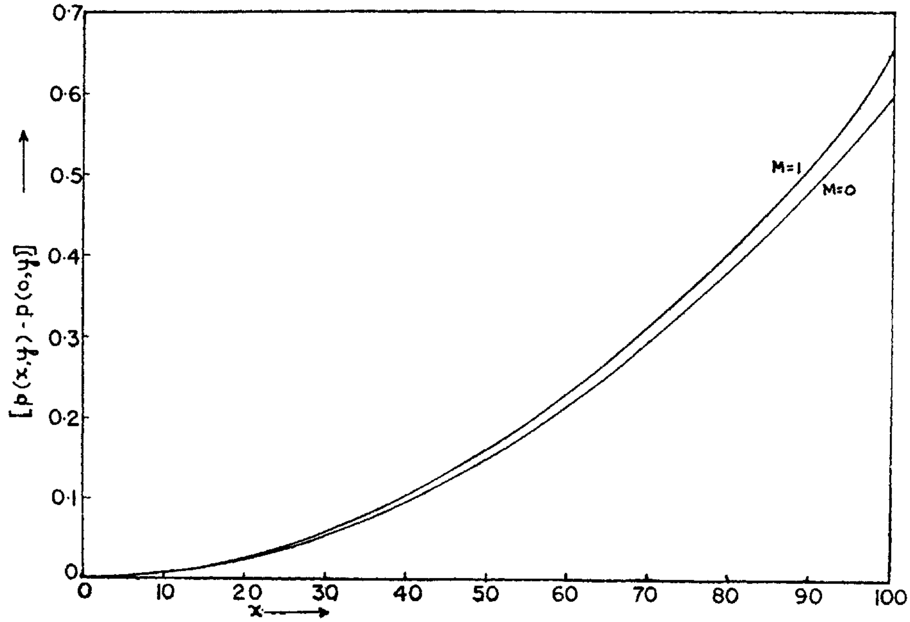

FIG. 3. Axial pressure increase v. length in flow direction for $\lambda = 0{\cdot}1$ and $R = 100$.

The coefficient of skin friction at the stationary plate is shown in Fig. 4 for $R = 100$ and for various values of $\lambda = 0{\cdot}1$, $0{\cdot}2$, $0{\cdot}5$ and $M = 0, 1, 2$.

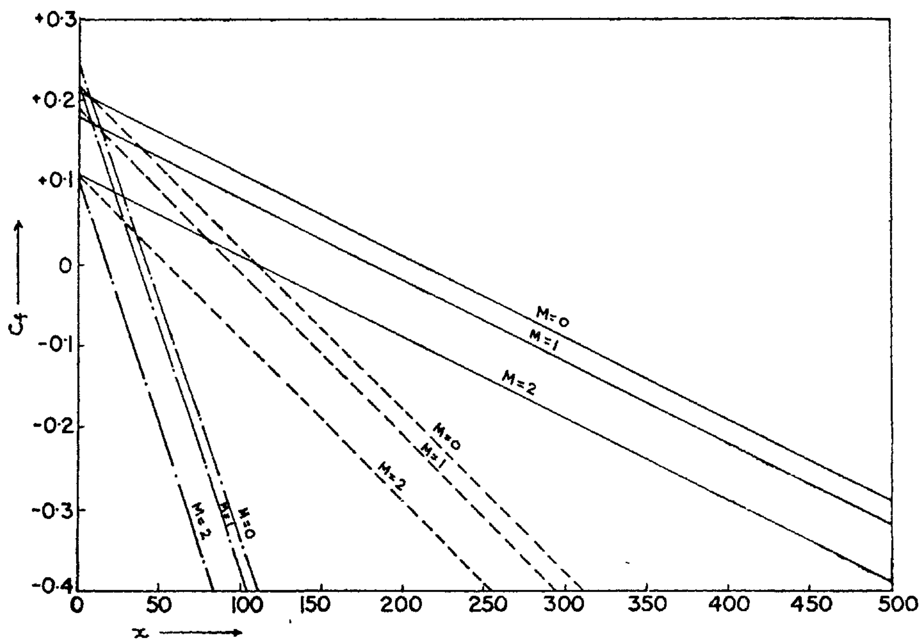

FIG. 4. Variation of coefficient of friction v. length in flow direction for $\lambda = 0{\cdot}1$ ——, $\lambda = 0{\cdot}2$ - - - -, $\lambda = 0{\cdot}5$ –·–·– and $R = 100$.

It is interesting to note that the coefficient of skin friction decreases with increase of suction parameter or with increase of Hartmann number.

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