ESTIMATION OF THE EFFECT OF CHANGES IN POPULATION PARAMETERS ON YIELD AND OPTIMUM AGE OF EXPLOITATION

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Methods of determining changes in potential yield and corresponding optimum age of exploitation following small changes in population parameters are described. Such methods for estimating changes in the yield or in the optimum age of exploitation for a given fishing mortality have not found to be advantageous over their exact estimation.

INTRODUCTION

The growth and mortality estimates of fish populations are generally based on average values and hence these parameters are subject to errors. Small fluctuations in growth and mortality rates are common features of most marine fish populations. Approximate estimation of the changes in yield, etc., following small fluctuations in population parameters using differential calculus is often easier than their exact estimation. Krishnan Kutty and Qasim (1968) have briefly discussed the error in the estimation of optimum age of exploitation and potential yield. This aspect is examined further in a wider context in this paper.

CHANGE IN YIELD FOLLOWING CHANGES IN POPULATION PARAMETERS

If the length-weight exponent is not accurate (Krishnan Kutty 1972) the estimate of the asymptotic weight, W_∞ will also be affected. Similarly errors in the estimates of the Bertalanffy growth constants, K and t_0 can occur especially in the case of small-sized, fast-growing tropical fishes like the oil sardine with a short life span. Often growth data are available for these species only after they have attained considerable portion of their growth. When the Bertalanffy equation is fitted to such data K is often underestimated and t_0 overestimated. Beverton and Holt (1957) have briefly dealt with the effect of changes in the main population parameters on yield. They have shown that the yield is considerably reduced when M is high or when there is an increase in K. Since variations in natural mortality are more common than the changes in K, the influence on predicted yield caused by the effect of changes in M is also important. An approximate estimation of the changes in yield by taking differentials, following either an error or a change in the estimated natural mortality is possible. But tho methds is not particularly labour saving as compared to the exact estimation by substituting the new values of M or other parameters in the Beverton-Holt yield equation. This is especially so when more than one variable are to be considered. Further, variations in M may be quite wide. Approximate methods are sufficiently accurate only for small changes in the variable.

Optimum Age of Exploitation for a Given F

Krishnan Kutty (1968, 1970) has described equations for the eumetric fishing curve and its complement. Like the yield equation these are somewhat complex. Since these are also implicit functions, an approximate estimation of errors or changes in the estimated age of exploitation for a given fishing mortality is not very advantageous over exact evaluation.

Optimum Age of Exploitation and Potential Yield

The equations for the optimum age of exploitation (t_y) and the corresponding potential yield (Y) given by Krishnan Kutty and Qasim are :

$$e^{k(t_y - t_o)} = \frac{nK+M}{-M} \quad \ldots \quad (1)$$

and

$$Y=RW_\infty e^{-M(t_y - t_p)} \left[1 - e^{-k(t_y - t_o)} \right]^n \quad (2)$$

where K and t_0 are growth constants in the Bertalanffy equation, n=length-weight exponent, R=number of recruits, t_p=age at recruitment and W_∞=asymptotic weight. From equation (1) it follows that

$$t_y = \frac{ln\left(1+\frac{nK}{M}\right)}{K}+t_0 \quad \ldots \quad (3)$$

Using Holt's substitutions in equations (1) and (2) so that the element of time does not directly enter into these equations they can be rewritten as

$$l_{cy}=L_\infty\left(1-\frac{M'}{n+M'}\right) \quad \ldots \quad (4)$$

and

$$Y = RW_\infty \left(\frac{l_c}{L_\infty}\right)^n\left(\frac{L_\infty-l_c}{L_\infty-l_r}\right)^{M'} \quad \ldots \quad (5)$$

where l_{cy}=optimum size corresponding to t_y, L_∞ = asymptotic length, $M'=\frac{M}{K}$ l_r=size at recruitment and l_c=size at exploitation (*see* Krishnan Kutty 1970).

Changes in the Optimum Age of Exploitation

When one variable alone changes — From the equation (3) the factors that can influence t_y are n, K, M and t_o. Krishnan Kutty and Qasim (1968) have shown that the error in t_y following a change in the L/Wt exponent is obtained from the relationship

$$\triangle t_y = \frac{\triangle n}{M+nK} \quad \ldots \quad (6)$$

where n takes the value used for estimating t_y. If the precise value of the exponent is greater than the value used for estimating t_y from the equation (3), then

$\triangle n$ or the difference between the observed value and the value used is negative, thereby showing that the estimated t_y will be an under estimate and the error will approximately be equal to $\frac{\triangle n}{M+nk}$ years.

Taking differentials of the equation (4)

$$\triangle l_{cy} = L_\infty M' (n+M')^{-2} \triangle n$$

$$= \frac{L_\infty M'}{(n+M')^2} \triangle n \quad \ldots \quad (7)$$

Equation (7) gives the error in optimum size of exploitation for a small error in the L/Wt exponent. The approximate change in t_y following a change in K, M and t_o are given respectively by

$$\triangle t_y = \frac{\frac{nK}{M+nK} - ln\left(1+\frac{nK}{M}\right)}{K^2} \triangle K \quad \ldots \quad (8)$$

$$\triangle t_y = \frac{-n}{M(M+nK)} \triangle M \quad \ldots \quad (9)$$

and $$\triangle t_y = \triangle t_o \quad \ldots \quad (10)$$

Better approximations—Smith (1966) has described methods of obtaining better approximations to any desired level, when only one variable is subject to change. For instance, the equation for the second approximation is given by

$$\triangle y = (D_x y) \triangle x + \tfrac{1}{2}(D_x^2 y)(\triangle x)^2 \quad .. \quad (11)$$

Thus when n changes a better approximation of $\triangle t_y$ is obtained from

$$\triangle t_y = \frac{\triangle n}{M+nK} - \frac{K(\triangle n)^2}{2\,(M+nK)^2} \quad .. \quad (12)$$

where n corresponds to the initial value before any change is effected. Similarly a second approximation of $\triangle t_y$ can be calculated when K or M varies. From equation (3) $D_t\ t_y$ is a constant so that second approximation is not required.

Applying equations (6) and (12) to the North Sea plaice (for data, see Beverton and Holt 1957) when n is taken to be 3.5 instead of 3.0, first approximation of the error in t_y is

$$\triangle t_y = \frac{0.5}{0.1+3.5\times0.095} = 1.15607 \text{ years}$$

when n=3.5, t_y from equation (3) is 14.59995 years so that the corrected t_y is

14.59995−1.15607 years=13.44388 years.

Second approximation of error in t_y using equation (12) is

$$\triangle t_y = \frac{0.5}{0.1+3.0\times0.095} - \frac{0.095\times0.25}{2(0.1+3.0\times.095)^2} \text{ years}$$
$$=1.21859 \text{ years.}$$

Hence, according to second approximation when n is taken to be 3.5 instead of 3.0 t_y is over estimated by 1.21859 years. so that the corrected estimate is

$$14.59995 - 1.21859 \text{ years.} = 13.38136 \text{ years.}$$

The exact value of t_y from equation (3) using $n=3.0$ is 13.37522 years. Thus second approximation shows the error better than the first.

Error when more than one variable changes— Approximate estimation of the error can also be obtained readily when more than one variables are subject to change. Thus, from the equation (3) when K and t_o are made to vary simultaneously,

$$\triangle t_y = \frac{\partial t_y}{\partial K}\triangle K + \frac{\partial t_y}{\partial t_o}\triangle t_o \quad \ldots \quad (13)$$

so that

$$\triangle t_y = \left[\frac{nK}{M+nK} - ln\left(1+\frac{nK}{M}\right)\right]\frac{\triangle K}{K^2} + \triangle t_o \quad \ldots \quad (14)$$

which is also obtained by adding equations (8) and (10). Similarly, the other approximations are obtained by adding the proper differentials. It can be seen that when three or more variables are involved, exact evaluation of the change is easier than finding approximations.

Changes in potential yield—Any change in the potential yield due to changes in appropriate parameters are obtained similarly from the equation (2). Thus taking logarithm and finding differentials, the error in Y following an error in n is given by

$$\triangle Y = Y ln\left\{1-e^{-K(t_y-t_o)}\right\}\triangle n \quad \ldots \quad (15)$$

where Y and t_y may be replaced by the respective functions or by their values obtained by substituting the altered value of n. Estimation of the absolute error using the equation (15) requires the estimation of Y. This can be avoided by estimating the per cent error in Y following a change in n. Thus the percent error in Y is given by

$$\frac{\triangle Y}{Y}\times 100 = 100\, ln\left\{1-e^{-K(t_y-t_o)}\right\}\triangle n \quad \ldots \quad (16)$$

Similarly, the per cent change in Y, following a change in K, M, W_∞, t_o and t_y are respectively

$$\frac{\triangle Y}{Y}\times 100 = \frac{100n(t_y-t_o)e^{-K(t_y-t_o)}}{1-e^{-K(t_y-t_o)}}\triangle K \quad \ldots \quad (17)$$

where t_y is the optimum age of exploitation for the altered value of K,

$$\frac{\triangle Y}{Y}\times 100 = 100\,(t_p - t_y)\triangle M \quad \ldots \quad (18)$$

$$\frac{\triangle Y}{Y}\times 100 = \frac{100\triangle W_\infty}{W_\infty} \quad \ldots \quad (19)$$

$$\frac{\triangle Y}{Y} \times 100 = \frac{-100Kne^{-K(t_y-t_0)}}{1-e^{-K(t_y-t_0)}} \triangle t_0 \qquad \ldots \qquad (20)$$

and

$$\frac{\triangle Y}{Y} \times 100 = \left(\frac{100Kne^{-K(t_y-t_0)}}{1-e^{-K(t_y-t_0)}} - M \right) \triangle t_y \qquad \ldots \qquad (21)$$

There is no advantage to find the equations for approximate estimations of the changes in Y when more than one variable changes, since the resulting equation is not simpler than the equation (2).

In order to examine the accuracy of approximate estimations an example has been worked out below by estimating the value of Y, using the calculated per cent error for the North Sea plaice. If W_∞ is taken to be 3000g instead of 2867 g, $\triangle W_\infty = 133$ g, and the per cent error in the potential yield using equation (19) is=4.43333. This means that Y estimated from the equation (2) using $W_\infty = 3000$g, will be 4.3333 per cent higher than the actual value. The value of potential yield/recruit (Y/R) when W_∞ is put 3000 g is 463.3831. Hence the corrected $\frac{Y}{R}$ is

$$463.3831 - 463.3831 \times 0.0443333 \text{ g} = 442.8398 \text{ g}$$

The exact value of $\frac{Y}{R}$ from equation (2) using 2867 for W_∞ is 442.8397 g.

Discussion

Small changes in population parameters have been commonly found in many marine populations. Beverton and Holt (1957, 1959) and Holt (1962) have shown that very often there is a functional relationship between several variables. These authors have pointed out that a relation may exist between the growth parameters and W_∞ or between K and M. In addition, any change in the length-weight exponent might influence W_∞. Thus, the yield, the optimum age of exploitation and the potential yield may all be influenced simultaneously by more than one variable, following a change in any one related variable. This provides the need for a better understanding of the relationships between population parameters.

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