# STUDY OF THE INTERNAL STRUCTURE OF A PLANET IN GENERAL RELATIVITY

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In the present paper, we shall study the internal structure of a single zone planet under the assumptions that (*i*) the internal distribution of matter is homogeneous and preserves the spherical symmetry, (*ii*) the planet is in steady state under gravitational equilibrium and (*iii*) the total pressure $P$ and density $\rho$ of matter at any point of the planet obey a power law of the form $P=K\rho^{1-1/n}-s$.

## INTRODUCTION

It is well-known that if a planet is to resemble in constitution with that of the Earth, which is considered as four concentric spheres, then the pressure and density are related by a power law (an equation of state, including that of incompressible matter) :

$$P = K\rho^{b} - s, \qquad \text{...(1)}$$

where

$$K = \frac{a}{b}\,\rho_u^{-(1+n-i)}\,;\; s = \frac{a}{b} \text{ and}$$

$$n = \text{polytropic index} = \frac{1}{b-1}, \qquad \text{...(2)}$$

where $a$, $b$ and $\rho_u$ (an effective uncompressed mass density $\rho$ for which $P = 0$) are constants having prescribed values in different zones. The density of matter at any point depends essentially on the pressure together with the relevant constants for the zones with which we are concerned. It does not seem necessary to give here the details of a planet pertaining to periods of the growth of the core: the total gravitational potential energy $\Omega$; and the gradual (or rapid) formation of the core etc. ,as the same have been dealt with by Runcorn (1962, 1965), Stacey (1963), Birch (1965), Shimazu (1965), and Fujii and Uyeda (1966), Urey (1952), and Iriyama (1970). The author (Sharma 1972, 1973) has recently studied the "planetary structures in general relativity" in general and the "internal structure of Mars in general relativity" in particular with a view to extend Lyttleton's (1963) studies of planets from classical to general relativity. Planets' composition are such that the relation (2) holds; the energy density $\epsilon$ of matter satisfies the relation (Tooper 1964)

$$\epsilon = \rho' c^2 \qquad \text{...(3)}$$

where $\rho'$ is the total mass density and $c^2$ is the square of the velocity of light. Here we shall study the structure of a planet assuming that the energy density, the rest mass density and the pressure $P$ are related by the equation (Tooper 1965)

$$\epsilon = \rho c^2 + nP, \qquad ...(4)$$

where $P$ defines the total pressure (not necessarily gas pressure). The material undergoes adiabatic compression as the central region of the original earth (which attains a temperature of 1200°K). The importance and usefulness of the treatment of the present problem would be more clear if we try to go through two papers "chemical evidence relative to the origin of solar system" (Urey 1966) and "thermal convection within the earth's mantle" (Iriyama 1971).

The field equations of the problem have been given in the next section. In the third section, we shall deduce Emden's equations of relativity in dimensionless form which will be followed by homology transformations (as applied to the relativistic Lane-Emden equations giving rise to another class of solutions) and solutions of the relativistic Lane-Emden equations. A few parametric quantities, which describe the structure of a planet, have been given in the section titled, "Parameters.... configurations", Section on "Energies of the Planet" contains derivations for the energies of the planet. The central phase-velocity of sound has been discussed in the last section.

## Field Equations of the Problems

For a spherically symmetric distribution of matter, devoid of any rotation and magnetic field, space-time metric is of the form

$$-ds^2 = -e^{\nu} c^2 dt^2 + r^2 (d\theta^2) + \sin^2 \phi \, d\phi^2) + e^{\lambda} dr^2, \qquad .. (5)$$

where according to the assumption ($ii$), $\nu$ and $\lambda$ are functions of $r$ only. The Einstein field equations, appropriate to the metric (5), are

$$e^{-\lambda}\left(\frac{1}{r}\frac{d\nu}{dr} + \frac{1}{r^2}\right) - \frac{1}{r^2} = \frac{KP^2}{c^2}, \qquad ...(6)$$

$$e^{-\lambda}\left(\frac{1}{r}\frac{d\lambda}{dr} - \frac{1}{r^2}\right) + \frac{1}{r^2} = K\rho, \qquad ...(7)$$

and

$$\frac{e^{-\lambda}}{2}\left\{\frac{d^2\nu}{dr^2} + \frac{1}{2}\left(\frac{d\nu}{dr} - \frac{d\lambda}{dr}\right)\left(\frac{d\nu}{dr} + \frac{2}{r}\right)\right\} = \frac{KP}{c^2}, \qquad ...(8)$$

where $K = 8\pi G/c^2$, $G = 6.67 \times 10^{-8} gm^{-1} cm^3 \sec^2$ is the Newtonian constant of gravitation. The metric components $e^{-\lambda}$ and $e^{\nu}$ are defined by (Schwarzschild exterior solutions)

$$\left.\begin{aligned} P &= \rho c^2 = 0 \\ e^{\nu} &= e^{-\lambda} = 1 - \frac{2GM_r}{c^2 r} \end{aligned}\right\} \qquad (r \geqslant R), \qquad ...(9)$$

$M_r = M$ (total mass at $r = R$). The non-vanishing $r$-component of the energy-momentum tensor is

$$\frac{dp}{dr} + 0.5\,(P + \epsilon)\,\frac{d\nu}{dr} = 0. \qquad ...(10)$$

## Relativistic Lane-Emden Equations with Possible Solutions

We can discuss the planetary structures in general relativity by first inserting Eq. (1) into fundamental equation of hydrostatic support [Tolman-Oppenheimer-Volkoff(TOV) equation]

$$r^2\,\frac{dP}{dr} = -\,\frac{G}{c^2}\,\frac{\left(M_r + \dfrac{4\pi\, r^3 P}{c^2}\right)(P + \epsilon)}{\left(1 - \dfrac{2G\, M_r}{c^2 r}\right)}. \qquad ...(11)$$

The mass is defined by

$$\frac{dM_r}{dr} = 4\,\pi\, r^2\,\frac{\epsilon}{c^2}\,. \qquad ...(12)$$

Let us define the three new dimensionless variables $\xi$, $\theta$ and $\nu$ by the equations,

$$r = \xi\, N^{-1}; \qquad N^2 = \frac{4\,\pi\, G\, \lambda_1}{(n+1)\,\alpha c^2}\,; \qquad ...(13)$$

$$\rho = \lambda_1\, \theta^n;\ \text{and}\ M_r = \frac{4\pi\, \lambda_1}{N^3}\, \nu\,(\xi), \qquad ...(14)$$

where $\xi$, $\lambda_1$, $\theta$ and $\nu\,(\xi)$ denote the "reduced" radius, central density, "reduced" temperature and 'reduced" mass respectively, then Eqs. (1), (2), (4) and first equation in (14) transform Eq. (10) to the form

$$2\,\alpha\,(n+1)\,\frac{d\theta}{dr} + [(\alpha\theta - \beta\theta^{-n})\,(n+1) + 1]\,\frac{d\nu}{dr} = 0, \qquad ...(15)$$

where

$$\alpha = \frac{K\lambda_1^{1/n}}{c^2}; \quad \beta = \frac{s}{c^{2\,(n+1)}}\left(\frac{K}{\alpha}\right)^n = \frac{s}{\lambda_1\, c^2}\,. \qquad ..(16)$$

With the help of Eqs. (1), (2), (4), (13), (14), and (16), we deduce, from Eqs. (11) and (12), the fundamental equations of the problem :

$$\frac{1 - 2\alpha\,(n+1)\,\nu/\xi}{1 + (n+1)(\alpha\theta - \beta\theta^{-n})}\,\xi^2\,\frac{d\theta}{d\xi} + \nu(\xi) + \alpha(1 - \alpha^{-1}\,\beta\theta^{-n-1})\,\xi^3\,\theta^{n+1} = 0, \qquad ...(17)$$

and

$$\frac{dv}{d\xi} = \xi^2\theta^n \left[1+n\left(\alpha\theta-\beta\theta^{-n}\right)\right] \qquad \text{...(18)}$$

known as the Emden's equations of relativity indices $\alpha$, $\beta$ and polytropic index $n$. First equation in (16) defines the ratio of $P_c + s$ to $c^2$. For both $\alpha$ and $\beta$ tending to zero non-relativistic case obtains, then we have the following classical Emden's differential equation of index $n$ :

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = \theta^n. \qquad \text{...(19)}$$

Equations (17) and (18) can be solved (provided values of $\alpha$, $\beta$ and $n$ are known) subject to the boundary conditions

$$\left.\begin{array}{l} \theta(0) = 1;\ \dfrac{d\theta}{d\xi} = 0, \quad v(0) = 0 \text{ at } \xi=0 \\ \qquad \theta(\xi_1) = 0 \text{ at } \xi = \xi_1. \end{array}\right\} \qquad \text{...(20)}$$

(*i*) *Place of homology transformations* — The homology theorem for non-relativistic Lane-Emden equation makes it clear that if $\theta(\xi)$ is a solution of the Lane-Emden equation of index $n$, then $A_1^{2/n-1}\,\theta(A_1\xi)$ is also a solution. Hence, accordingly, in our case, the following transformation laws must hold :

$$\xi\rightarrow A_1\xi;\ \theta\rightarrow A_2\theta;\ v=A_3\,v,$$

where the scalar constants $A_1$, $A_2$ and $A_3$ are related with one another by the equations

$$A_3 = A_1A_2 = A_1^3\ A_2^n = A_1^{3-n/1-n}, \qquad \text{...(21)}$$

$$A_2 = A_1^{2/1-n}. \qquad \text{...(22)}$$

Under such a scale transformation the relativistic Lane-Emden equations, of indices $n$, $\alpha$ and $\beta$, give rise to another class of solutions with indices $n$, $\alpha A_2$ and $BA_2^{-n}$. Incidentally, the same law also holds for the relativistic Lane-Emden Eqs. (23) and (24) of author's previous work (Sharma 1972).

(*ii*) *Solutions of the relativistic Lane-Emden equation in closed form for index* $n = 0$ (*case of incompressibility of matter*) — It does not seem possible to obtain closed solutions of Eqs. (17) and (18) for non-zero values of $n$; for $n = 0$ solutions of (18) and (17) are given by

$$3v = \xi_3 \qquad \text{...(23)}$$

$$\alpha\theta = \frac{(1-\beta)(3\alpha-3\beta+1)(1-\frac{2}{3}\alpha\,\xi^2)^{1/2}-(1-3\beta)(\alpha-\beta+1)}{3(\alpha-\beta+1)-(1-\frac{2}{3}\alpha\xi^2)^{1/2}(3\alpha-3\beta+1)}\,. \qquad \ldots(24)$$

The value of $\xi_1$ is given by

$$\xi_1 = \left[\frac{3}{2\alpha} - \left\{\frac{(1-3\beta)(\alpha-\beta+1)}{3(\alpha-\beta)+1}\right\}^2 \frac{3}{2\alpha(1-\beta)^2}\right]^{\frac{1}{2}}. \qquad \ldots(25)$$

From Eq. (1), first equation in Eqs. (14), (16) and (24), we obtain

$$\frac{P}{\rho_c\,c^2} + \beta = \alpha\beta = \frac{(1-\beta)(3\alpha-3\beta+L)(1-Y)^{1/2}-(1-3\beta)(\alpha-\beta+1)}{3(\alpha-\beta+1)-(1-Y)^{1/2}\,(3\alpha-3\beta+1)}\,, \qquad \ldots(26)$$

$\gamma$ being proportional to the square of $\xi$; that is,

$$\gamma = \tfrac{2}{3}\,\alpha\xi^2 = \delta\,r^2, \delta = \frac{8\pi\,G\,\rho_c}{3c^2}\,. \qquad \ldots(27)$$

The central pressure is given by

$$\left(\frac{P}{\rho_c\,c^2}\right)_c = \alpha-\beta, \qquad \ldots(28)$$

where suffix $c$ refers to the central value.

As the boundary of the planet is reached, pressure, density and temperature tend to vanish, hence

$$y^2 = \gamma = 1 - \frac{1}{(1-\beta)^2}\left\{\frac{(1-3\beta)\,(\alpha-\beta+1)}{3(\alpha-\beta)+1}\right\}^2 = \frac{8\pi\,G\,\rho_c}{3c^2}\,R^2 \qquad \ldots(29)$$

for $\xi = \xi_1$, $r = R$. The total gravitational mass is given by

$$M = \frac{4}{3}\,\pi\,\rho\left(\frac{\alpha c^2}{4\pi\,G\rho_c}\right)^{\frac{3}{2}}\left[\frac{3}{2\,\alpha} - \left\{\frac{(1-3\beta)\,(\alpha-\beta+1)}{3(\alpha-\beta)+1}\right\}^2\frac{3}{2\alpha(1-\beta)^2}\right]^{\frac{3}{2}}$$

$$= \left(\frac{3c^6}{32\pi\,G^3\,\rho_c^{\,3}}\right)^{\frac{1}{2}}\,\rho\left[1 - \frac{1}{(1-\beta)^2}\left\{\frac{(1-3\beta)(\alpha-\beta+1)}{3(\alpha-\beta)+1}\right\}^2\right]^{\frac{3}{2}}. \qquad \ldots(30)$$

From the above closed solution for $n = 0$, we infer that for the case of liquid core, $b$ assumes very large value. Consequently the value of $a$ would also change. This change can be ascribed to the general relativistic effects that are being taken into account.

The proper mass (or baryon number $L$) is expressed as

$$m_N'L = \int\frac{4\pi\,r^2 m_N'\,\bar{n}\,dr}{[1-2G\,m_r/c^2r]^{1/2}}\,, \qquad \ldots(31)$$

the integration of which yields

$$2\pi \ m_N' \bar{n} \left(\frac{3\rho}{8\pi G}\right)^{\frac{3}{2}} [\text{Sin}^{-1} \gamma - \gamma (1-\gamma^2)]. \quad \ldots(32)$$

By applying the scale transformation law $R \to A_4 R$, $M \to A_4 M$, we obtain

$$\frac{2GM}{c^2R} = \gamma^2 \text{ (an invariant quantity)}, \quad \ldots(33)$$

which would mean

$$\rho \to A_4^{-2} \rho ; \bar{n} \to A_4^{-2} \bar{n}, \quad \ldots(34)$$

which shows that [as is clear from Eq. (32)]

$$L \to A_4^{-3} L.$$

Hence, from the scale transformation law, we infer that the planet gets transformed into one of different baryon numbers.

Since for non-zero values of $n$ it is not possible to obtain solutions of Eqs. (17) and (18) in closed form, one may take resort to the methods of numerical integrations. This is performed by following the well-known Runge-Kutta method. For given values of the constants $n \left(= 2/5, = \frac{5}{17}\right)$, $\alpha$ and $\beta$, numerical integration would start from centre $\xi = 0$ (where $\theta = 1$, $\nu = 0$) onwards by proceeding by finer small intervals, the values of $\xi$, $\theta(\xi)$, $\nu(\xi)$, $d\theta/d\xi$, $d\nu/d\xi$ are tabulated till $\theta(\xi)$ attains the value $\theta(\xi_1)$ (which tends to zero) At this point, we have $\xi = \xi_1$, $\nu(\xi) = \nu(\xi_1)$, $d\theta/d\xi = (d\theta/d\xi)_1$ and $(d\nu/d\xi) = (d\nu/d\xi)_1$. The author, however, is not interested in numerical integrations of the field equations.

## Parameters Describing the Structure of Panetary Configurations

(*i*) *The mass-radius relation* — For given values of $\alpha$(or $\beta$) and $n$, the ratio of gravitational radius to the coordinate radius, as deduced from Eqs. (13), (14) and (16), with the boundary conditions $\xi = \xi_1$ at $r = R$, is given by

$$\frac{2\,GM}{c^2R} = 2\alpha\,(n+1)\ \frac{\nu(\xi_1)}{\xi_1}. \quad \ldots(35)$$

The expression (35) can be re-expressed as

$$\frac{GM}{c^2R} = \alpha(n+1)\ \frac{\nu(\xi_1)}{\xi_1}, \quad \ldots(35a)$$

where

$$\bar{r} = \int_0^r e^{\lambda(r)/2}\, dr$$

$$e^{-\lambda(r)} = \frac{\xi - 2\alpha\,(n+1)\, v\,(\xi)}{\xi}$$

$$\bar{\xi} = \int_0^{\xi} \left[ \frac{\xi - 2\alpha\,(n+1)\, v\,(\xi)}{\xi} \right]^{-\frac{1}{2}} d\xi; \text{ for } \bar{r} = \bar{R},\ \bar{\xi}_1 = N\bar{R}. \qquad \ldots(36)$$

A particular case of interest. for the incompressible matter (liquid core), is obtained by putting $n = 0$ in equations (35), (35*a*) and (36).

(*ii*) Expressions for the radius $r$ (or $R$) and mass $M(r)$ (or $M$), which are directly proportional to $\xi$ (or $\xi_1$) and $v\,(\xi)$ (or $v\,(\xi_1)$, are

$$r = B\alpha^{\frac{1-n}{2}}\, \xi = B_1\, \beta^{\frac{n-1}{2n}}\, \xi;\ R = B\alpha^{\frac{1-n}{2}}\, \xi_1 = B_1\, \beta^{\frac{n-1}{2n}}\, \xi_1 \qquad \ldots(37)$$

and

$$M(r) = D\alpha^{\frac{3-n}{2}}\, v(\xi) = D_1\, \beta^{\frac{n-3}{2n}}\, v(\xi);\ M(R) = D\alpha^{\frac{3-n}{2}}\, v(\xi_1) = D_1 \beta^{\frac{n-3}{2n}}\, v(\xi_1). \qquad \ldots(38)$$

In the foregoing expressions the values of the constants $B$, $B_1$, $D$ and $D_1$, which depend on two disposable constants $K$ and $n$, are given by

$$B = \left[\frac{K^n\,(n+1)}{4\pi\, G}\right]^{\frac{1}{2}} c^{1-n},\quad B_1 = \left[\frac{(n+1)\, K}{4\pi\, G}\right]^{\frac{1}{2}} \left(\frac{c}{s^2}\right)^{\frac{n-1}{n}}, \qquad \ldots(39)$$

$$D = \left[\frac{K^n\,(n+1)^3}{4\pi\, G^3}\right]^{\frac{1}{2}} c^{3-n},\quad D_1 = \left[\frac{K^3\,(n+1)^3}{4\pi\, G^2}\right]^{\frac{1}{2}} \left(\frac{c}{s^2}\right)^{\frac{n-3}{n}}. \qquad \ldots(40)$$

(*iii*) *Explicit expressions for $e^{\nu}$ (for the liquid core case)* — For $n \neq 0$, we are unable to find expressions for $e^{\nu}$ in closed form. Therefore the explicit form of the solutions for the metric component $e^{\nu}$ for the liquid core, as deduced from Eq. (15), are given by

$$e^{\nu(r)} = \left(1 - \frac{2GM}{c^2 R}\right)\left(\frac{1-\beta}{1+\alpha\theta-\beta}\right)^2. \qquad \ldots(41)$$

$$e^{\nu(R)} = \left(1 - 2\alpha\, v\,(\xi_1)/\xi_1\right)\left(\frac{1-\beta}{1+\alpha\theta-\beta}\right)^2,\ (e^{\nu}<1). \qquad \ldots(42)$$

(*iv*) *Density distributions* — The total mass, rest mass and the microscopic kinetic energy densities are related with the central density by the following expressions :

$$\rho' = \rho_c \left[ \frac{\theta^n \{1+n(\alpha\theta-\beta\theta^{-n})\}}{1+n(\alpha-\beta)} \right]; \qquad ...(43)$$

$$\rho = \rho_c' \frac{\theta^n}{1+n(\alpha-\beta)}; \qquad ...(44)$$

and

$$\rho_K = \rho_c' \frac{n(\alpha\theta^{n+1}-\beta)}{1+n(\alpha-\beta)} \qquad ...(45)$$

as obtained by substitutions of (1), (2), first equation of (14), and (16) into

$$\rho' = \frac{\epsilon}{c^2} = \rho + \rho_K = \rho + \frac{nP}{c^2}.$$

The liquid core-case is obtained for $n = 0$.

(*v*) *Pressure distribution* — The total pressure, in terms of central pressure and $\theta$, is expressed as

$$P = (s+P_c)\,\theta^{n+1} - s \qquad ...(46)$$

Hence for $n = 0$,

$$P = P_c\theta + s\,(\theta - 1). \qquad ...(46a)$$

ENERGIES OF THE PLANET

Expressions for the total energy $E$, rest energy $E_{0g}$ and total microscopic kinetic energy $E_{0k}$ are

$$E = D_1 c^2 \beta^{\frac{n-3}{2n}} v(\xi_1),$$

$$\left.\begin{aligned} E_{0g} &= D_1\, \beta^{\frac{n-3}{2n}} c^2 \int_0^{\xi_1} \frac{\theta^n \xi^2}{[1-2\alpha\,(n+1)\, v(\xi)/\xi]^{1/2}}\, d\xi \\ &= D_1\, \beta^{\frac{n-3}{2n}} c^2 F_1(\xi_1) \end{aligned}\right\} \qquad ...(47)$$

$$E_{0k} = D_1\, \beta^{\frac{n-3}{2n}} c^2 \int_0^{\xi_1} \frac{n\alpha\,(\theta^{n+1}-\beta\alpha^{-1})\,\xi^2}{[1-2(n+1)\alpha v/\xi]^{1/2}}\, d\xi \qquad ...(48)$$

$$= D_1 \beta^{\frac{n-3}{2n}} c^2 F_1(\xi_1),$$

where

$$F_1(\xi_1) = \int_0^{\xi_1} \frac{\theta^n \xi^2}{[1-2\alpha(n+1)\, v(\xi)/\xi]^{1/2}} d\xi, \qquad \text{...(48}a\text{)}$$

and

$$F_2(\xi_1) = \int_0^{\xi_1} \frac{n\alpha\,(\theta^{n+1}\,\beta\alpha^{-1})\xi^2}{[1-2(n+1)\,\alpha v/\xi]^{1/2}} d\xi.$$

The negative gravitational potential energy is defined as the sum of $E_{0k}$ and $E_{0g}$ minus $E$. The expression for the binding energy $E_{BE}$ is given by

$$E_{BE} = E_{0g} - E = D_1\, c^2\, \beta^{\frac{n-3}{2n}} [F_1(\xi_1) - v(\xi_1)] \qquad \text{...(49)}$$

which can be re-expressed in the form

$$E_{BE} = E_{0g}\left[1 - \frac{v(\xi_1)}{F_1(\xi_1)}\right]. \qquad \text{...(50)}$$

## The Central-Phase Velocity of Sound

The general expression for the ratio of the squares of the speed of sound to that of light is given by

$$\frac{v_s^2}{c^2} = \frac{dP}{d\epsilon} = P\gamma' \frac{d\rho}{\rho} \frac{1}{d\epsilon} = \frac{n+1}{n} \frac{P}{P+\epsilon} \qquad \text{...(51)}$$

which, with the help of Eqs. (1), (4) and the first equation of (16), can be rewritten as

$$\frac{v_s^2}{c^2} = \frac{n+1}{n} \frac{\alpha\theta - \beta\theta^{-n}}{(\alpha\theta - \beta\theta^{-n})(1+n)+1}. \qquad \text{...(52)}$$

Now, at the center $\theta = 1$, therefore, if we put $n = 0$, we get

$$v_s = \pm c\left(\frac{n+1}{n} \frac{\alpha-\beta}{(\alpha-\beta)(n+1)+1}\right)^{\frac{1}{2}}. \qquad \text{...(53)}$$

Since for the liquid-core case ($n = 0$) $\alpha \to \infty$ and $\beta \to 0$, hence $v_s \to \pm\infty$.

For prescribed non-zero values of $n$, we infer that, for large values of $\alpha - \beta$,

$$\frac{v_s^2}{c^2} = \frac{1}{n}, \text{ when } \alpha - \beta \gg 1 \qquad \text{...(54)}$$

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