A POINT EXPLOSION IN AN ARBITRARY ATMOSPHERE WITH RADIATIVE HEAT TRANSFER

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The authors have considered the propagation of a strong shock in an arbitrary atmosphere taking into account the effects of radiation heat conduction. They have shown that the radiating flow approaches an isothermal one as the shock advances in the upward direction.

INTRODUCTION

In this paper we have considered the propagation of a strong shock in an arbitrary atmosphere taking into account the effects of radiation heat conduction under Rosseland diffusion approximation. Using the approximate integral method of Laumbach and Probstein (1970) we have obtained the detailed flow field behind the shock propagating in upward direction at different altitudes from the sea level. The solution, though dependent on the altitude of the burst point, can still be scaled for an arbitrary explosion energy.

ASSUMPTIONS AND BASIC EQUATIONS

Initially, the atmosphere is considered to be at zero temperature and pressure.

The atmospheric density distribution is taken as

$$\rho_0 = \rho_B \, f\left(\frac{h}{\triangle}\right), \qquad \ldots(1)$$

where ρ_0 is the atmospheric density, $\triangle$ the scale height of the atmosphere which is taken constant, h the altitude measured from the burst point and ρ_B the density at the burst point before explosion.

In terms of Lagrangian coordinate r_0 which is defined as the coordinate r of a fluid particle at the burst time $t = 0$, Eq. (1) becomes

$$\rho_0 = \rho_B f\,(r_0 \cos\theta/\triangle). \qquad \ldots(2)$$

Assuming the initial flow to be locally radial, the position $R(t, \theta)$, of the shock front, which is symmetric about the vertical axis with the explosion point 0 as the origin, is shown in Fig. 1.

Neglecting the external forces and assuming the shock radius to be large as compared to the radiation mean free path, the equations governing the flow of an inviscid gas are

$$\rho_0 \, r_0^2 \, dr_0 = \rho r^2 dr \qquad \ldots(3)$$

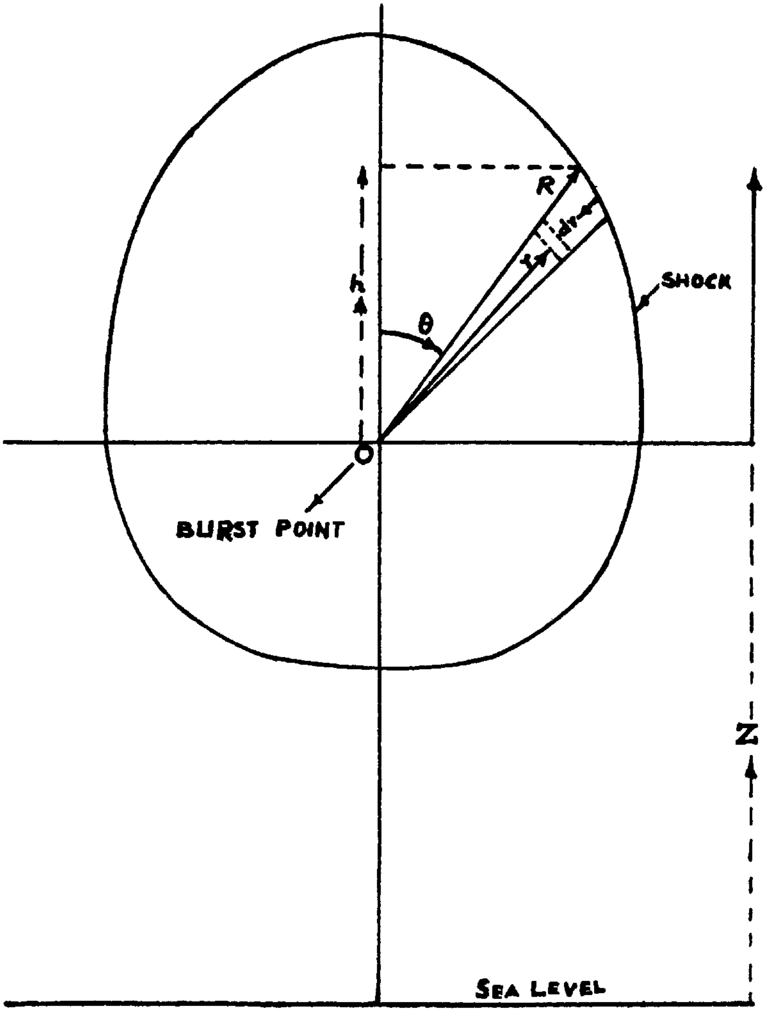

FIG. 1. Flow geometry

$$\frac{\partial^2 r}{\partial t^2} + \frac{r^2}{\rho_0 r_0^2} \frac{\partial p}{\partial r_0} = 0 \qquad \ldots(4)$$

$$\frac{p}{\gamma - 1} \frac{\partial}{\partial t}\left(\ln \frac{p}{\rho^\gamma} \right) = - \frac{\rho}{\rho_0 r_0^2} \frac{\partial}{\partial r_0}\left(r^2 q \right) \qquad \ldots(5)$$

and the equation of state for air obeying the perfect gas law is

$$p = \Gamma \rho T, \qquad \ldots(6)$$

where p is the pressure, T the absolute temperature, $\gamma = \frac{c_p}{c_p}$ = constant, Γ an appropriately selected gas constant and the radiation heat flux q, under Rosseland diffusion approximation, is given by

$$q = -16/3\, \lambda\, \sigma T^3 \frac{\partial T}{\partial r}. \qquad \ldots(7)$$

Here λ is the Rosseland mean free path and σ the Stefan-Boltzmann constant.

The Rankine-Hugoniot conservation conditions across the strong shock are

$$\left.\begin{aligned} u_s &= (1-\beta)\dot{R}, \\ p_s &= (1-\beta)\rho_0 \dot{R}^2, \\ \frac{\rho_0}{\rho_s} &= \beta = \frac{\gamma-1}{\gamma+1}\left[1 + \frac{2q_s}{p_s\dot{R}}\right], \end{aligned}\right\} \qquad \ldots(8)$$

where u is the particle velocity, $\dot{R}$, the shock velocity and subscript 's' denotes the conditions just behind the shock. By integrating Eq. (3) from the burst point to the shock front and making use of Eq. (2) and the relations $\beta = \frac{\rho_0(R)}{\rho_s}$, we obtain the value of β as

$$\beta = \left[3\int_0^1 \frac{\rho_s}{\rho}\, \frac{f(r_0\cos\theta/\Delta)}{f(R\cos\theta/\Delta)} \left(\frac{r_0}{R}\right)^2 d\left(\frac{r_0}{R}\right)\right]^{-1}. \qquad \ldots(9)$$

The momentum Eq. (4) is expressed in the integral form as

$$p(r_b, t; \theta) - p_s(R;\theta) = \int_{r_0}^{R} \frac{1}{r^2}\frac{\partial^2 r}{\partial t^2}\rho_0 r_0^2\, dr_0. \qquad \ldots(10)$$

Making use of Eq. (3) the energy equation for a given polar angle is written as

$$\frac{E}{4\pi} = \int_0^R \frac{p}{\gamma-1} r^2 dr + \int_0^R \left(\frac{\partial r}{\partial t}\right)^2 \rho_0 r_0^2\, dr_0, \qquad \ldots(11)$$

where E denotes the total constant energy of the flow field.

By following the method of Laumbach and Probstein (1970) and using $\lambda = a\,(\rho_g/\rho)\, T^{-17/6}$ we obtain the basic equations

$$\frac{F(\eta)}{2}\frac{d(\eta'^2)}{d\eta} + G(\eta)\,\eta'^2 - \bar{H}(\eta)\,(\eta'^2)^{4/3} = 1 \text{ for } 0 \leqslant \theta \leqslant \frac{\pi}{2} \qquad \ldots(12)$$

$$\frac{F(\eta)}{2}\frac{d(\eta'^2)}{d\eta} + G(\eta)\,\eta'^2 + \bar{H}(\eta)\,(\eta'^2)^{4/3} = -1 \text{ for } \frac{\pi}{2} \leqslant \theta \leqslant \pi. \qquad \ldots(13)$$

where

$R\cos\theta/\Delta = \eta,\ r_0\cos\theta/\Delta = \bar{\eta}$

$$F(\eta) = \frac{1}{3}\,\frac{\eta(1-\beta)^2}{(1-2\beta)(\gamma-1)}\int_0^{\eta}\bar{\eta}^2 f(\bar{\eta})\,d\bar{\eta}$$

$$G(\eta) = \frac{\eta^3(1-\beta)f(\eta)}{3(\gamma-1)} + F(\eta)\left[\frac{2\beta}{\eta} + \frac{\beta}{1-\beta}\frac{d}{d\eta}\ln f(\eta) - \frac{2}{1-\beta}\frac{d\beta}{d\eta} + \frac{3}{2}\frac{(1-2\beta)(\gamma-1)}{\eta}\right]$$

$$\bar{H}(\eta) = \frac{f^2(\eta)\,\{\gamma(\beta-1)+\beta+1\}\,F(\eta)}{2K\beta^{1/6}(1-\beta)^{7/6}(\gamma-1)}$$

$$K = \frac{16}{3}\left(\frac{\rho_g}{\rho_B}\right)\frac{\sigma a}{\rho_B\Gamma^{7/6}}\left(4\pi\rho_{B,E}\right)^{1/3}$$

$\bar{t} = t\left[\dfrac{E\,|\cos^5\theta\,|}{4\pi\rho_B\Delta^5}\right]^{1/2}$; and dash denotes differentiation with respect to $\bar{t}$.

The pressure, temperature and density distributions are obtained as

$$\frac{p}{p_s} = 1 + \frac{1}{f(\eta)}\left[\frac{1-\beta}{1-2\beta}\frac{\eta''}{\eta^2\eta'^2} + \frac{2\beta(1-\beta)}{(1-2\beta)\eta^2}\frac{d}{d\eta}\left\{\text{in } f(\eta)\right\} - \frac{2}{\eta^2(1-2\beta)}\frac{d\beta}{d\eta} - \frac{f^2(\eta)\,|\,\eta\,\eta'^{2/3}\,|\,\{\gamma(\beta-1)+\beta+1\}}{2K\eta^3\beta^{1/6}(1-\beta)^{1/6}(1-2\beta)(\gamma-1)}\right]\int_{\eta}^{\eta}\bar{\eta}^2 f(\bar{\eta})\,d\bar{\eta} \qquad \ldots(14)$$

$$\left(\frac{T}{T_s}\right)^{7/6} = 1 + \frac{7}{6}\,\frac{f^2(\eta)\,|\,\eta\,\eta'^{2/3}\,|}{K\beta^{13/6}(1-\beta)^{1/6}(\gamma-1)}\left[\frac{\beta}{1+\beta}\left\{1-\left(\frac{r}{R}\right)^{\frac{1+\beta}{\beta}}\right\}\left\{\frac{\gamma(\beta-1)+\beta+1}{2} + \frac{1}{3}\frac{\rho_b}{\rho_s}\frac{p_b}{p_s}\frac{\eta}{\eta'}\frac{d}{dt}\ln\left(\frac{p}{\rho\gamma}\right)_b\right\} - \frac{1}{6}\frac{\rho_b}{\rho_s}\frac{p_b}{p_s}\frac{\eta}{\eta'}\frac{d}{d\bar{t}}\,\text{In}\left(\frac{p}{\rho\gamma}\right)_b\left\{1-\left(\frac{r}{R}\right)^2\right\}\right] \qquad \ldots(15)$$

$$\left(\frac{\rho_s}{\rho}\right)^{7/6} = \left(\frac{p_s}{p}\right)^{7/6} + \left(\frac{p_s}{p}\right)^{7/6}\cdot\left(\frac{T}{T_s}\right)^{7/6}. \qquad \ldots(16)$$

Here the subscript b refers to conditions at the burst point where both r and r_0 are zero.

'β' is determined through Eqs. (9) and (16) by an iterative method and then the basic Eq. (12) is integrated.

With β and $\frac{\rho_s}{\rho}$ known, the relation between r and r_0 can be found by integrating the continuity equation (3) and this gives

$$\frac{r}{R} = \left[3\beta \int_0^{\frac{r_0}{R}} \frac{\rho_s}{\rho} \frac{f(\bar{\eta})}{f(\eta)} \left(\frac{\bar{r}_0}{R}\right)^2 d\left(\frac{\bar{r}_0}{R}\right)\right]^{\frac{1}{3}}. \qquad \ldots(17)$$

Differentiating Eq. (17) with respect to t, we obtain the velocity distribution as

$$\frac{u}{u_s} = \left(\frac{R}{r}\right)^2 \frac{\beta}{1-\beta} \int_0^{\frac{r_0}{R}} \frac{\rho_s}{\rho^2} \left[\frac{\bar{r}_0}{R} \frac{\partial \rho}{\partial\left(\frac{r_0}{R}\right)} - \frac{\eta}{\eta'} \frac{\partial \rho}{\partial \bar{t}}\right] \frac{f(\bar{\eta})}{f(\eta)} \left(\frac{\bar{r}_0}{R}\right)^2 d\left(\frac{\bar{r}_0}{R}\right), \qquad \ldots(18)$$

where $\bar{r}_0$ is a dummy variable

Discussion and Results

In order to solve Eq. (12), we have taken the initial condition $R = \left(\frac{25\, AE}{16\pi\, \rho_B}\right)^{1/5} t^{2/5}$ from the solution of the point explosion problem with radiating flow in a uniform density atmosphere. Transforming this initial condition in terms of η and $\bar{t}$ we obtain

$$\eta'^2 = \frac{A}{\eta^3},$$

where A is determined from the solution of the equation

$$\left(B - \frac{3}{2}\right) A - \left[\frac{\gamma(\beta-1)+\beta+1}{2(\gamma-1)K\beta^{1/6}(1-\beta)^{7/6}}\right] A^{4/3} = \frac{9(1-2\beta)(\gamma-1)}{(1-\beta)^2},$$

B being a function of β (see Laumbach & Probstein 1970).

We have carried on the computational work for the ambient density distribution of Treve and Manley (1972) given by

$$f(z) = \frac{1.225}{(A_0 + A_1 Z + A_2 Z^2 + \ldots\ldots + A_{11} Z^{11})^4}\ Kg/m^3,$$

where Z is the altitude from the sea level and

$A_0 = +\,0.1000000000 \times 10^{-1}$, $A_1 = +\,0.3393495800 \times 10^{-1}$
$A_2 = -\,0.3433553057 \times 10^{-2}$, $A_3 = +\,0.5497466428 \times 10^{-3}$
$A_4 = -\,0.3228358326 \times 10^{-4}$, $A_5 = +\,0.1106617734 \times 10^{-5}$
$A_6 = -\,0.2291755793 \times 10^{-7}$, $A_7 = +\,0.2902146443 \times 10^{-9}$
$A_8 = -\,0.2230070938 \times 10^{-11}$, $A_9 = +\,0.1010575266 \times 10^{-13}$
$A_{10} = -\,0.24822089627 \times 10^{-16}$, $A_{11} = +\,0.2548769715 \times 10^{-19}$.

The burst point as indicated in Fig. 1 is located at a height 6 Δ above sea level. The values of γ and K are taken as 1.2 and 0.6 respectively as these are appropriate values corresponding to the altitute of the burst point. Figs. 2, 3 and 4 show the

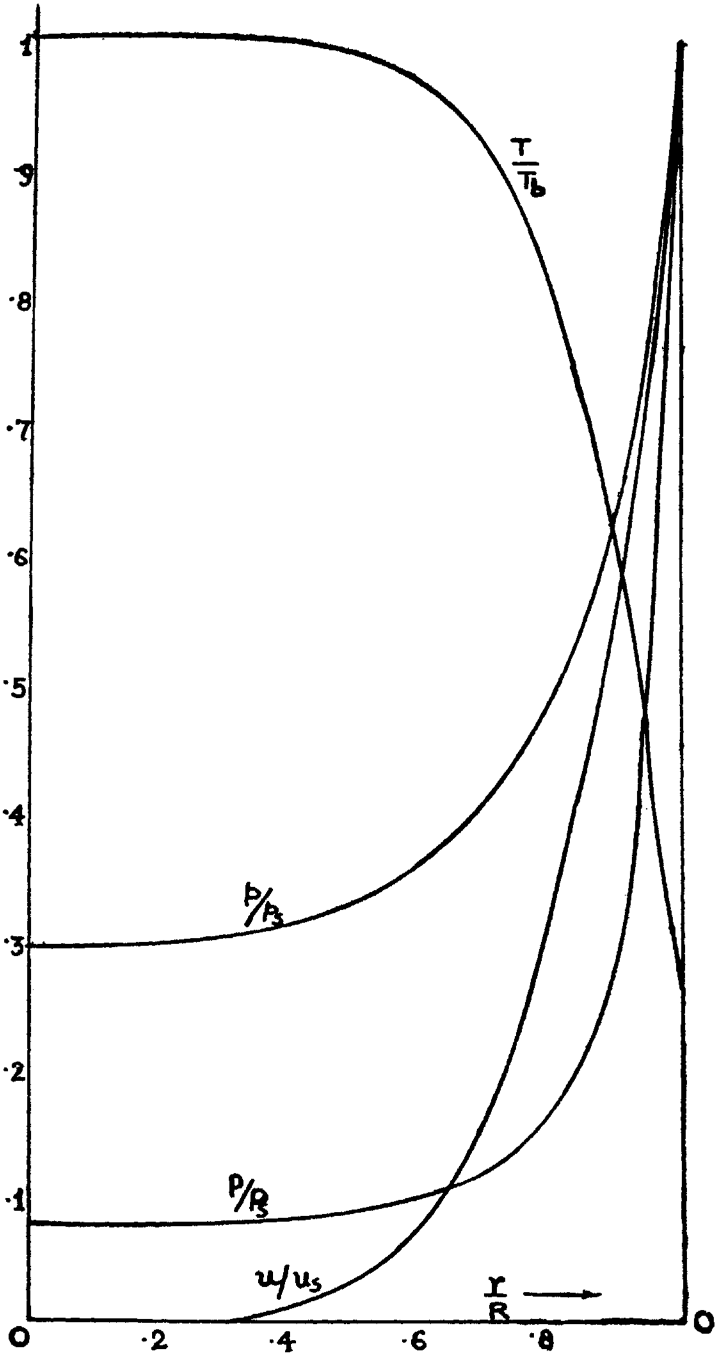

FIG. 2. Flow variables distributions in upward direction at $\frac{R \cos \theta}{\Delta} = 0.6$ for $\gamma=1.2$ and $K = 0.6$

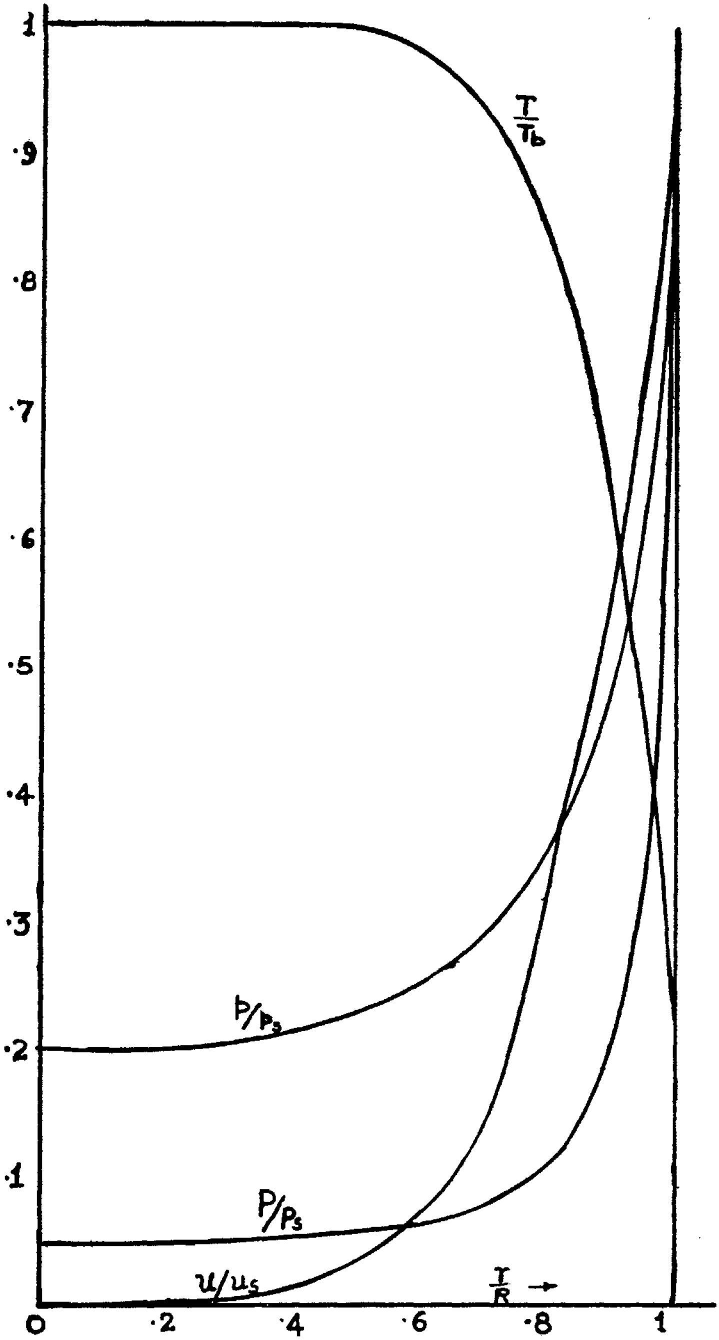

FIG. 3. Flow variables distributions in upward direction at $\frac{R\cos\theta}{\Delta} = 0.8$ for $\gamma=1.2$ and $K = 0.6$

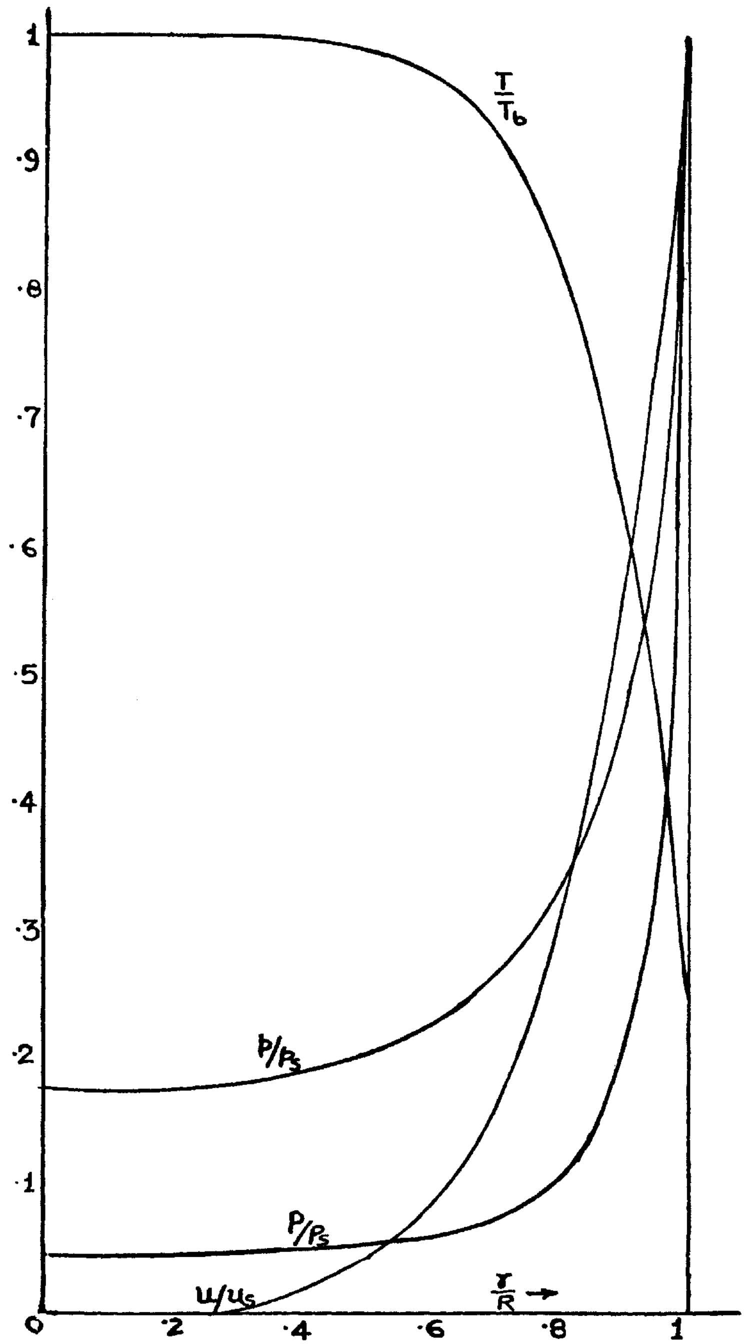

FIG. 4. Flow variables distributions in upward direction at $\frac{R \cos \theta}{\Delta} = 1$ for $\gamma = 1.2$ and $K = 0.6$

distributions of flow variables for the upward propagating shock at $R \cos \theta/\Delta = 0.6$, 0.8, and 1.0 respectively. The flow variables pressure, density and velocity are reduced by their values just behind the shock front while the temperature is reduced by its value at the burst point. Comparing the Figs. 2, 3 and 4 we observe that pressure and density profiles are getting closer to each other while the temperature remains almost constant in the vicinity of the burst point and this range of constancy (flattening of the temperature profile) increases as the shock advances upward. This is in conformity with the observation of Laumbach and Probstein (1970) that the radiating flow approaches an isothermal one as the shock advances in the upward direction. But the flattening of the temperature profile in our case is expected to be much slower as compared to that of the exponential density distribution because of the fact that

$T_b/T_s = 6.4$, for uniform density atmosphere

$T_b/T_s = 3.4$, for our arbitrary density distribution at $\eta = 0.6$

$T_b/T_s = 1.35$, for exponential density distribution

at $\eta = 2$ (see Laumbach & Probstein 1970).

The variation of β and shock velocity $\dot{R}$ with the shock position is shown below

η	β	$n' = \dot{R}[4\pi\rho_B \Delta^3/E \mid \cos^3\theta \mid]^{1/2}$
0.6	0.206	3.889
0.8	0.192	2.912
1.0	0.181	2.185

The variation of β in our case is small due to the fact that within the range $(0.6 \leqslant \eta \leqslant 1)$ of our calculation the ambient density distribution falls much slower than the exponential density distribution (see Fig. 5).

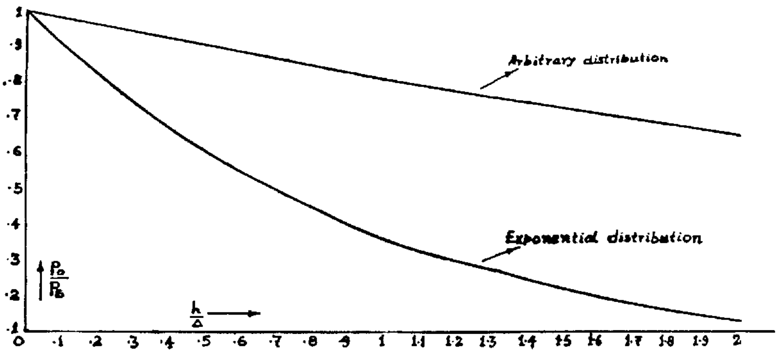

FIG. 5. Atmospheric density distribution with exponential and arbitrary variations

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