DIRECT ANALYSIS OF RESISTIVITY KERNEL FUNCTION OVER A TWO-LAYER EARTH WITH A TRANSITIONAL BOUNDARY

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The problem of resistivity stratification for a layered Earth model with transitional boundary is discussed theoretically in the present work. It makes use of Kernel function in the integral expression (Stefanesco and Schlumberger 1930) for potential for a stratified earth. The Kernel functions are evaluated for two types of conductivity inhomogeneity in the transition layer, using a new matrix method (Upadhyay and Sri Niwas 1971). The analysis of Kernel function is performed by Pekeris (1940) method.

The relevant results are :

(i) The kernel functions for a sharp and a transition boundary possess similar pattern of variation;

(ii) The analysis of the kernel curve for a sharp boundary gives a closely true picture of resistivity stratification. In the case of transitional boundary, the analysis does not reveal the actual presence of continuous conductivity variation. On the other hand, one is led to interpret the transition boundary as made up of two homogeneous layers.

The analysis presented herein is restricted to the theoretical study of kernel function for transitional boundary and the determination of resistivity stratification, starting from the integral expression for the potential.

INTRODUCTION

In the resistivity interpretation of electrical measurements, the assumption of stratified homogeneous earth is more common. At all times the assumption of homogeneity does not closely represent the actual earth conditions. Mallick and Roy (1968) have carried out the geo-electrical measurement in dug wells section with different orientation of electrodes in peninsular India region. They include a typical section in such region consisting of transported soil cover of a few meters thickness, underlain by a transitional layer of weathered granite or basalt of gradually increasing freshness and followed finally by the unaltered highly resistive bed rock. The top soil cover is more or less uniform with resistivity of a few ohm-meters within the weathered layer. This resistivity increases with depth until the unweathered rock with a resistivity of 500 to 1000 ohm-meters is reached at a depth of about 15 to 20 meters.

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Mallick and Roy (1968) solved the problem of resistivity sounding on a two-layer earth with transitional boundary. They gave the master curves to interpret the resistivity measurements indirectly by curve-matching. But it must be admitted that the resolving power of the curve-matching procedure is not very high. It is virtually impossible to estimate in advance the expected accuracy of these curve-fitting process. A direct procedure of interpretation was investigated by Langer (1933) who showed that 'if the conductivity of the ground is a function of depth only, then the potential about point current electrode uniquely determines this function.' Thus the inverse potential problem for horizontally stratified models, appears to have a unique solution. Pekeris (1940) developed a direct method of interpretation to obtain a resistivity stratification from the kernel curve, using Langer (1933) approach.

Koefoed (1965, 1966) discussed the problem of obtaining kernel curve from the apparent resistivity observations made in the field. He also outlined Pekeris (1940) method for determining resistivity stratification from the Kernel curve. Also, it was shown that for one set of apparent resistivity observation, only one Kernel curve is possible. However, the second part of analysis i.e., the determination of resistivity stratification from the kernel curve was found to involve ambiguity in interpretation. The present analysis is pertinent to this aspect of Koefoed's work. The results are based on a theoretical analysis for a geologically relevant situation. The discussion of the result follow at the end.

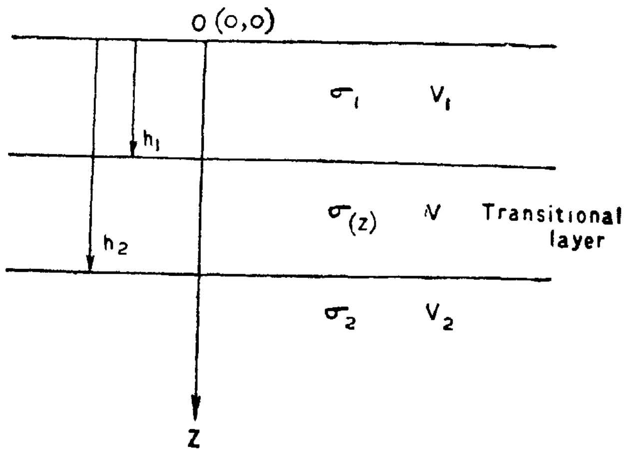

FIG. 1. Geometry of the assumed model

THEORY

Layered configuration is taken as shown in Fig. 1. 'O' is the position of the point source and is also the origin of the cylindrical co-ordinate system. Positive z-axis of the co-ordinate system is taken vertically downward. Upper and bottom layers are considered to be isotropic homogeneous. The intermediate layer possesses conductivity inhomogeneity in the z-direction, which varies as follows :

Case I $\sigma(z)=\sigma_0[1+k(z-h_1)]^n \qquad (h_1<z<h_2) \qquad (1)$

Case II $\sigma(z)=\sigma_0[e^{k(z-h_1)}] \qquad (h_1<z<h_2) \qquad (2)$

The basic equation which will be used in the present work is potential at the surface due to point source.

$$V=q\left[\frac{1}{r}+2\int_0^\infty k(\lambda)\,Jo(\lambda r)\,d\lambda\right] \qquad \text{(Stefanesco and Schlumberger 1930)} \qquad (3)$$

where $q=\frac{I}{2\pi\sigma_1}$, $K(\lambda)=\frac{A_1(\lambda)}{q}$, the Kernel function, I is the current strength and σ_1 is the resistivity of the top layer.

The potential distribution in the top, transition and bottom layers are governed respectively by :

$$\nabla^2 V_1=0 \qquad (4)$$

$$\nabla^2 V+\frac{1}{\sigma}\cdot\frac{\partial\sigma}{\partial z}\cdot\frac{\partial V}{\partial z}=0 \qquad (5)$$

$$\nabla^2 V_2=0 \qquad (6)$$

whose solutions are given by :

$$V_1=q\int_0^\infty e^{-\lambda z}J_0(\lambda r)\,d\lambda+\int_0^\infty A_1(\lambda)\,(e^{-\lambda z}+e^{\lambda z})\,J_0(\lambda r)\,d\lambda \qquad (7)$$

$$V=\int_0^\infty\left[A_2(\lambda)\,I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\left\{1+k(z-h_1)\right\}\right)+B_2(\lambda)\,{}^{I-\left(\frac{n-1}{2}\right)}_{K-\left(\frac{n-1}{2}\right)}\right.$$

$$\left.\left(\frac{\lambda}{k}\left\{1+k(z-h_1)\right\}\right)\right]\times\left[1+k(z-h_1)\right]^{\left(\frac{1-n}{2}\right)}J_0(\lambda r)d\lambda \qquad \text{(Case I)} \qquad (8a)$$

or

$$V=\int_0^\infty\left[A_2(\lambda)\,e^{p_1\lambda z}+B_2(\lambda)\,e^{p_2\lambda z}\right]J_0(\lambda r)d\lambda \qquad \text{(Case II)} \qquad (8b)$$

and

$$V_2=\int_0^\infty A_3(\lambda)\,e^{-\lambda z}\,J_0(\lambda r)d\lambda \qquad (9)$$

Where the combinations $I_{\left(\frac{n-1}{2}\right)}$ and $I_{-\left(\frac{n-1}{2}\right)}$ is applicable when $\left(\frac{n-1}{2}\right)$ is neither zero nor an integer; and the combination $I_{\left(\frac{n-1}{2}\right)}$ and $K_{\left(\frac{n-1}{2}\right)}$ applies to the case when $\left(\frac{n-1}{2}\right)$ assumes either zero or an integer value. $A_1(\lambda)$, $A_2(\lambda)$,

$B_2\lambda$ and $A_3(\lambda)$ are the constants.
And p_1 and p_2 are given by

$$p_1=\frac{-k+\sqrt{k^2+4\lambda^2}}{2} \tag{10}$$

$$p_2=\frac{-k-\sqrt{k^2+4\lambda^2}}{2} \tag{11}$$

The boundary conditions are the continuity of potential and normal component of current density at each interface, i.e.,

$$\begin{aligned} V_1 &= V \text{ and } \sigma_1\frac{\partial V_1}{\partial z}=\sigma(z)\frac{\partial V}{\partial z} \text{ at } z=h_1 \\ V &= V_2 \text{ and } \sigma(z)\frac{\partial V}{\partial z}=\sigma_2\frac{\partial V_2}{\partial z} \text{ at } z=h_2 \end{aligned} \tag{12}$$

Using these boundary conditions to the equations 7, 8a and 9 we have

$$qe^{-\lambda h_1}+A_1(\lambda)\left(e^{-\lambda h_1}+e^{\lambda h_1}\right)=A_2(\lambda)\, I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\right)+B_2(\lambda)\, \begin{matrix} I_{-\frac{n-1}{2}} \\ K_{\frac{n-1}{2}} \end{matrix}\left(\frac{\lambda}{k}\right) \tag{13}$$

$$-qe^{-\lambda h_1}+A_1(\lambda)\left(-e^{-\lambda h_1}+e^{\lambda h_1}\right)=\left[A_2(\lambda) I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}\right) + B_2(\lambda) \begin{matrix} I_{-\left(\frac{n+1}{2}\right)} \\ -K_{\left(\frac{n+1}{2}\right)} \end{matrix}\left(\frac{\lambda}{k}\right)\right]\cdot\frac{\sigma_0}{\sigma_1} \tag{14}$$

$$A_2(\lambda)\, x^{\frac{1-n}{2}}\, I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}x\right)+B_2(\lambda)x^{\frac{1-n}{2}} \begin{matrix} I_{-\frac{n-1}{2}} \\ K_{\frac{n-1}{2}} \end{matrix}\left(\frac{\lambda}{k}x\right)=A_3e^{-\lambda h_2} \tag{15}$$

$$\left[A_2(\lambda)\, x^{\frac{1-n}{2}}\, I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}x\right)+B_2(\lambda)x^{\frac{1-n}{2}} \begin{matrix} I_{-\frac{n+1}{2}} \\ -K_{\frac{n+1}{2}} \end{matrix}\left(\frac{\lambda}{k}x\right)\right]\cdot\frac{\sigma_0^1}{\sigma_2}=-A_3e^{-\lambda h_2} \tag{16}$$

Where : $x=[1+k(h_2-h_1)]$ and $\sigma_0^1=\sigma_0\,[1+k(h_2-h_1)]$

We solve the above set of equation by matrix method to obtain the value of kernel function

From equation (13) and (14) we have

$$\begin{bmatrix} 1 \\ A_1(\lambda) \end{bmatrix} = \begin{bmatrix} qe^{-\lambda h_1}\{e^{-\lambda h_1}+e^{\lambda h_1}\} \\ -qe^{-\lambda h_1}\{-e^{-\lambda h_1}+e^{\lambda h_1}\} \end{bmatrix}^{-1} \times$$

$$\begin{bmatrix} I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\right) & \frac{I_{-\left(\frac{n-1}{2}\right)}}{K_{\left(\frac{n-1}{2}\right)}}\left(\frac{\lambda}{k}\right) \\ I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}\right)\cdot\frac{\sigma_0}{\sigma_1} & \frac{I_{-\left(\frac{n+1}{2}\right)}}{-K_{\left(\frac{n+1}{2}\right)}}\left(\frac{\lambda}{k}\right)\cdot\frac{\sigma_0}{\sigma_1} \end{bmatrix} \begin{bmatrix} A_2(\lambda) \\ B_2(\lambda) \end{bmatrix} \tag{17a}$$

and from equations (15) and (16) we have.

$$\begin{bmatrix} A_2(\lambda) \\ B_2(\lambda) \end{bmatrix} = \begin{bmatrix} x^{\frac{1-n}{2}} I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}x\right) & x^{\frac{1-n}{2}} \frac{I_{-\frac{n-1}{2}}}{K_{\frac{n-1}{2}}}\left(\frac{\lambda}{k}x\right) \\ x^{\frac{1-n}{2}} I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}x\right)\cdot\frac{\sigma_0{}^1}{\sigma_1} & x^{\frac{1-n}{2}} \frac{I_{-\left(\frac{n+1}{2}\right)}}{-K_{\left(\frac{n+1}{2}\right)}}\left(\frac{\lambda}{k}x\right)\cdot\frac{\sigma_0{}^1}{\sigma_2} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} e^{-\lambda h_2} & 0 \\ -e^{-\lambda h_2} & 0 \end{bmatrix} \begin{bmatrix} A_3(\lambda) \\ B_3(\lambda) \end{bmatrix} \tag{17b}$$

Combining equations (17*a*) and (17*b*) we have.

$$\begin{bmatrix} 1 \\ A_1(\lambda) \end{bmatrix} = \begin{bmatrix} qe^{-\lambda h_1}(e^{-\lambda h_1}+e^{\lambda h_1}) \\ -qe^{-\lambda h_1}(-e^{-\lambda h_1}+e^{\lambda h_1} \end{bmatrix}^{-1} \begin{bmatrix} I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\right) & \frac{I_{-\frac{n-1}{2}}}{K_{\frac{n-1}{2}}}\left(\frac{\lambda}{k}\right) \\ I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}\right)\cdot & \frac{I_{-\frac{n+1}{2}}}{-K_{\frac{n+1}{2}}}\left(\frac{\lambda}{k}\right)\cdot\frac{\sigma_0}{\sigma_1} \end{bmatrix} \times$$

$$\begin{bmatrix} x^{\frac{1-n}{2}} I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}x\right) & x^{\frac{1-n}{2}} \frac{I_{-\frac{n-1}{2}}}{K_{\left(\frac{n-1}{2}\right)}}\left(\frac{\lambda}{k}x\right) \\ x^{\left(\frac{1-n}{2}\right)} I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}x\right).\frac{\sigma_0^1}{\sigma_1} & x^{\frac{1-n}{2}} \frac{I_{-\frac{n+1}{2}}}{-K_{\frac{n+1}{2}}}\left(\frac{\lambda}{k}x\right).\frac{\sigma_0^1}{\sigma_2} \end{bmatrix}^{-1} \begin{bmatrix} e^{-\lambda h_2} & 0 \\ -e^{-\lambda h_2} & 0 \end{bmatrix} \begin{bmatrix} A_3(\lambda) \\ B_3(\lambda) \end{bmatrix} \tag{17c}$$

This gives the kernel function for the case I as

$$K(\lambda) = \frac{P_1 \lambda e^{-2\lambda h_1}}{1 - P_1 \lambda e^{-2\lambda h_1}} \tag{18}$$

Where

$$P_1(\lambda) = \frac{\left[I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\right) + I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}\right).\frac{\sigma_0}{\sigma_1} \right] \left[\frac{-I-\left(\frac{n+1}{2}\right)}{K\left(\frac{n+1}{1}\right)}\left(\frac{\lambda}{k}x\right).\frac{\sigma_0^1}{\sigma_2} - \frac{I-\left(\frac{n-1}{2}\right)}{K\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}x\right) \right] +}{\left[I_{\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\right) - I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}\right).\frac{\sigma_0}{\sigma_1} \right] \left[\frac{-I-\left(\frac{n+1}{2}\right)}{K\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}x\right).\frac{\sigma_0^1}{\sigma_2} - \frac{I-\left(\frac{n-1}{2}\right)}{K\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}x\right) \right] +}$$

$$\frac{\left[\frac{I-\left(\frac{n-1}{2}\right)}{K\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\right) + \frac{-I-\left(\frac{n+1}{2}\right)}{K\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}\right).\frac{\sigma_0}{\sigma_1} \right] \left(I_{\frac{n+1}{2}}\left(\frac{\lambda}{k}x\right).\frac{\sigma_0^1}{\sigma_2} + I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}x\right) \right)}{\left(\frac{I-\left(\frac{n-1}{2}\right)}{K\left(\frac{n-1}{2}\right)}\left(\frac{\lambda}{k}\right) - \frac{-I-\left(\frac{n+2}{2}\right)}{K\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}\right).\frac{\sigma_0}{\sigma_1} \right) \left(I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}x\right).\frac{\sigma_0^1}{\sigma_2} + I_{\left(\frac{n+1}{2}\right)}\left(\frac{\lambda}{k}x\right) \right)}$$

Similarly, we can obtain the kernel function for the case II repeating the same procedure as for case I, i.e.,

$$K(\lambda) = \frac{P_2(\lambda) e^{-2\lambda h_1}}{1 - P_2(\lambda) e^{-2\lambda h_1}} \tag{19}$$

Where

$$P_2(\lambda) = \frac{\left(\frac{\lambda\sigma_1}{\sigma_0} + p_1\right) - \left(\frac{\lambda + p_1\frac{\sigma_0''}{\sigma_2}}{\lambda + p_2\frac{\sigma_0''}{\sigma_2}}\right)\left(\frac{\lambda\sigma_1}{\sigma_0} + p_2\right) e^{(p_1-p_2)(h_2-h_1)}}{\left(\frac{\lambda\sigma_1}{\sigma_0} - p_1\right) - \left(\frac{\lambda + p_1\frac{\sigma_0''}{\sigma_2}}{\lambda + p_2\frac{\sigma_0''}{\sigma_2}}\right)\left(\frac{\lambda\sigma_1}{\sigma_0} - p_2\right) e^{(p_1-p_2)(h_2-h_1)}}$$

Where

$$\sigma_0'' = \sigma_0 \, exp[k(h_2 - h_1)]$$

ANALYSIS OF THE KERNEL FUNCTION

Here we would analyse the kernel function for a particular case of linear variation of conductivity i.e., when $n=1$ in equation (1). The kernel function in this case is obtained by putting $\sigma_0 = \sigma_1$, $\sigma_0^1 = \sigma_2$ and $k = \frac{\alpha}{\sigma_1}$ in equation (18), i.e.

$$K(\lambda) = \frac{P_3(\lambda)e^{-2\lambda h_1}}{1-P_3(\lambda)e^{-2\lambda h_1}} \tag{20}$$

Where

$$P_3(\lambda) = \frac{\left[I_0\left(\frac{\lambda\sigma_1}{\alpha}\right)+I_1\left(\frac{\lambda\sigma_1}{\alpha}\right)\right]\left[K_1\left(\frac{\lambda\sigma_2}{\alpha}\right)-K_0\left(\frac{\lambda\sigma_2}{\alpha}\right)\right]+\left[K_0\left(\frac{\lambda\sigma_1}{\alpha}\right)-K_1\left(\frac{\lambda\sigma_1}{\alpha}\right)\right]}{\left[I_0\left(\frac{\lambda\sigma_1}{\alpha}\right)-I_1\left(\frac{\lambda\sigma_1}{\alpha}\right)\right]\left[K_1\left(\frac{\lambda\sigma_2}{\alpha}\right)-K_0\left(\frac{\lambda\sigma_2}{\alpha}\right)\right]+\left[K_0\left(\frac{\lambda\sigma_1}{\alpha}\right)+K_1\left(\frac{\lambda\sigma_1}{\alpha}\right)\right]}$$

$$\frac{\left[I_1\left(\frac{\lambda\sigma_2}{\alpha}\right)+I_0\left(\frac{\lambda\sigma_2}{\alpha}\right)\right]}{\left[I_1\left(\frac{\lambda\sigma_2}{\alpha}\right)+I_0\left(\frac{\lambda\sigma_2}{\alpha}\right)\right]}$$

Where

$$\alpha = \frac{\sigma_2-\sigma_1}{h_2-h_1}$$

We obtain variation of kernel function against λ assuming $\frac{\sigma_2}{\sigma_1}=11$, $h_2-h_1=0$ and 4 and $h_1=1$ and their graphical representation is produced in Figure 2. The value of the kernel function can be calculated from the formula (20) or it may directly be obtained by integrating the potential measurement conducted in the field.

In order to apply the Pekeris (1940) method of interpretation to the curve obtained in Fig. 2 we have to obtain a function $f_1(\lambda)$ defined below.
Taking Hankel transform of equation (3) we get.

$$1+2K(\lambda) = \lambda\int_0^\infty V(r)J_0(\lambda r)dr$$

or (21)

$$K(\lambda) = \frac{1}{2}\left[\lambda\int_0^\infty V(r)J_0(\lambda r)d\lambda - 1\right] \tag{22}$$

The modified kernel function $G(\lambda)$ is given by

$$G(\lambda) = \frac{K(\lambda)}{1+K(\lambda)} \tag{23}$$

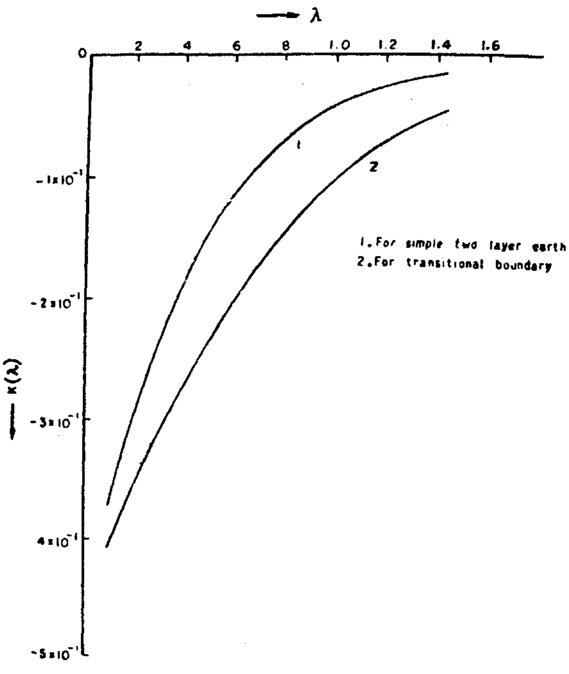

FIG. 2. Graphical representation of K (λ)

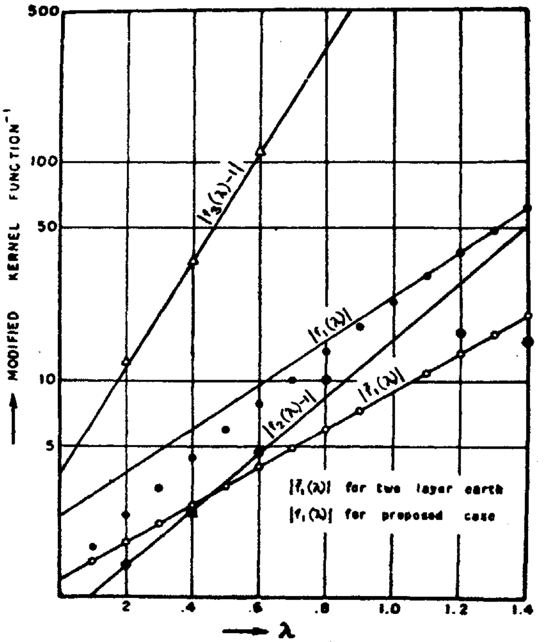

FIG. 3. Analysis of K (λ)

$$\frac{1}{G(\lambda)} = \frac{1+K(\lambda)}{K(\lambda)} = f_1(\lambda) \tag{24}$$

or

$f_1(\lambda) = \{P_3(\lambda)\}^{-1} \ e^{2\lambda h_1}$ (Obtained from equation 20)

According to Pekeris (1940) procedure if we plot $|f_1(\lambda)|$ on a log scale against λ on a linear scale we should get a straight line, if the measurements are being made on a two-layer earth model. The intercept of the straight line on the ordinate would give $\frac{1}{k_1} = \left[\frac{\sigma_1+\sigma_2}{\sigma_1-\sigma_2}\right]$ and its slope would give the thickness of the upper layer. The graphical plotting of $|f_1(\lambda)|$ is shown in Fig 3. It should be noted that graphical representation of the kernel function is similar to that of kernel for a simple two layer case. From Fig. 3 it is observed that we do not get a straight line, for the kernel calculated from equation (20). Now we proceed further as suggested by Pekeris (1940) to calculate $|f_2(\lambda)-1|$ and $|f_3(\lambda)-1|$ when we get a straight line closely. Following again the procedure outlined by Pekeris we find that the kernel curve corresponds to a four layer model earth. The calculated values of resistivity stratifications are presented in Table I :

TABLE I

Layers	Thickness	Conductivity ratio
Top layer	1.15	$2.50 = \sigma_2/\sigma_1$
Second layer	1.50	$3.60 = \sigma_3/\sigma_2$
Third layer	2.80	$1.35 = \sigma_4/\sigma_3$
Bottom layer	Extending to infinity.	

Comparing the actual model assumed $\left[h_1 = 1, \sigma_1 = \frac{1}{11}\sigma_2, \ h_2 = 5 \right]$ from the result obtained in Table I, the following points are observed.

The Pekeris method of interpretation replaces the inhomogeneous intermediate layer by two homogeneous layers having different conductivities. The total thickness of this layer (1.50+2.80=4.3, as calculated) is quite in agreement with the assumed value (4.00). The conductivity ratio of top and bottom layer is 12.1 (as calculated) against assumed value 11.

The calculated value of thicknesses and conductivity ratio of the top and bottom layer are different from the actual values by a small amount. These information, again, are reproduced in the Fig. 4.

DISCUSSION AND CONCLUDING REMARKS

This paper deals with the practical occurrence of conductivity inhomogeneity in geologic section and its interpretation by direct method. As discussed by Koefoed

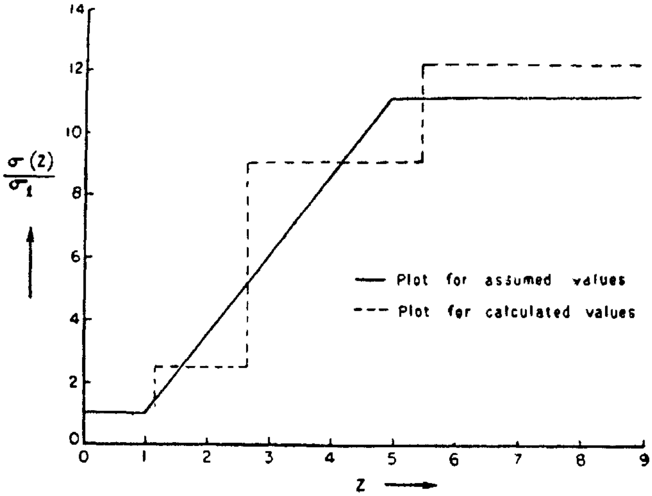

FIG. 4. Plot of calculated and assumed values for layer thickness and conductivity ratios

(1965, 1966), the direct method involves two steps, i.e., the determination of the kernel function from apparent resistivity observations and analysis of the kernel function to reveal the resistivity stratification. The content of the present paper concerns only to the second part, assuming that the kernel curve for transitional boundary is available.

The analysis proceeds by taking a model inhomogeneity representation (equations 1 and 2) compatible with the geologically relevant situation, considered by Mallick and Roy (1968). The potential in the different layers are written down. By making use of the boundary conditions, the value of the kernel functions (equations 18 and 19) are derived. Analysis of the kernel function is performed as discussed by Pekeris (1940). The main results are :

The kernel curve for a two-layer earth with sharp boundary, and that for transition boundary (*see* Fig. 2) in between, possess similar characteristics. However, on analyzing the kernel curve for a transition boundary, using Pekeris method, the transition layer is shown to be replaced by two homogeneous layers with different conductivity values. As such, the method does not reveal the presence of actual inhomogeneity, but brings out an equivalent model.

This theoretical analysis is significant in showing that, by looking at the kernel curves (Fig. 2) one cannot in general distinguish between two distinct cases of sharp and transition boundaries. The interpretation of kernel curve for sharp boundary proves useful. However, the method fails in case of inhomogeneity. If one assumes an earth model as depicted in Fig. 4 by the dotted line, and determines the corresponding kernel curve, it would closely fit with curve 2 of Fig. 2 (for a transition boundary). This is pertinent to the phenomenon of 'Principle of Equivalence'. Although the problem of calculation of kernel curve from apparent resistivity observation has been overcome to a great extent (Koefoed 1965, 1966), the determination of resistivity stratification from kernel curve remains in the same stage, as was developed by Pekeris (1940).

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