RADIAL OSCILLATIONS OF COMPOSITE POLYTROPES—PART I

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(Communicated by C. S. Ghosh, F.N.I.)

(*Received 22 January 1968*)

Composite polytropic configurations have been obtained by taking polytropic indices as 1·5 and 3 respectively in the core and envelope. Some numerical solutions of Emden equation have also been developed.

1. INTRODUCTION

The pulsational properties of a star mainly depend on the density distribution in the star and the ratio of specific heat of the material of the star. The polytropic models for different n afford a convenient series of models for the study of the pulsational properties. However, for most of the models a better approximation to the density distribution of a star can be achieved by the composite polytropes as the latter depends on more than one parameter. So it appears quite possible to obtain some conclusions regarding the effect of density distribution on the pulsational properties of a star by considering composite models with different indices and with interfaces at different radii.

With this object in view a number of composite polytropes have been constructed by the author and their pulsational properties investigated. The present paper obtains the configuration of four composite polytropes having indices 1·5 and 3 respectively in the core and the envelope and with different positions of the interface. Milne's method (1932) for fitting a core of one index with an envelope of another index has been employed.

2. COMPOSITE CONFIGURATION

For a composite polytrope the configuration will consist of a core satisfying the Emden equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n \quad \ldots \quad (1)$$

for a value n_1 of the index n, surrounded by an envelope satisfying eqn. (1) for another index n_2. The boundary conditions at the centre still remains the same. So the core will be described by an Emden solution of index n_1. The boundary conditions for the solution in the envelope are determined by the continuity of pressure and density across the interface. So the solution in the envelope is not necessarily an Emden solution.

Using variables ϕ and η for the core and θ and ξ for the envelope Chandrasekhar (1939) gave the equation of fit as

$$u(n_1;\ \eta) = u(n_2;\ \xi):\quad V(n_1;\ \eta) = V(n_2;\ \xi) \qquad \ldots \quad (2)$$

where

$$u = \frac{\xi\theta^n}{\theta'} \qquad \ldots \quad (3)$$

and

$$V = (n+1)\,\frac{\xi\theta'}{\theta}. \qquad \ldots \quad (4)$$

3. Fitting Solution to Composite Configuration

From any solution of Emden eqn. (1) a whole continuous family of solutions can be derived by the application of 'homology theorem' which states that if $\theta = f(\xi)$ is a solution of Emden eqn. (1) of index n, then $\theta = A^{2/n-1}f(A\xi)$ is also a solution of the same equation, where A is an arbitrary real number and is known as homology constant.

Let us consider a polytropic configuration consisting of polytropes of indices $n = 1{\cdot}5$ in the core and $n = 3$ in the envelope. For the sake of convenience we take the surface of configuration at $\xi = 1$. We shall first obtain the solution corresponding to Emden solution $n = 3$, but with surface at $\xi = 1$.

We know that first zero of Emden function of index 3 is $\xi_0 = 6{\cdot}89689$. If we denote this solution by $\theta = f(\xi)$, then by 'homology theorem' any other solution will be $\theta = Af(A\xi)$. The latter solution will vanish at ξ_1 where $A\xi_1 = \xi_0$. So by taking $A = \xi_0$, we get a solution vanishing at $\xi = 1$. For any value of A, we have

$$\left(\xi^2\frac{d\theta}{d\xi}\right)_{\xi=\xi_1} = \{\xi^2A^2f'(A\xi)\}_{\xi=\xi_1} = \xi_0^2\,f'(\xi_0)$$
$$= -2{\cdot}01824 \text{ from Emden tables.} \qquad \ldots \quad (5)$$

Therefore, the solution $\theta = f(\xi)$ computed for

$$\xi = 0\,;\ \theta = 1,\quad \frac{d\theta}{d\xi} = 0 \qquad \ldots \quad (6)$$

may be converted into solution for which

$$\xi = 1\,;\ \theta = 0,\quad \frac{d\theta}{d\xi} = -2{\cdot}01824 \qquad \ldots \quad (7)$$

by taking $\theta = \xi_0\,f(\xi_0\xi)$. At the centre θ will be now

$$\theta = \xi_0\,f(0) = 6{\cdot}89689. \qquad \ldots \quad (8)$$

The quantity $\xi^2\,\dfrac{d\theta}{d\xi}$ introduced in eqn. (5) is a homology invariant. It is a particular case of the invariant

$$\omega_n = -\xi_0^{(n+1)/(n-1)}\left(\frac{d\theta}{d\xi}\right)_{\xi=\xi_0}. \qquad \ldots \quad (9)$$

TABLE I

Solutions of Emden equation for the envelope

ξ	θ	$-d\theta/d\xi$	ξ	θ	$-d\theta/d\xi$
FOR THE INTERFACE $\xi_i = 0{\cdot}2094$, $a = 2{\cdot}11920$, $A = 7{\cdot}14286$					
1·00	0·00000	2·11920	0·58	1·51349	5·97157
0·98	0·04325	2·20658	0·56	1·63642	6·32485
0·96	0·08830	2·29947	0·54	1·76664	6·69992
0·94	0·13527	2·39833	0·52	1·90457	7·09682
0·92	0·18427	2·50366	0·50	2·05065	7·51515
0·90	0·23546	2·61597	0·48	2·20531	7·95384
0·88	0·28896	2·73586	0·46	2·36893	8·41099
0·86	0·34495	2·86394	0·44	2·54185	8·88368
0·84	0·40358	3·00086	0·42	2·72435	9·36761
0·82	0·46505	3·14735	0·40	2·91659	9·85689
0·80	0·52954	3·30416			
0·78	0·59729	3·47211	0·38	3·11861	10·34358
0·76	0·66851	3·65205	0·36	3·33025	10·81738
0·74	0·74345	3·84489	0·34	3·55113	11·26519
0·72	0·82239	4·05160	0·32	3·78058	11·67075
0·70	0·90562	4·27316	0·30	4·01755	12·01427
0·68	0·99343	4·51059	0·28	4·26058	12·27222
0·66	1·08615	4·76495	0·26	4·50768	12·41724
0·64	1·18415	5·03725	0·24	4·75631	12·41822
0·62	1·28777	5·32851	0·22	5·00323	12·24058
0·60	1·39742	5·63968	0·20	5·24449	11·84652
FOR THE INTERFACE $\xi_i = 0{\cdot}3915$, $a = 2{\cdot}62357$, $A = 5{\cdot}54789$					
1·00	0·00000	2·62357	0·68	1·22734	5·53676
0·98	0·05354	2·73174	0·66	1·34102	5·83377
0·96	0·10931	2·84674	0·64	1·46081	6·14739
0·94	0·16746	2·96911	0·62	1·58703	6·47736
0·92	0·22813	3·09945	0·60	1·72000	6·82298
0·90	0·29149	3·23837	0·58	1·86004	7·18303
0·88	0·35773	3·38653	0·56	2·00741	7·55554
0·86	0·42702	3·54466	0·54	2·16233	7·93771
0·84	0·49958	3·71340	0·52	2·32496	8·32557
0·82	0·57563	3·89356	0·50	2·49535	8·71386
0·80	0·65541	4·08588	0·48	2·67347	9·09566
0·78	0·73916	4·29114	0·46	2·85908	9·46217
0·76	0·82714	4·51010	0·44	2·05178	9·80233
0·74	0·91966	4·74349	0·42	3·25091	10·10253
0·72	1·01698	4·99201	0·40	3·45551	10·34637
0·70	1·11944	5·25628	0·38	3·66425	10·51436
FOR THE INTERFACE $\xi_i = 0{\cdot}6160$, $a = 4{\cdot}03626$, $A = 4{\cdot}48214$					
1·00	0·00000	4·03626	0·78	1·13544	6·55756
0·98	0·08237	4·20268	0·76	1·26970	6·87152
0·96	0·16818	4·37957	0·74	1·41038	7·19852
0·94	0·25763	4·56772	0·72	1·55772	7·53679
0·92	0·35096	4·76789	0·70	1·71191	7·88373
0·90	0·44843	4·98083	0·68	1·87310	8·23572
0·88	0·55029	5·20724	0·66	2·04134	8·58793
0·86	0·65681	5·44774	0·64	2·21658	8·93405
0·84	0·76829	5·70281	0·62	2·39861	9·26607
0·82	0·88503	5·97279	0·60	2·58706	9·57398
0·80	1·00731	6·25778			

TABLE I—*concld.*

ξ	θ	$-d\theta/d\xi$	ξ	θ	$-d\theta/d\xi$
FOR THE INTERFACE $\xi_i = 0{\cdot}8250$, $a = 8{\cdot}07311$, $A = 3{\cdot}93939$					
1·00	0·00000	8·07311	0·90	0·89667	9·94915
0·98	0·16475	8·40596	0·88	1·09999	10·38605
0·96	0·33637	8·75950	0·86	1·31222	10·83869
0·94	0·51528	9·13458	0·84	1·53361	11·30180
0·92	0·70190	9·53137	0·82	1·76431	11·76779

The value of ω_n for Emden solution is denoted by ω_n^0. Let the integrations be carried out inwards from the surface to the centre giving any prescribed value to ω_n. It is known that when $\omega_n < \omega_n^0$, then $\theta(\xi) \to \infty$ as $\xi \to 0$; and when $\omega_n > \omega_n^0$, $\theta(\xi)$ rises to a maximum and decreases to zero before reaching the centre.

In order to get composite polytropes suitable for our purpose we calculated a number of solutions for the envelope taking the boundary conditions as

$$\xi = 1\,;\ \theta = 0,\ \ \frac{d\theta}{d\xi} = -a, \qquad \ldots \quad \ldots \quad \ldots \quad (10)$$

where a is some definite number greater than ω_n^0. Different values of a will give interfaces at different radii; and some good guessing was needed to choose values which gave interfaces at approximately $\xi = 0{\cdot}2$, $0{\cdot}4$, $0{\cdot}6$ and $0{\cdot}8$.

After some value of a was chosen, a numerical solution of eqn. (1) for the envelope was developed for the index 3. The Emden solution for the core was taken from the British Association Mathematical Tables (1932). Now u, V curves both for the core as well as for the envelope were drawn and their point of intersection was taken to be the interfacial point of the composite model. After obtaining the interface, the values of ϕ for the core were recalculated by 'homology theorem'. The values of θ and $\frac{d\theta}{d\xi}$ for these models are given in Table I.

ACKNOWLEDGEMENTS

The author is highly grateful to Prof. C. Prasad for suggesting the problem and for constant encouragement and guidance. The research fellowship from the Council of Scientific and Industrial Research is acknowledged gratefully.

REFERENCES

British Association Mathematical Tables, Vol. 2 (1932). Cambridge University Press, London.

Chandrasekhar, S. (1939). An Introduction to the Study of Stellar Structure. Dover Publications Inc., 173.

Milne, E. A. (1932). The analysis of stellar structure. *Mon. Not. R. Astr. Soc.*, **92**, 610.