HYDROMAGNETIC STABILITY OF A STRATIFIED FLUID

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The dynamic stability of a rotating viscous hydromagnetic fluid configuration of variable density has been investigated to include the combined influence of the effects of F.L.R. and finite magnetic resistivity. The prevalent magnetic field is assumed to be uniform and vertical. A variational principle is shown to characterize the problem. Making use of the existence of the variational principle, an explicit solution has been obtained for a semi-infinite fluid layer in which the density varies exponentially along the vertical. It is found that the magnetic resistivity has a destabilizing influence while the coriolis forces, viscosity and F.L.R. exhibit the stabilizing character.

INTRODUCTION

The problem of the instability arising in a horizontal layer of an incompressible, inviscid fluid of variable density, stratified in the vertical direction, is well-known in hydrodynamics. This problem was studied by Rayleigh (1883) who showed that the stratification is stable or unstable according as the density decreases everywhere or increases anywhere in the vertically upward direction. Several attempts have been made to investigate the influence of other operative forces, e.g., those due to the presence of a magnetic field, viscosity and coriolis forces, on this stability problem. Chandrasekhar (1961) has given a detailed account of the various investigations of this problem.

It is well-known that the single fluid, idealized hydromagnetic equations are, strictly speaking, valid only in the limit that the Larmor radii of the charged particles are effectively zero and the corresponding Larmor frequencies are regarded as infinitely large. However, in many astrophysical situations such as the solar corona, and interstellar and interplanetary plasmas, the approximation of zero Larmor radius and infinite Larmor frequency is not valid. It, therefore, becomes interesting to examine the modifications to the fluid equations if one relaxes the above approximation.

Rosenbluth *et al.* (1962), Roberts and Taylor (1962) and Jukes (1964) have all pointed out the stabilizing influence of the finite ion Larmor radius, which exhibits itself in the form of a magnetic viscosity in the fluid equations, on plasma instabilities.

Ariel (1971) has investigated the influence of F.L.R. on the Rayleigh-Taylor instability of a rotating hydromagnetic configuration of variable density, in which the prevalent magnetic field is parallel to the direction of gravity. He has considered the case of an inviscid and ideally conducting fluid.

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In this paper the authors have investigated the combined influence of the effects of the finiteness of the ion Larmor radius, coriolis forces and magnetic resistivity on the dynamic stability of a viscous plasma of varying density. A variational principle is shown to characterize the problem and the dispersion relation has been obtained, through the variational methods for a fluid in which the density has an exponential stratification along the vertical direction.

Perturbation Equations

The relevant linearized perturbation equations, governing the motion of an infinite horizontal strata of a viscous, finitely conducting, rotating fluid of variable density, are :

$$\rho \frac{\partial u}{\partial t} = - \nabla \delta\Pi + \frac{1}{4\pi} (\nabla \times h) \times H + 2 \rho (u \times \Omega) + g\delta\rho$$

$$+ \mu \nabla^2 \mu + (\nabla \mu . \nabla) u + (\nabla \mu) . (\nabla u), \quad \ldots (1)$$

$$\frac{\partial}{\partial t} \delta\rho = - (u . \nabla) \rho, \quad \ldots (2)$$

$$\frac{\partial h}{\partial t} \nabla \times (u \times H) + \eta \nabla^2 h, \quad \ldots (3)$$

$$\nabla . u = 0, \nabla . h = 0, \quad \ldots (4)$$

where $u (u, v, w)$, $\delta.\Pi$ $\delta\rho$ and $h (h_x, h_y, h_z)$ denote the perturbations, respectively, in velocity, stress tensor Π, density ρ and magnetic field H, Ω is rotation, $\mu(z)$ is the coefficient of viscosity, $g(0, 0, -g)$ is gravity and η is magnetic resistivity. We assume here that the prevalent magnetic field acts along the vertical direction, i.e., $H = (0, 0, H_0)$. Then the components of stress tensor Π, taking into account the finiteness of the ion Larmor radius, are (Robert & Taylor 1962) :

$$\left.\begin{aligned}
\Pi_{xx} &= p - \rho\nu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right), \\
\Pi_{yy} &= p + \rho\nu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right), \\
\Pi_{zz} &= p \\
\Pi_{yz} &= \pi_{zy} = 2 \rho\nu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \\
\Pi_{zx} &= \pi_{xz} = -2 \rho\nu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \\
\Pi_{xy} &= \pi_{yx} = \rho\nu \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right),
\end{aligned}\right\} \quad \ldots (5)$$

where p is the scalar part of the pressure and $\rho\nu = \frac{NT}{4\omega_H}$, where ω_H is ion gyration frequency while N and T are, respectively, the number density and temperature of the ions.

Analysing in terms of normal modes we seek solutions, of the above equations, whose dependence on x, y, z and t is of the form

$$F(z) \exp\, [ik_x x + ik_y y + nt], \qquad \dots (6)$$

where $F(z)$ is some function of z, k_x and k_y are horizontal wave numbers, $k_x^2 + k_y^2 = k^2$, and n is the frequency of the perturbation. Taking the direction of rotation also to be vertical, i.e., $\Omega = (0, 0, \Omega)$ and using Eq. (6) in Eq. (1) to (4) we obtain :

$$n\,\rho u - 2\,\rho v\Omega = -\,ik_x\delta p - \rho\nu k^2 v + 2\nu D\,[\rho\,(Dv + ik_y w)] + \frac{H_0}{4\pi}(Dh_x - ik_x\,h_z) + \mu\,(D^2 - k^2)\,u + (ik_x w + Du)\,D\mu \qquad \dots (7)$$

$$n\,\rho v + 2\,\rho\,u\Omega = -\,ik_y\delta p + \rho\nu k^2 u - 2\,\nu D\,[\rho\,(Du + ik_x w)] + \frac{H_0}{4\pi}(Dh_y - ik_y h_z) + \mu(D^2 - k^2)\,v + (ik_y w + Dv)\,D\mu\,v \qquad \dots (8)$$

$$n\,\rho\,w = -\,D\delta p - 2\rho\nu D\,(ik_y u - ik_x v) - g\delta\rho + \mu\,(D^2 - k^2)w + 2\,(Dw)\,D\mu \qquad \dots (9)$$

$$\delta\rho = -\,\frac{w\,(D\rho)}{n}, \qquad \dots (10)$$

$$nh_x = H_0 Du + \eta\,(D^2 - k^2)\,h_x, \qquad \dots (11)$$

$$nh_y = H_0 Dv + \eta\,(D^2 - k^2)\,h_y, \qquad \dots (12)$$

$$nh_z = H_0 Dw + \eta\,(D^2 - k^2)\,h_z. \qquad \dots (13)$$

$$ik_x u + ik_y v + Dw = 0, \qquad \dots (14)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \qquad \dots (15)$$

where D stands for $\frac{d}{dz}$.

Eliminating some of the variables from the Eqs. (7) to (15) we finally obtain the following four equations in w, h_z, ζ and ξ :

$$n\,[\rho k^2 w - D(\rho\, Dw)] + (\nu k^2 - 2\Omega)\, D(\rho\zeta) - 2\nu (D^2 + k^2)(\rho D\zeta)$$

$$- gk^2 \frac{w(D\rho)}{n} + \frac{H_0}{4\pi}(D^2 - k^2)\, Dh_z + \mu (D^2 - k^2)^2 w$$

$$+ (D^2\mu)(D^2 + k^2) w + 2\,(D\mu)(D^2 - k^2)\, Dw = 0, \qquad \ldots (16)$$

$$\rho n\zeta - \mu (D^2 - k^2)\zeta - (D\mu)\, D\zeta + \rho\,(\nu k^2 - 2\,\Omega)\, Dw$$

$$- 2\,\nu D\,[\rho\,(D^2 + k^2)\, w] - \frac{H_0}{4\pi}\, D\xi = 0 \qquad \ldots (17)$$

$$[n - \eta\,(D^2 - k^2)]\xi = H_0 D\zeta \qquad \ldots (18)$$

$$[n - \eta\,(D^2 - k^2)] h_z = H_0 Dw \qquad \ldots (19)$$

where ζ and ξ are, respectively, the vertical components of the vectors Curl $\underset{\sim}{u}$ and Curl $\underset{\cdot}{h}$ and are given by

$$\zeta = ik_x v - ik_y u, \quad \xi = ik_x h_y - ik_y h_x \qquad \ldots (20)$$

We have thus a set of four equations [Eqs. (16)—(19)] in four variables w, h_z, ζ and ξ which are to be solved subject to the appropriate boundary conditions, depending upon whether the bounding surfaces are free or rigid.

Boundary Conditions

We assume that the considered fluid is semi-infinite in the sense that it is infinitely extending along the two horizontal directions x and y, and along the vertical it is confined between two free surfaces, at $z = 0$ and $z =$ d. At the free surfaces the tangential stress components P_{xz} and P_{yz} must vanish, leading to the conditions

$$Dh_z = 0,\ \xi = 0 \quad \text{at } z = 0 \quad \text{and} \quad z = d. \qquad \ldots (21)$$

Also, we have, at a free surface

$$D\zeta = 0,\ w = 0. \qquad \ldots (22)$$

Variational Principle

Suppose that the solutions w_i, h_i, ζ_i and ξ_i belong to the characteristic value n_i, and w_j, h_j, ζ_j and ξ_j, belong to the characteristic value n_j (where, for convenience, we have dropped the suffix z on h).

Multiplying Eq. (16) for i by w_j and integrating with respect to z over the whole vertical extent L of the fluid we obtain

$$n_i \int_L [\rho k^2 w_i - D(\rho\, Dw_i)]\, w_j dz + (\nu k^2 - 2\Omega) \int_L [D(\rho \zeta_i)]\, w_j\, dz$$

$$- 2\nu \int_L [(D^2 + k^2)\,(\rho\, D\, \zeta_i)]\, w_j dz - \frac{gk^2}{n_i} \int_L (D\, \rho)\, w_i\, w_j\, dz$$

$$+ \frac{H_0}{4\pi} \int_L [(D^2 - k^2) Dh_i]\, w_j\, dz + \int_L \mu\, [(D^2 - k^2)^2\, w_i]\, w_j\, dz$$

$$+ \int_L (D^2 \mu)\, [(D^2 + k^2)\, w_i]\, w_j\, dz + 2\int_L (D\mu)\, [(D^2 - k^2)\, D\, w_i]\, w_j\, dz = 0. \quad \dots (23)$$

Performing integration by parts and using the boundary conditions, we get

$$n_i \int_L [\rho\, k^2\, w_i\, w_j - D(\rho\, D\, w_i)\, w_j]\, dz = n_i \int_L \rho\, [D\, w_i\, D\, w_j + k^2\, w_i\, w_j]\, dz, \quad \dots (24)$$

$$(\nu k^2 - 2\Omega) \int_L [D\,(\rho\, \zeta_i)]\, w_j\, dz - 2\nu \int_L [D^2 + k^2)\, (\rho\, D\, \zeta_i)]\, w_j\, dz$$

$$= n_j \int_L \rho\, \zeta_i\, \zeta_j\, dz + \int_L \mu[D\, \zeta_i\, D\, \zeta_j + k^2\, \zeta_i\, \zeta_j]\, dz$$

$$+ \frac{n_j}{4\pi} \int_L \xi_i\, \xi_j\, dz + \frac{\eta}{4\pi} \int_L [D\, \xi_i\, D\, \xi_j + k^2\, \xi_i\, \xi_j]\, dz, \quad \dots (25)$$

$$\frac{H^0}{4\pi} \int_L [(D^2 - k^2)\, Dh_i]\, w_j\, dz$$

$$= \frac{n_j}{4\pi} \int_L [Dh_i\, Dh_j + k^2 h_i h_j]\, dz + \frac{\eta}{4\pi} \int_L [(D^2 - k^2)\, h_i]\, [D^2 - k^2)\, h_j]\, dz\,, \quad \dots (26)$$

$$\int_L \mu\, [(D^2 - k^2)^2\, w_i]\, w_j\, dz + \int_L (D^2 \mu)\, [(D^2 + k^2)\, w_i]\, w_j\, dz$$

$$+ 2 \int_L (D\mu)\, [(D^2 - k^2)\, D\, w_i]\, w_j\, dz$$

$$= \int_L \mu[D^2\, w_i\, D^2\, w_j + 2k^2\, Dw_i\, Dw_j + k^4\, w_i\, w_j]\, \mathrm{d}z$$

$$+ k^2 \int_L (D^2\, \mu)\, w_i\, w_j\, dz. \quad \dots (27)$$

Combining (23) to (27) and putting $i = j$ we get

$$n\, (I_1 + I_2) + I_3 + \frac{n}{4\pi} I_4 + \frac{\eta}{4\pi} I_5 - \frac{gk^2}{n}\, I_6 + \frac{n}{4\pi}\, I_7$$

$$+ \frac{\eta}{4\pi} I_8 + I_9 + k^2 I_{10} = 0 \quad \dots (28)$$

where

$$I_1 = \int_L \rho\, [(Dw)^2 + k^2w^2]dz, \qquad \ldots (29)$$

$$I_2 = \int_L \rho\, \zeta^2 dz, \qquad \ldots (30)$$

$$I_3 = \int_L \mu\, [(D\zeta)^2 + k^2\zeta^2]\, dz, \qquad \ldots (31)$$

$$I_4 = \int_L \xi^2 dz, \qquad \ldots (32)$$

$$I_5 = \int_L [(D\xi)^2 + k^2\, \xi^2\,]\, dz, \qquad \ldots (33)$$

$$I_6 = \int_L (D\, \rho)\, w^2 dz, \qquad \ldots (34)$$

$$I_7 = \int_L [(Dh)^2 + k^2h^2]\, dz, \qquad \ldots (35)$$

$$I_8 = \int_L [(D^2-k^2)h]^2\, dz, \qquad \ldots (36)$$

$$I_9 = \int_L \mu[(D^2w)^2 + 2k^2\,(Dw)^2 + k^4w^2]\, dz, \qquad \ldots (37)$$

$$I_{10} = \int_L (D^2\mu)\, w^2\, dz. \qquad \ldots (38)$$

Considering now the change δn in n consequent upon the arbitrary variations δw, δh, $\delta\zeta$ and $\delta\xi$ in the corresponding physical quantities w, h, ζ and ξ, compatible with the boundary conditions, and retaining only the linear terms in the small quantities δw, δh, etc., we have

$$-\delta n\, [I_1 + I_2 + \frac{1}{4\pi} I_4 + \frac{gk^2}{n^2} I_6 + \frac{1}{4\pi} I_7]$$

$$= n\,(\delta I_1 + \delta I_2) - \delta I_3 + \frac{n}{4\pi}\, \delta I_4 - \frac{\eta}{4\pi}\, \delta I_5 - \frac{gk^2}{n} \delta I_6$$

$$-\frac{n}{4\pi}\, \delta I_7 + \frac{\eta}{4\pi}\, \delta I_8 + \delta I_9 + k^2 \delta I_{10}, \qquad \ldots (39)$$

where

$$\tfrac{1}{2}\, \delta I_1 = \int_L [\rho\, k^2 w - D\,(\rho\, Dw)]\delta w dz, \qquad \ldots (40)$$

$$\tfrac{1}{2}\, \delta I_2 = \int_L \rho\zeta\, \delta\zeta\, dz \qquad \ldots (41)$$

$$\tfrac{1}{2}\,\delta I_3 = \int_L \zeta\,[\mu(D^2-k^2)\,\delta\zeta + (D\mu)\,D\,(\delta\zeta)]\,dz, \qquad \ldots (42)$$

$$\tfrac{1}{2}\,\delta I_4 = \int_L \xi\,\delta\xi\,dz, \qquad \ldots (43)$$

$$\tfrac{1}{2}\,\delta I_5 = \int_L [(D^2-k^2)\,\xi]\,\delta\xi\,dz, \qquad \ldots (44)$$

$$\tfrac{1}{2}\,\delta I_6 = \int_L w\,(D\rho)\,\delta w dz, \qquad \ldots (45)$$

$$\tfrac{1}{2}\,\delta I_7 = \int_L [(D^2-k^2)h]\,\delta h dz, \qquad \ldots (46)$$

$$\tfrac{1}{2}\,\delta I_8 = \int_L [(D^2-k^2)^2 h)\,\delta h dz, \qquad \ldots (47)$$

$$\tfrac{1}{2}\,\delta I_9 = \int_L [\mu(D^2-k^2)^2 w + (D^2\mu)\,D^2 w + 2\,(D\mu)(D^2-k^2)Dw]\delta w dz, \qquad \ldots (48)$$

$$\tfrac{1}{2}\,\delta I_{10} = \int_L (D^2\mu)w\;\delta w dz. \qquad \ldots (49)$$

The integrals δI_1 to δI_{10} follow from I_1 to I_{10}, on performing integration by parts. Also the perturbations δw, δh, $\delta\zeta$ and $\delta\xi$ are connected by the relations [see Eqs. (17) to (19)]

$$\rho\delta n\,\zeta + \rho n\delta\zeta - \mu(D^2-k^2)\,\delta\zeta - D\mu\;D\delta\zeta + \rho(\nu k^2 - 2\Omega)D\delta w$$

$$-2\nu\,D[\rho\,(D^2+k^2)\,\delta w] - \frac{H_0}{4\pi}D\delta\xi = 0, \qquad \ldots (50)$$

$$\delta n\;\xi + n\delta\xi = -\eta\,(D^2-k^2)\delta\xi = H_0 D\delta\zeta \qquad \ldots (51)$$

$$\delta n\;h + n\delta h - \eta\,(D^2-k^2)\delta h = H_0 D\delta w\;. \qquad \ldots (52)$$

Combining Eqs. (39) to (52) and simplifying, we obtain

$$-\tfrac{1}{2}\,\delta n\,[I_1 - I_2 + \frac{1}{4\pi}I_4 + \frac{gk^2}{n^2}I_6 - \frac{1}{4\pi}I_7]$$

$$= \int_L [n\{\rho k^2 w - D(\rho Dw)\} + (\nu k^2 - 2\Omega)\,D(\rho\zeta) - 2\nu(D^2+k^2)\,(\rho D\zeta)$$

$$-\frac{gk^2}{n}w(D\rho) + \frac{H_0}{4\pi}\,(D^2-k^2)\,Dh + \{\mu(D^2-k^2)^2 w +$$

$$(D^2\mu)\,(D^2+k^2)w + 2(D\mu)\,(D^2-k^2)Dw\}\;\delta w\;dz. \qquad \ldots (53)$$

We observe that the quantity occuring as the coefficient of $\delta\omega$ under the sign of integration in Eq. (53) vanishes if Eq. (16) is satisfied. Thus a necessary and sufficient condition that δn be zero to the first order for small arbitrary variations δw, δh, etc. [connected by the relations (50) to (52)] and compatible with the boundary conditions, is that w, h, etc. be the solutions of the characteristic value problem. Thus a variational procedure for obtaining the approximate solution of the present problem is possible.

A Fluid Layer of Variable Density

We now make use of the existence of variational principle to obtain the explicit solution of the problem of a continuously stratified fluid of depth d in which the undisturbed density ρ varies exponentially along the vertical, i.e.,

$$\rho = \rho_0 \exp(\beta z) , \qquad \dots (54)$$

where ρ_0 is the density at the lower boundary and β is a constant. We assume that μ is also stratified exponentially along the vertical, i.e.,

$$\mu = \mu_0 \exp(\beta z), \qquad \dots (55)$$

where μ_0 is the coefficient of viscosity at $z = 0$.
We also assume that

$$| \beta d | << 1, \qquad \dots (56)$$

which implies that the difference between the densities at any two neighbouring points is much less than the average density.

Appropriate to the boundary conditions (21) to (22), let us take the trial solutions for w, h, ζ and ξ as

$$\left.\begin{aligned} w(z) &= A \sin lz, \quad h(z) = B \cos lz, \\ \zeta(z) &= C \cos lz, \quad \xi(z) = F \sin lz, \end{aligned}\right\} \qquad \dots (57)$$

where A, B, C and F are constants and $l = \frac{\pi m}{d}$, m being an integer.

Substituting these trial solutions (57) in Eq. (28) and eliminating the constants A, B, C and F with the help of Eqs. (17) to (19) we get, after some suitable simplifications, the following dispersion relation :

$$n^5 + n^4[2(l^2 + k^2)(\eta + \nu_0)]$$

$$+ n^3\left[\left\{ 2\left(l^2 V_0^2 + \eta\nu_0 \ (l^2 + k^2)^2\right) - \frac{g\beta k^2}{l^2 + k^2}\right\}\right.$$

$$\left. + \frac{\{2\Omega + \nu(k^2 - 2l^2)\}^2 l^2}{(l^2 + k^2)} + (l^2 + k^2)^2 (\eta + \nu_0)^2\right]$$

$$+ n^2 \left[\left\{ 2\left(l^2 + V_0^2 + \eta\nu_0 (l^2 + k^2)^2 \right) - \frac{g\beta k^2}{l^2 + k^2}\right\} (l^2 + k^2)(\eta + \nu_0)\right.$$

$$\left. - g\beta k^2\eta + 2\left\{ 2\Omega + \nu (k^2 - 2l^2)\right\}^2 l^2\eta \right]$$

$$+ n\left[\left(l^2V_0^2 + \eta\,\nu_0\,(l^2 + k^2)^2\right)\left\{ l^2V^2_0 + \eta\nu_0\,(l^2 + k^2)^2 - \frac{g\beta k^2}{l^2 + k^2}\right\}\right.$$

$$\left. - g\beta k^2\eta\,(\eta + \nu_0)(l^2 + k^2) + \left\{2\Omega + \nu(k^2 - 2l^2)\right\}^2 l^2 (l^2 + k^2)\,\eta^2\right]$$

$$- \left[g\beta k^2\eta\left\{ l^2 V_0^2 + \eta\nu_0 (l^2 + k^2)^2 \right\}\right] = 0, \qquad \ldots (58)$$

where $V_0 = H_0/\sqrt{4\pi\rho_0}$ is the Alfven Velocity and $\nu_0 = \mu_0/\rho_0$.

Discussion

(*i*) *Stable Stratification* ($\beta < 0$)

Applying Hurwitz's criterion to Eq. (58) for the case $\beta < 0$ we find that, as all the terms of this equation are then positive, all the roots of n are either real and negative, or there is one negative real root and the remaining roots are complex with negative real parts, implying stability in each case. Thus the system remains stable whether the effects of viscosity, finite conductivity, rotation and F.L.R. are included separately or jointly.

(*ii*) *Unstable Stratification* ($\beta > 0$)

Making the substitutions

$$\left.\begin{aligned} \sigma &= \frac{n}{lV_0},\ x = \frac{k}{l},\ B = \frac{g\beta}{l^2V_0^2}, \\ E &= \frac{\eta l}{V_0},\ S = \frac{\Omega}{lV_0}, \\ N &= \frac{\nu l}{V_0},\ N_0 = \frac{\nu_0 l}{V_0}, \end{aligned}\right\} \qquad \ldots (59)$$

in Eq. (58) we get its non-dimensional form as

$$\sigma^5 + \sigma^4 \left[2(E + N_0)(1 + x^2\right] + \sigma^3 \left[\left\{2\left(1 + EN_0 (1 + x^2)^2\right)\right.\right.$$

$$- \frac{B x^2}{1+x^2}\Big\} + \frac{(N (x^2 - 2)+ 2S)^2}{1 + x^2} + (1+x^2)^2 (E + N_0)^2 \Big]$$

$$+ \sigma^2 \Big[\Big\{ 2 \Big(1 + E\, N_0 (1 + x^2)^2 \Big) - \frac{Bx^2}{1 + x^2} \Big\} (1 + x^2)\, (E + N_0)$$

$$- BE x^2 + 2E \Big(N (x^2 - 2) + 2S\Big)^2 \Big]$$

$$+ \sigma \Big[\Big\{1 + E N_0\, (1 + x^2)^2 \Big\}\Big\{ 1 + E N_0 (1 + x^2)^2 - \frac{B x^2}{1 + x^2} \Big\}$$

$$- B\, E (E + N_0)\, x^2 (1 + x^2) + \Big(N (x^2 - 2) + 2S \Big)^2 (1+x^2)\, E^2 \Big]$$

$$- B\, E\, x^2 \Big(1 + E\, N\, (1+ x^2)^2\Big) = 0. \qquad \dots (60)$$

Here the parameters E, N_0, S, B and N measure, respectively, the effects of finite conductivity, viscosity, rotation, buoyancy foices and F.L.R., in terms of the magnetic field.

Before we discuss the dispersion relation (60) let us first consider some of its special cases.

First Special Case : $E = 0$, $N_0 = 0$ — For the case of no viscosity and infinite electrical conductivity the dispersion relation (60) reduces to the quartic equation

$$\sigma^4 + \sigma^2 \Big[2 + \frac{(N (x^2 - 2) + 2S)^2 - B\, x^2}{1 + x^2} \Big] + \Big(1 - \frac{B\, x^2}{1 + x^2}\Big) = 0. \qquad \dots (61)$$

This case has been discussed by Ariel (1971). From Eq. (61) it follows that the configuration is unstable for all wave numbers larger than x^* where x^* is given by

$$x^* = (B - 1)^{-1/2}. \qquad \dots (62)$$

Clearly x^* is independent of the effects of rotation and F.L.R.

Second Special Case : $E = 0$ — For an infinitely conducting viscous fluid equation (60) reduces to

$$\sigma^4 + 2N_0 (1 + x^2)\, \sigma^3 + \sigma^2 \Big[N_0^2 (1 + x^2)^2 + 2 + \frac{(N(x^2-2)+2S)^2 - B\, x^2}{1+x^2} \Big]$$

$$+ \sigma N_0 (1 + x^2) \Big(2 - \frac{B\, x^2}{1+x^2}\Big) + \Big\{1 - \frac{B\, x^2}{1+x^2}\Big\} = 0. \qquad \dots (63)$$

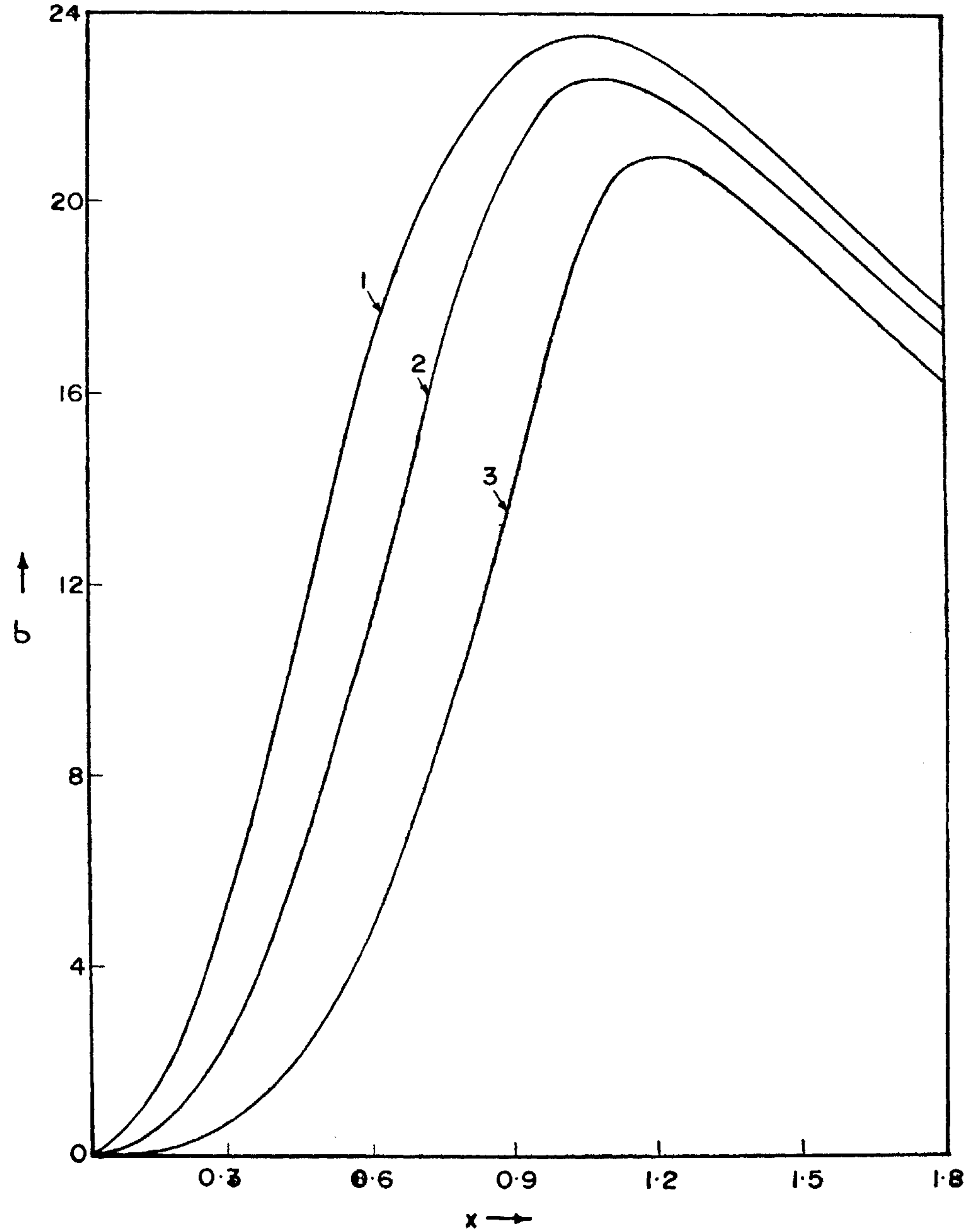

FIG. 1. Plot of growth rate σ (real positive values) against wave number x for different values of the parameters E, N S and N_0 taking $B = 5$. The curves 1 – 3 correspond to the values :

(1) $E = 1$, $N_0 = 5$, $S = 1$, $N = 2$

(2) $E = 1$, $N_0 = 5$, $S = 1$, $N = 5$

(3) $E = 1$, $N_0 = 5$, $S = 1$, $N = 10$

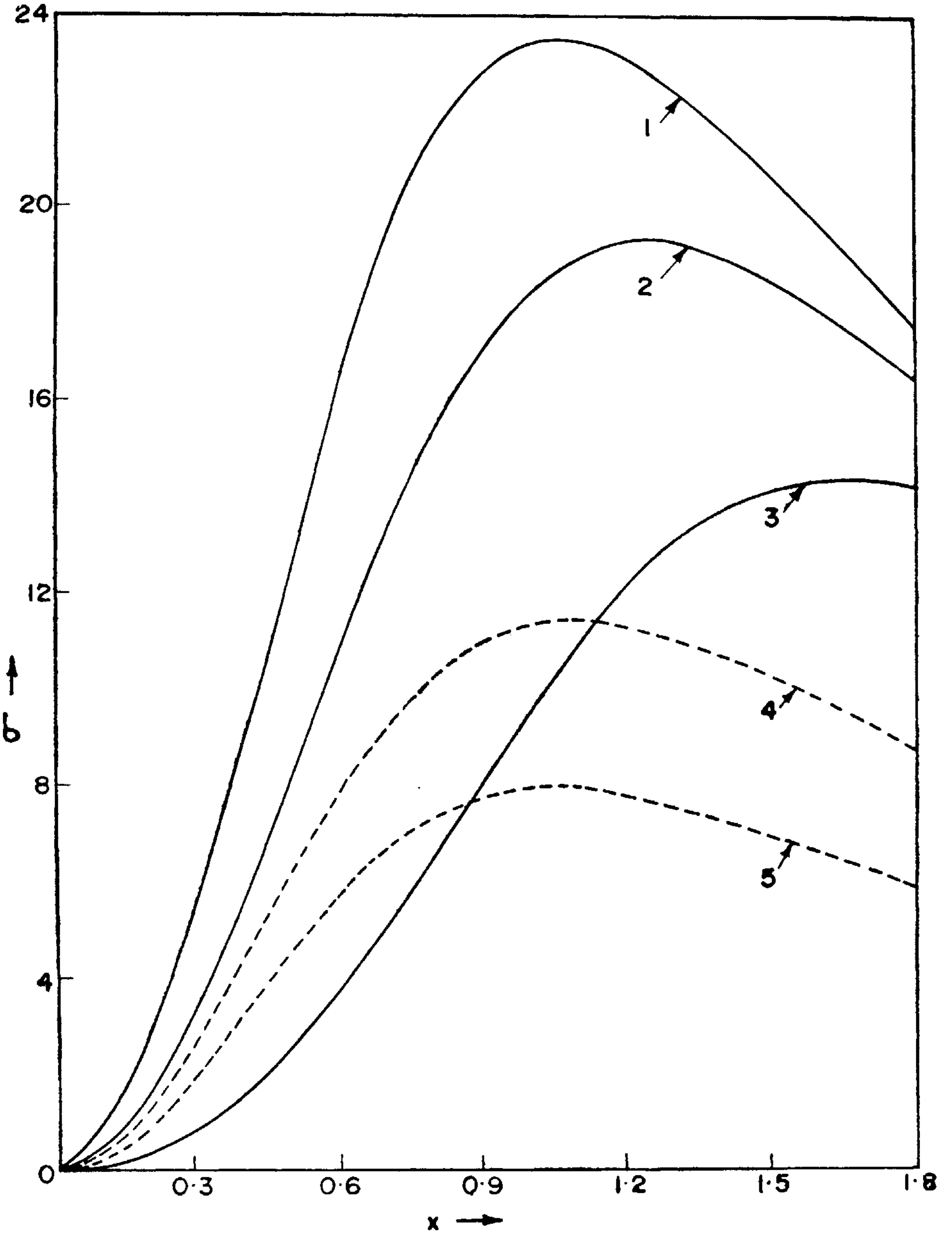

FIG. 2. Plot of growth rate σ against wave number x for different values of the parameters E, N_0, S and N, taking $B = 5$. The curves 1 – 5 correspond to the values :

(1) $E = 1, \quad N_0 = 5, \quad S = 1, \quad N = 2$

(2) $E = 1, \quad N_0 = 5, \quad S = 5, \quad N = 2$

(3) $E = 1, \quad N_0 = 5, \quad S = 10, \quad N = 2$

(4) $E = 1, \quad N_0 = 10, \quad S = 5, \quad N = 2$

(5) $E = 1, \quad N_0 = 15, \quad S = 5, \quad N = 2$

Applying Hurwitz's criterion to this equation we find that the system is stable in this case also for all wave numbers smaller than x^* given by (62). Thus we find that the stability criterion is not affected even by the inclusion of the effect of viscosity.

General Case — Applying Hurwitz's criterion to Eq. (60) we find that the dispersion relation, being a polynomial of the fifth degree, must possess at least one real root, the sign of which is opposite to that of the last term in it. Since the last term is always negative for $\beta > 0$, Eq. (60) possesses at least one positive real root for all wave numbers x. The considered system is consequently unstable for all wave numbers. Thus we find that the effect of the inclusion of finite electrical conductivity is destabilizing as it renders the system unstable in the wave number range $x < x^*$. We thus see that, irrespective of whether or not the effects of F.L.R., rotation and viscosity, the configuration is unstable at all wave numbers if electrical conductivity is finite.

To study the influence of the effects of finite ion Larmor radius, viscosity and rotation on the growth rate, numerical calculations were performed to locate the roots of σ against x, from Eq. (60), for several values of the parameters involved. These calculations are presented in Figs. 1 and 2, where we plot the growth rate (positive real values of σ) against wave number x, for $E = 1$; $N_0 = 5, 10, 15$; $S = 1, 5, 10$ and $N = 2, 5, 10$. taking $B = 5$. From Fig. 1, we see that as the parameter N (F.L.R.) increases, σ (the growth rate) decreases for fixed values of other parameters. The influence of F.L.R. is, therefore, stabilizing. We also observe from Fig. 2 that the growth rate decreases as either S (rotation) or N_0 (viscosity) or both increase. This shows that the influence of the coriolis forces as well as that of viscosity is stabilizing.

We may thus conclude that the effect of finite electrical conductivity is destabilizing. On the other hand the effects of F.L.R., rotation and viscosity are found to be stabilizing.

A physical reasoning for the apparent ineffectiveness of the finite Larmor radius to modify the stability criterion (62) can be given as follows. For the magnetic field acting along the vertical direction, the ion Larmor motion is in a horizontal plane, and hence it is not likely to play any part in any displacement of the particles between the points of different densities. On the other hand, for a horizontal orientation of the magnetic field, as Ariel and Bhatia (1969, 1970) have shown, the transverse perturbations are stabilized by F.L.R., for in this case the ion Larmor motion decisively displaces the particles between points of different densities.

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