MAGNETO-THERMO-ELASTIC INTERACTIONS IN AN INFINITE SOLID DUE TO TRANSIENT HEAT SOURCES

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The object of the present paper is to determine the distribution of stress, strain, temperature, electric and magnetic fields in an infinite solid if it is subjected to a transient heat source. The solutions are first obtained in terms of Fourier and Laplace transforms. The inverse transforms are studied in particular cases.

INTRODUCTION

The studies in magneto-elasticity which has largely been possible due to the merger of the theories of electro-magnetism and elasticity are of recent origin. Apart from these, attempts are also being made in recent times to accommodate the thermal field in the interplay of electro-magnetic and mechanical fields. These studies are considered to be important primarily because of their diverse applications in various branches of geophysics, optics and in the devices such as aero-magnetic flutter and also in plasmatrons, plasmaguides, etc. Though there exists in abundance a literature of magneto-elasticity, vide Kaliski (1960*a*, *b* and 1961), Sinha (1965, 1966, 1967), Paria (1961), Giri (1966), Das (1967), the subject of magneto-thermo-elasticity is still in its formative period. The initial attempts in this direction have been constituted by Nowacki and Kaliski (1963), Nowacki (1962, 1963), Paria (1962, 1964), Wilson (1966). The papers of Sinha (1964) and Murthy (1966) should also be mentioned in this connection. The present paper is a similar attempt aiming at the determination of the effects of thermal and electro-magnetic conditions on the distribution of temperature, stresses as well as the amount of the induced magnetic field. The solution is achieved by making use of the equations of Maxwell, equation of elasticity and equation of heat conduction. It is found that the use of Fourier-Laplace transforms facilitates the solution of the problem.

FORMULATION OF THE PROBLEM, FUNDAMENTAL EQUATIONS

Let us consider an infinite elastic solid body in which an *initial* magnetic field is impressed and having a uniform temperature. There is no initial stress or strain. The body is subjected to a time-decaying heat source placed on a plane extending in all directions. The plane is taken as y-z plane, z-axis being along the original magnetic field H_3. The x-axis is perpendicular to this plane. The displacement $\vec{u}$ has the components (u, o, o), u is a function of x and t.

The problem being one of magneto-thermo-elasticity, we have to solve Maxwell's electromagnetic equations, Fourier equation of heat conduction together with the equation of elasticity given by Hooke-Duhamel.

Maxwell's equations reduce to

$$\left.\begin{aligned} &\operatorname{curl} \vec{H} = \vec{j}, \quad \operatorname{div} \vec{B} = 0 \\ &\operatorname{curl} \vec{E} = -\frac{\partial B}{\partial t}, \quad \vec{B} = \mu_e \vec{H} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

in absence of displacement current.

These equations should be supplemented by Ohm's law given by

$$\vec{j} = \sigma \left[\vec{E} + \frac{\partial \vec{u}}{\partial t} \times \vec{B} \right] - k_0 \operatorname{grad} T \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

The Hooke-Duhamel law can be written as

$$\tau_{ij} = 2\mu e_{ij} + (\lambda e - \beta T)\delta_{ij} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where

$$\left.\begin{aligned} e_{ij} = \tfrac{1}{2}(u_{i,\,j} + u_{j,\,i}), \quad e = \operatorname{div} \vec{u} \\ \delta_{ij} = 1 \quad \text{for } i = j \text{ and } \delta_{ij} = 0 \quad \text{for } i \neq j \end{aligned}\right\} \quad \ldots \quad \ldots \quad (4)$$

The Fourier law of heat conduction is

$$k \nabla^2 T + Q = \rho c_v \frac{\partial T}{\partial t} + T_0 \beta \frac{\partial e}{\partial t} + \pi_0 \operatorname{div} \vec{j} \quad \ldots \quad \ldots \quad (5)$$

The equation of motion of the solid is

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \tau_{ik}}{\partial x_k} + \left(\vec{j} \times \vec{B}\right)_i \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

The symbols used here have their usual meanings as in Paria (1962). Q represents the intensity of the heat source, T_0 the initial uniform temperature of the solid, and T the perturbed temperature over and above T_0.

From eqns. (3), (4) and (6) we get

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = \mu \nabla^2 \vec{u} + (\lambda + \mu) \operatorname{grad} e - \beta \operatorname{grad} T + \left(\vec{j} \times \vec{B}\right) \quad \ldots \quad \ldots \quad (7)$$

Solution of the Problem

We can assume for this problem from symmetry that all the physical quantities must be functions of x and t.

Let us take $Q = q_0\delta(x)e^{-\Omega t}(\Omega > 0)$ where $\delta(x)$ is the Dirac-delta function and q_0 the strength of the source. We get the following equations as given by Paria (1964):

$$E_x = \frac{k_0}{\sigma}\cdot\frac{\partial T}{\partial x}, \quad H_x = 0, \quad j_x = 0 \qquad (8)$$

$$\frac{\partial E_y}{\partial x} = -\mu_e\frac{\partial H_z}{\partial t}, \quad j_y = -\frac{\partial H_z}{\partial x}, \quad H_y = 0 \qquad (9)$$

$$E_z = 0, \quad j_z = 0 \qquad (10)$$

$$\frac{\partial h_z}{\partial t} = \nu_H\frac{\partial^2 h_z}{\partial x^2} - H_3\frac{\partial^2 u}{\partial x\partial t} \qquad (11)$$

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial T}{\partial x} - \mu_e H_3\frac{\partial h_z}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \qquad (12)$$

$$k\frac{\partial^2 T}{\partial x^2} + q_0\delta(x)e^{-\Omega t} = \rho c_v\frac{\partial T}{\partial t} + T_0\beta\frac{\partial^2 u}{\partial x\partial t} \qquad (13)$$

where $\nu_H = \frac{1}{\mu_e\sigma}$ is the magnetic viscosity and $H_z = H_3 + h_z$. (h_z is so small that its square and higher powers and its product with u can be neglected).

Equations (11), (12) and (13) will determine h_z and u, T and hence from eqn. (8) we get E_x and from third equation of (9), j_y can be determined.

We now proceed to obtain the solution of the problem by means of transforms.

Let us define the Laplace and Fourier transforms of a function $f(x, t)$ by $\bar{f}$ and f', respectively where

$$\bar{f} = \int_0^\infty f(x,t)\exp(-pt)dt, \quad \bar{f}' = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\bar{f}.\exp(i\xi x)dx. \ (\text{Re}\ p > 0)$$

From equations (11), (12) and (13) we get

$$(p+\nu_H\xi^2)\bar{h}_z' = H_3 ip\xi\bar{u}' \qquad (14)$$

$$\mu_e H_3 i\xi\bar{h}_z' = \{\rho p^2 + (\lambda+2\mu)\xi^2\}\bar{u}' - \beta i\xi\bar{T}' \qquad (15)$$

$$\frac{q_0}{\sqrt{2\pi}} = \{\rho c_v p(p+\Omega) + (p+\Omega)k\xi^2\}\bar{T}' - T_0\beta i\xi p(p+\Omega)\bar{u}' \qquad (16)$$

Solving these equations we get

$$\bar{T}' = \frac{q_0}{\sqrt{2\pi}}\cdot\frac{L_1}{L_2}$$

$$\bar{u}' = \frac{q_0}{\sqrt{2\pi}}\cdot\frac{\beta i\xi(p+\nu_H\xi^2)}{L_2}$$

$$\bar{h}_z' = -\frac{q_0 H_3 p\beta\xi^2}{\sqrt{2\pi}L_2}$$

where

$$L_1 = \nu_H(\lambda+2\mu)\xi^4+p\left(\nu_H\rho p+\lambda+2\mu+\mu_e H_3^2\right)\xi^2+\rho p^3$$

$$L_2 = \{\rho c_v p(p+\Omega)+(p+\Omega)k\xi^2\}L_1+T_0\beta^2\xi^2 p(p+\Omega)(p+\nu_H\xi^2)$$

Putting

and

$$\left.\begin{aligned} R_H &= \frac{\mu_e H_3^2}{\rho c_1^2}, \quad \epsilon_T = \frac{T_0\beta^2}{\rho^2 c_v \cdot c_1^2} \\ k_1 &= \frac{k}{\rho c_v} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

The above relations can be put in the form

$$\bar{T}' = \frac{q_0}{\rho c_v\sqrt{2\pi}} \cdot \frac{M_1}{M_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

$$\bar{u}' = \frac{q_0}{\rho c_v\sqrt{2\pi}} \cdot \frac{\beta i\xi(p+\nu_H\xi^2)}{\rho M_2} \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

$$\bar{h}_z' = -\frac{q_0}{\rho c_v\sqrt{2\pi}} \cdot \frac{H_3 p\beta\xi^2}{\rho M_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where

and

$$\left.\begin{aligned} M_1 &= \left(\nu_H c_1^2\xi^2+\xi_1^2\right)\left(\xi^2+\xi_1^2\right) \\ \xi_1 &= \tfrac{1}{2}p\left(\nu_H p+c_3^2+\sqrt{N}\right) \\ \nu_H c_1^2\xi_2^2 &= \tfrac{1}{2}p\left(\nu_H p+c_3^2-\sqrt{N}\right) \\ N &= \nu_H p^2-2\nu_H p\left(2c_1^2-c_3^2\right)+c_3^4 \\ c_1^2 &= \frac{\lambda+2\mu}{\rho}, \quad c_3^2 = c_1^2(1+R_H) \\ M_2 &= (p+k_1\xi^2)M_1+c_1^2 p\xi^2(p+\nu_H\xi^2)\epsilon_T \end{aligned}\right\} \quad \ldots \quad \ldots \quad (21)$$

Using the second and third equations of (9) and the first equation of (8) and taking transforms, we get

$$\left.\begin{aligned} \bar{j}_y' &= -\frac{q_0}{\rho c_v\sqrt{2\pi}} \cdot \frac{H_3 p\beta i\xi^3}{\rho M_2} \\ \bar{E}_y' &= \frac{q_0}{\rho c_v\sqrt{2\pi}} \cdot \frac{\mu_e H_3\beta i\xi p}{\rho M_2} \\ \bar{E}_x' &= -\frac{K_0}{\sigma} \cdot \frac{q_0}{\rho c_v\sqrt{2\pi}} \cdot \frac{i\xi M_1}{M_2} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

By taking the inverse transforms in eqns. (18)–(20) and (22) we can obtain T, u, h_z, E_x, E_y, j_y. The other components of $\vec{H}$, $\vec{E}$ and $\vec{j}$ have already been proved to be zero. The only non-vanishing stress component is τ_{xx} which can be derived from eqn. (3) in the form

$$\tau_{xx} = (\lambda+2\mu)\frac{\partial u}{\partial x} - \beta T$$

Taking the transforms we obtain

$$\bar{\tau}'_{xx} = -\frac{q_0}{\rho c_v\sqrt{2\pi}}\cdot\frac{\beta p}{M_2}\cdot\{\xi^2(\nu_H p + c_1^2 R_H) + p^2\} \qquad \text{(23)}$$

For a perfect conductor we have $\sigma \to \infty$ so that $\nu_H = 0$. Therefore,

$$\left.\begin{aligned} M_1 &= p(p+\Omega)[c_3^2 k_1\xi^4 + p\xi^2(c_5^2+k_1 p)+p^3] \\ M_2 &= p(p+\Omega)(c_3^2 k_1\xi^2+\xi_3^2)(\xi^2+\xi_4^2) \end{aligned}\right\} \qquad \text{(24)}$$

where

$$c_5^2 = c_3^2 + c_1^2\epsilon_T = c_1^2(1+R_H+\epsilon_T) \qquad \text{(25)}$$

$$\xi_3^2 = \tfrac{1}{2}p(k_1 p + c_5^2 + \sqrt{R}) \qquad \text{(26)}$$

$$\xi_4^2 = \frac{p}{2k_1 c_3^2}(k_1 p + c_5^2 - \sqrt{R}) \qquad \text{(27)}$$

$$R = k_1^2 p^2 - 2k_1 p(2c_3^2 - c_5^2) + c_5^4 \qquad \text{(28)}$$

Therefore, it follows from eqn. (18) that

$$\bar{T}' = \frac{q_0}{\rho c_v\sqrt{2\pi}\,.\,(p+\Omega)}\left(\frac{A}{c_3^2 k_1\xi^2+\xi_3^2} + \frac{B}{\xi^2+\xi_4^2}\right) \qquad \text{(29)}$$

where

$$A = \tfrac{1}{2}c_3^2\left(1 - \frac{k_1 p - c_5^2}{\sqrt{R}}\right) \qquad \text{(30)}$$

$$B = \frac{1}{2k_1}\left(1 + \frac{k_1 p - c_5^2}{\sqrt{R}}\right) \qquad \text{(31)}$$

Taking the inverse Fourier transform of $\bar{T}'$, we get

$$\bar{T} = \frac{q_0}{2\rho c_v p^{\frac{1}{2}}(p+\Omega)}\left[\frac{A}{c_3\sqrt{k_1}}\cdot\xi_4\cdot\exp\left(\frac{-\xi_3 x}{c_3\sqrt{k_1}}\right) + B\xi_3\exp(-\xi_4 x)\right] \qquad \text{(32)}$$

Similarly from eqn. (20) we get

$$\bar{h}'_z = -\frac{q_0 H_3\beta}{\rho^2 c_v\sqrt{2\pi}\,.\,p(p+\Omega)}\cdot\frac{1}{\sqrt{R}}\left(\frac{\xi_3^2}{c_3^2 k_1\xi^2+\xi_3^2} - \frac{\xi_4^2}{\xi^2+\xi_4^2}\right)$$

The inverse Fourier transform then gives

$$\bar{h}_z = -\frac{q_0 H_3\beta}{2\rho^2 p(p+\Omega)\sqrt{R}}\left[\frac{\xi_3}{c_3\sqrt{k_1}}\exp\left(\frac{-\xi_3 x}{c_3\sqrt{k_1}}\right) - \xi_4\exp(-\xi_4 x)\right] \qquad \text{(33)}$$

From eqn. (23) we get $\bar{\tau}'_{xx}$. Putting $\nu_H = 0$ and then taking the inverse Fourier transform we have

$$\bar{\tau}_{xx} = -\frac{q_0\beta}{2\rho c_v p^{3/2}(p+\Omega)}\left[\frac{A_1\xi_4}{c_3\sqrt{k_1}}\exp\left(\frac{-\xi_3 x}{c_3\sqrt{k_1}}\right)+B_1\xi_3\exp(-\xi_4 x)\right] \quad \ldots \quad (34)$$

where

$$A_1 = \frac{c_1^2}{p\sqrt{R}}\left[R_H\xi_3^2-(1+R_H)p^2k_1\right] \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

$$B_1 = -\frac{1}{p\sqrt{R}}(c_1^2R_H\xi_4^2-p^2) \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

Temperature Field for Small Thermo-elastic Coupling

ϵ_T is called the thermo-elastic coupling factor which is very small for common metals. We shall expand various expressions containing ϵ_T retaining only its first power.

From eqn. (28) we have

$$\sqrt{R} = (k_1p-c_3^2)+\frac{k_1p+c_3^2}{k_1p-c_3^2}\cdot c_1^2\epsilon_T \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

Hence from eqns. (26) and (27) we get

$$\xi_3 = p\sqrt{k_1}\left[1+\frac{c_1^2\epsilon_T}{2(k_1p-c_3^2)}\right] \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

$$\xi_4 = \sqrt{\frac{p}{k_1}}\left[1-\frac{c_1^2\epsilon_T}{2(k_1p-c_3^2)}\right] \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

Also from eqns. (30) and (31) we obtain

$$A = \frac{c_1^2c_3^2\epsilon_T\,.\,k_1p}{(k_1p-c_3^2)^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

$$B = \frac{1}{k_1}\left[1-\frac{c_1^2k_1p\epsilon_T}{(k_1p-c_3^2)^2}\right] \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

With these approximated values, we have from eqn. (32)

$$\frac{2\rho c_v\bar{T}}{q_0} = \frac{1}{\sqrt{k_1p}\,.\,(p+\Omega)}\cdot e^{-x\sqrt{\frac{p}{k_1}}}+\frac{c_1^2\epsilon_T}{p+\Omega}\left[\frac{c_3e^{-\frac{px}{c_3}}}{(k_1p-c_3^2)^2}+\frac{x}{2k_1(k_1p-c_3^2)}\cdot e^{-x\sqrt{\frac{p}{k_1}}}\right.$$

$$\left.-\frac{1}{4}\left\{\frac{1}{(\sqrt{k_1p}-c_3)^2}+\frac{1}{(\sqrt{k_1p}+c_3)^2}\right\}\times\frac{1}{\sqrt{k_1p}}e^{-x\sqrt{\frac{p}{k_1}}}\right] \quad \ldots \quad \ldots \quad (42)$$

Taking the inverse Laplace transform, vide Erdelyi (1954), and introducing the dimensionless quantity $\eta = \frac{xc_1}{k_1}$, we obtain

$$\frac{2\rho c_v k_1}{q_0 c_1} . T = e^{-\Omega t} . \frac{2k_1}{c_1^2\sqrt{\pi}} . \int_0^{\frac{c_1}{\sqrt{k_1}}\sqrt{t}} e^{-\frac{\eta^2}{4\tau^2}+\frac{\Omega k_1}{c_1^2}\tau^2} d\tau + \epsilon_T F(\eta, t, R_M) \quad .. \quad (43)$$

where

$$R_M = 1 + R_H \quad .. \quad .. \quad .. \quad .. \quad (44)$$

and

$$F(\eta, t, R_M) = -\frac{e^{-\Omega t}}{\sqrt{\pi}} . \frac{2k_1}{c_1^2} . \int_0^{\frac{c_1}{\sqrt{k_1}}\sqrt{t}} \tau^2 . e^{\left(\frac{\Omega k_1}{c_1^2}\tau - \frac{\eta^2}{4\tau^2}\right)} d\tau$$

$$+\frac{k_1}{c_1^2} e^{-\Omega t} \int_0^{\frac{c_1}{\sqrt{k_1}}\sqrt{t}} \left[\tau . e^{\frac{\Omega k_1}{c_1^2}\tau} \left\{(\eta - \tau^2\sqrt{R_M}) e^{\tau^2 R_M - \eta\sqrt{R_M}}\right.\right.$$

$$\times \operatorname{erfc}\left(\frac{\eta}{2\tau} - \tau\sqrt{R_M}\right) + (\eta + \tau^2\sqrt{R_M}) . e^{\tau^2 R + \eta M\sqrt{R_M}}$$

$$\left.\left.\times \operatorname{erfc}\left(\frac{\eta}{2\tau} + \tau\sqrt{R_M}\right)\right\}\right] d\tau \quad \text{for } t < \frac{x}{c_3} = \frac{\eta k_1}{c_1 c_3} \quad .. \quad .. \quad (45)$$

and

$$= \frac{c_1 c_3}{k_1} \left[\frac{e^{-\Omega\left(t - \frac{\eta k_1}{c_1 c_3}\right)}}{\left(\Omega + \frac{c_3^2}{k_1}\right)^2} - \frac{1}{\left(\Omega + \frac{c_3^2}{k_1}\right)} . e^{\frac{c_3^2}{k_1}\left(t - \frac{\eta k_1}{c_1 c_3}\right)} + \frac{1}{\left(\Omega - \frac{c_3^2}{k_1}\right)} . \left(t - \frac{\eta k_1}{c_1 c_3}\right) . e^{\frac{c_3^2}{k_1}\left(t - \frac{\eta k_1}{c_1 c_3}\right)}\right]$$

$$-\frac{e^{-\Omega t}}{\sqrt{\pi}} . \frac{2k_1}{c_1^2} \int_{\frac{\eta k_1}{c_1 c_3}}^{\frac{c_1}{\sqrt{k_1}}\sqrt{t}} \tau^2 . e^{\left(\frac{\Omega k_1}{c_1^2}\tau - \frac{\eta^2}{4\tau^2}\right)} . d\tau$$

$$+\frac{k_1}{c_1^2} e^{-\Omega t} \int_{\frac{\eta k_1}{c_1 c_3}}^{\frac{c_1}{\sqrt{k_1}}\sqrt{t}} \left[\tau . e^{\frac{\Omega k_1 \tau}{c_1^2}} \left\{(\eta - \tau^2\sqrt{R_M}) e^{\tau^2 R_M - \eta\sqrt{R_M}} . \operatorname{erfc}\left(\frac{\eta}{2\tau} - \tau\sqrt{R_M}\right)\right.\right.$$

$$\left.\left.+(\eta + \tau^2\sqrt{R_M}) e^{\tau^2 R_M + \eta\sqrt{R_M}} . \operatorname{erfc}\left(\frac{\eta}{2\tau} + \tau\sqrt{R_M}\right)\right\}\right] \quad \text{for } t > \frac{\eta k_1}{c_1 c_3} \quad .. \quad .. \quad (46)$$

From eqn. (43), it follows that the first term on the right-hand side which decays with time exponentially represents the solution of the classical heat conduction equation while the function $F(\eta, t, R_M)$ is the perturbation due to the thermo-elastic coupling factor ϵ_T. The initial magnetic field R_M occurs only in the perturbed function.

Table I gives the values of $F(\eta, t, R_M)$ for different values of η, R_M when $\Omega t = 0{\cdot}72$, the conductor being aluminium.

TABLE I

Values of $F(\eta, t, R_M)$ in units of 10^{-11}

η / R_M	0·0	0·6	1·2	1·8	2·4	3·0	3·6
1·0	−3·487	−1·043	−0·179	0·016	0·015	0·004	0·003
2·0	−8·278	−2·526	−0·648	−0·089	−0·002	0·012	0·015

The continuous curve as in Fig. 1 represents the perturbation when the magnetic field is absent while the dotted curve the perturbation function when the magnetic field parameter $R_M = 2$. It is observed from the graph that the magnetic field reduces the temperature near the end ($\eta = 0$) of the solid while, in the rest of the region, temperature is not much affected by the presence of the magnetic field.

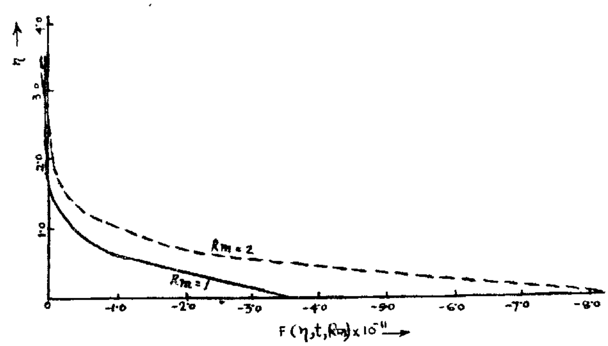

FIG. 1. Effect of magnetic field parameter on perturbation due to thermo-elastic coupling factor.

DISTRIBUTION OF STRESS AND INDUCED MAGNETIC FIELD FOR SMALL TIME

The stress and induced magnetic field are determined considering small values of time. In this case p is very large. Therefore, we can take

$$\sqrt{R} = k_1 p, \quad \xi_3 = \sqrt{k_1} \cdot p \left(1 + \frac{c_5^2}{4k_1 p}\right)$$

$$\xi_4 = \frac{c_5}{c_3}\sqrt{\frac{p}{2k_1}}$$

in place of relations (26), (27) and (28) respectively.

Relations (35) and (36) reduce to

$$A_1 = -c_1^2\left(1 - \frac{c_5^2 R_M}{2k_1 p}\right), \quad B_1 = \frac{1}{k_1}\left(1 - \frac{c_1^2 c_5^2 R_M}{2c_3^2 k_1 p}\right)$$

Equation (33) reduces to

$$\bar{h}_z = -\frac{q_0 H_3 \beta}{2\rho^2 c_v k_1 p^2 (p+\Omega) \cdot c_3} \left[p\left(1+\frac{c_5^2}{4k_1 p}\right) e^{-\frac{px}{c_3}\left(1+\frac{c_5^2}{4k_1 p}\right)} \right.$$

$$\left. -c_5 \sqrt{\frac{p}{2k_1}}\, e^{-(c_5/c_3)} \cdot \sqrt{\frac{p}{2k_1}} \cdot x \right] \quad \ldots \quad (47)$$

Taking the inverse Laplace transform, we get

$$h_z = -\frac{q_0 H_3 \beta}{2\rho^2 c_v k_1 c_3} \phi(\eta, t, R_M, \epsilon_T)$$

where

$$\eta = \frac{x c_1}{k_1}$$

and

$$\phi(\eta, t, R_M, \epsilon_T)$$

$$= -\frac{2k_1}{c_1^2}(\sqrt{R_M+\epsilon_T}) \cdot e^{-\Omega t} \int_0^{c_1\sqrt{\frac{t}{k_1}}} \left[\sqrt{\frac{2}{\pi}} \cdot \tau^2 \cdot \exp\left\{ \frac{\Omega k_1}{c_1^2}\tau^2 - \frac{\eta^2}{\tau^2}\left(1+\frac{\epsilon_T}{R_M}\right) \right\} \right.$$

$$\left. -\frac{\eta\tau}{2}\left(\sqrt{1+\frac{\epsilon_T}{R_M}}\right) \cdot e^{\frac{\Omega k_1}{c_1^2}\tau^2} \operatorname{erfc}\left(\frac{\eta}{2\sqrt{2}\tau} \cdot \sqrt{1+\frac{\epsilon_T}{R_M}} \right) \right] d\tau \text{ for } t < \frac{\eta k_1}{c_1 c_3} \quad \ldots \quad (48)$$

and

$$= \frac{1}{\Omega}\left\{ 1 - e^{-\Omega t + \frac{\eta k_1 \Omega}{c_1 c_3}} \right\} \cdot e^{-\frac{\eta}{4\sqrt{R_m}} \cdot (R_M+\epsilon_T)} + \frac{c_1^2}{4k_1\Omega}(R_M+\epsilon_T) \cdot e^{-\frac{\eta}{4\sqrt{R_m}} \cdot (R_M+\epsilon_T)}$$

$$\times \left\{ t - \frac{\eta k_1}{c_1 c_3} + \frac{1}{\Omega} e^{-\Omega t + \frac{\eta k_1 \Omega}{c_1 c_3}} - \frac{1}{\Omega} \right\}$$

$$-\frac{2k_1}{c_1^2}\sqrt{R_M+\epsilon_T} \cdot e^{-\Omega t} \cdot \int_{\frac{\eta k_1}{c_1 c_3}}^{c_1\sqrt{\frac{t}{k_1}}} \left[\sqrt{\frac{2}{\pi}} \cdot \tau^2 \exp\left\{ \frac{\Omega k_1}{c_1^2}\tau^2 - \frac{\eta^2}{\tau^2}\left(1+\frac{\epsilon_T}{R_M}\right) \right\} \right.$$

$$\left. -\frac{\eta\tau}{2}\sqrt{1+\frac{\epsilon_T}{R_M}} \cdot e^{\frac{\Omega k_1 \tau^2}{c_1^2}} \operatorname{erfc}\left(\frac{\eta}{2\sqrt{2}\tau} \cdot \sqrt{1+\frac{\epsilon_T}{R_M}} \right) \right] d\tau \text{ for } t > \frac{\eta k_1}{c_1 c_3} \quad \ldots \quad (49)$$

To study the effect of the magnetic field parameter on the induced field we can put $\epsilon_T = 0$ since ϵ_T occurs with R_M which is very large in comparison with ϵ_T.

Table II gives the values of $\phi(\eta)$ when $\Omega t = 0{\cdot}72$, $\epsilon_T = 0$, $R_M = 1{\cdot}5$.

TABLE II

Values of $\phi(\eta)$ in units of 10^{-12}

η	0	·1	·2	·3	·4	·5	·6
$\phi(\eta)$	1·043	0·797	0·904	0·911	0·943	0·944	1·060

From the graph (Fig. 2) it is easily seen that the effect of R_M at a certain instant of time gradually increases along the length of the solid beginning from a particular point of its length.

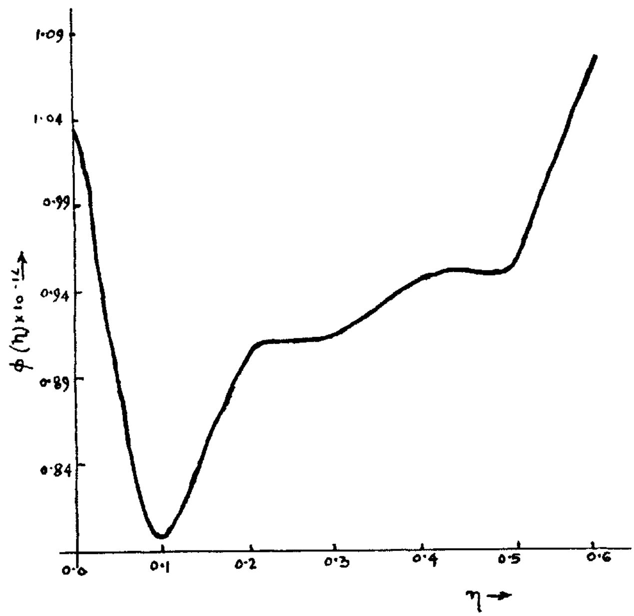

FIG. 2. Effect of magnetic field parameter on induced magnetic field.

Taking into consideration the approximations for small-time, equation (34) reduces to

$$\bar{\tau}_{xx} = \frac{q_0\beta}{2\rho c_v(p+\Omega)}\left[\frac{c_1^2 c_5}{\sqrt{2c_3^2 k_1 p}}\left(1-\tfrac{1}{2}R_H\cdot\frac{c_5^2}{k_1 p}\right)e^{-\frac{px}{c_3}-\frac{c_5^2 x}{4k_1 c_3}}\right.$$
$$\left.-\frac{1}{\sqrt{k_1 p}}\left\{1+\frac{c_5^2(c_3^2-2R_H c_1^2)}{4k_1 c_3^2 p}\right\}e^{-\frac{c_5}{c_3}x\cdot\sqrt{\frac{p}{2k_1}}}\right] \quad \ldots \quad \ldots \quad (50)$$

The inverse Laplace transform then gives

$$\tau_{xx} = \frac{q_0\beta}{2\rho c_v}\,\psi(\eta,\, t,\, R_M,\, \epsilon_T)$$

where

$$\psi(\eta, t, R_M, \epsilon_T) = \frac{2}{\sqrt{\pi}\,.\,c_1}\int_0^{c_1\sqrt{\frac{t}{k_1}}} .\, e^{-\Omega t+\frac{\Omega k_1}{c_1^2}\tau-\frac{R_M+\epsilon_T}{8}\cdot\frac{\eta^2}{\tau^2}} .\, d\tau$$

$$-\frac{1}{\sqrt{k_1}}\cdot\frac{R_M+\epsilon_T}{2R_M}\,.\,(2-R_M)\times\int_0^{c_1\sqrt{\frac{t}{k_1}}} e^{-\Omega t+\frac{\Omega k_1}{c_1^2}\tau} .\,\tau\left\{\frac{2\sqrt{k_1}\tau}{c_1\sqrt{\pi}}\right.$$

$$\left.\times e^{-\frac{R_M+\epsilon_T}{8}\cdot\frac{\eta^2}{\tau^2}}-\sqrt{\frac{R_M+\epsilon_T}{2R_M}}\,.\,\eta\,.\,\frac{\sqrt{k_1}}{c_1}\,\mathrm{erfc}\left(\sqrt{\frac{R_M+\epsilon_T}{2R_M}}\cdot\frac{\eta}{2\tau}\right)\right\}d\tau$$

$$\text{for } t<\frac{\eta k_1}{c_1c_3}$$

and

$$=\sqrt{\frac{R_M+\epsilon_T}{2}}\cdot\frac{c_1}{R_Mk_1\Omega}\cdot e^{-\frac{R_M+\epsilon_T}{4\sqrt{R_M}}\eta}\cdot\left\{1-e^{-\Omega t+\frac{\Omega k_1\eta}{c_1^2\sqrt{R_M}}}\right\}$$

$$-\frac{R_M-1}{2\sqrt{2R_M}}\cdot\frac{c_1^3\,.\,(R_M+\epsilon_T)^{\frac{3}{2}}}{k_1^2}\cdot\left\{\frac{1}{\Omega}\left(t-\frac{\Omega k_1\eta}{c_1^2\sqrt{R_M}}\right)\right.$$

$$\left.+\frac{1}{\Omega^2}\cdot e^{-\Omega t+\frac{\Omega k_1}{c_1^2}\cdot\frac{\eta}{\sqrt{R_M}}}-\frac{1}{\Omega^2}\right\}\cdot e^{-\frac{R_M+\epsilon_T}{4\sqrt{R_M}}\cdot\eta}$$

$$+\frac{2}{\sqrt{\pi}c_1}\int_{\frac{\eta k_1}{c_1c_3}}^{c_1\sqrt{\frac{t}{k_1}}} e^{-\Omega t+\frac{\Omega k_1}{c_1^2}\tau-\frac{R_M+\epsilon_T}{8}\cdot\frac{\eta^2}{\tau^2}} .\, d\tau$$

$$-\frac{1}{\sqrt{k_1}}\cdot\frac{R_M+\epsilon_T}{R_M}\,.\,(2-R_M)\,.\int_{\frac{\eta k_1}{c_1c_3}}^{c_1\sqrt{\frac{t}{k_1}}} e^{-\Omega t+\frac{\Omega k_1}{c_1^2}\cdot\tau} .\,\tau\left\{\frac{2\sqrt{k_1}\tau}{c_1\sqrt{\pi}}\right.$$

$$\left.\times e^{-\frac{R_M+\epsilon_T}{8}\cdot\frac{\eta^2}{\tau^2}}-\sqrt{\frac{R_M+\epsilon_T}{2R_M}}\,.\,\eta\,.\,\frac{\sqrt{k_1}}{c_1}\,.\,\mathrm{erfc}\left(\sqrt{\frac{R_M+\epsilon_T}{2R_M}}\cdot\frac{\eta}{2\tau}\right)\right\}d\tau$$

$$\text{for } t>\frac{\eta k_1}{c_1c_3}$$

Here also the thermo-elastic coupling factor ϵ_T occurs with R_M. Hence to find the effect of magnetic field parameter we put $\epsilon_T = 0$.

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