MAGNETOHYDRODYNAMIC FLOW NEAR A TIME-VARYING ACCELERATED POROUS PLATE

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The flow of an electically conducting incompressible viscous liquid due to the time-varying motion of an infinite porous plate has been studied in the presence of a uniform magnetic field. The magnetic lines of force are taken to be fixed relative to the plate. General expressions of the velocity and skin-friction have been obtained when the plate moves with a velocity $e^{at}\, t^{n}$. Several particular cases have been studied and the results have been compared with the help of some graphs and tables.

INTRODUCTION

The flow of a viscous incompressible and electrically conducting fluid in the presence of an external magnetic field, past an impulsively moving infinite plate has been studied by Rossow (1957), Kakutani (1958) and Ong and Nicholls (1959). They extended the problem to the case of oscillating infinite plate with and without the induced magnetic field produced by the current. Further extensions over the problem have been made by Gupta (1960), Nanda and Sundaram (1962), Soundalgekar (1965), Pop (1967), Mohapatra (1971), Mishra and Mohapatra (1973) and many others. The aim of this paper is to discuss the flow of an electrically conducting, incompressible, viscous liquid due to the time-varying motion of an infinite porous plate in the presence of a uniform magnetic field. The magnetic lines of force are taken to be fixed relative to the plate.

Extending the results of Mishra and Mohapatra (1973) including suction at the plate, the basic equation of motion is

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} + m(u - A\, e^{at}\, t^{n}) = \nu \frac{\partial^2 u}{\partial y^2}, \qquad \ldots(1)$$

where $m = \dfrac{\sigma B_0^2}{\rho}$ and v_0 is the constant suction velocity.

The boundary conditions are

$$\left.\begin{aligned} &u = 0, \text{ every where for } t \leqslant 0 \\ &u = Ae^{at}\, t^{n} \text{ at } y = 0 \\ &u = 0 \text{ or finite as } y \to \infty. \end{aligned}\right\} \qquad \ldots(2)$$

SOLUTION OF EQUATION

We shall solve equation (1) by the method of Laplace transforms :

The Laplace transform of the equation (1) and the boundary conditions (2) are given by

$$\frac{d^2\bar{u}}{dy^2}+\frac{v_0}{\nu}\frac{d\bar{u}}{dy}-\frac{(p+m)}{\nu}\bar{u}=-\frac{Am}{\nu}\frac{\Gamma(n+1)}{(p-a)^{n+1}} \qquad \ldots(3)$$

$$\left.\begin{aligned}\bar{u}(y,p) &= \frac{A\,\Gamma(n+1)}{(p-a)^{n+1}} \text{ as } y=0\\ \bar{u}(y,p) &= 0 \text{ or finite as } y\rightarrow\infty\end{aligned}\right\} \qquad \ldots(4)$$

Solving Eq. (3) with the boundary conditions (4) we get

$$\bar{u}(y,p)=\frac{A\Gamma(n+1)}{(p+m)\,(p-a)^{n+1}}\left[m+p\,e^{-\frac{S\eta}{2}-\frac{y}{\sqrt{\nu}}\left(\frac{v_0^2}{4\nu}+p+m\right)^{1/2}}\right] \qquad \ldots(5)$$

Inverting Eq. (5), we get

$$u(y,t)=A\Gamma(n+1)\left[e^{-\frac{S\eta}{2}}\{I_1(y,t,\alpha,n)-mI_2(y,t,\alpha,n)\}+mI_2(0,t,\alpha,n)\right] \qquad \ldots(6)$$

where

$$I_1(y,t,\alpha,n)=\frac{1}{2\pi i}\,e^{-\left(a_0^2+\frac{S^2}{4}\right)}\int_{B_{r_3}} e^{z^2t-bz}\frac{2zdz}{(z^2-\alpha)^{n+1}} \qquad \ldots(7)$$

$$I_2(y,t,\alpha,n)=\frac{1}{2\pi i}\,e^{-\left(a_0^2+\frac{S^2}{4}\right)}\int_{B_{r_3}} e^{z^2t-bz}\frac{2zdz}{\left(z^2-\alpha\right)^{n+1}\left(z^2-\frac{v_0^2}{4\nu}\right)} \qquad \ldots(8)$$

and

$$b=\frac{y}{\sqrt{\nu}},\quad p+m+\frac{v_0^2}{4\nu}=z^2,\quad \alpha=m+a+\frac{v_0^2}{4\nu}$$

$$a_0=\sqrt{mt},\quad S=\frac{v_0\sqrt{t}}{\sqrt{\nu}},\quad \eta=\frac{b}{\sqrt{t}},\quad \alpha t=a_0^2+\text{at}+\frac{S^2}{4}.$$

We can find the values of $I_1(y,t,\alpha,n)$ and $I_2(y,t,\alpha,n)$ for different integral values of n.

$$\text{Let } I_1(y,t,\alpha,0)=\frac{1}{2\pi i}\,e^{-\left(a_0^2+\frac{S^2}{4}\right)}\int_{B_{r_3}} e^{z^2t}\frac{2zdz}{(z^2-\alpha)}=F(\alpha) \qquad \ldots(9)$$

Differentiating Eq. (9) with respect to α, we get

$$I_1(y,t,\alpha,1) = \frac{dF}{d\alpha} + tF \qquad \ldots(10)$$

Differentiating Eq. (10) successively $(n-1)$ times with respect to α we get I_1 (y, t, α, n) for different integral values of n.

From McLachlan (1963), we can find the value

$$I_1(y,t,\alpha,0) = \frac{1}{2} e^{\alpha t} \left[e^{\eta\sqrt{\alpha t}} \operatorname{erfc}\left(\frac{\eta}{2} + \sqrt{\alpha t}\right) + e^{-\eta\sqrt{\alpha t}} \operatorname{erfc}\left(\frac{\eta}{2} - \sqrt{\alpha t}\right)\right] \qquad \ldots(11)$$

By use of Eqs. (10) and (11) we get

$$I_1(y,t,\alpha,1) = \frac{1}{2} te^{\alpha t}\left[e^{\eta\sqrt{\alpha t}} \operatorname{erfc}\left(\frac{\eta}{2} + \sqrt{\alpha t}\right)\left(\frac{\eta}{2\sqrt{\alpha t}} + 1\right) - e^{-\eta\sqrt{\alpha t}} \operatorname{erfc}\left(\frac{\eta}{2} - \sqrt{\alpha t}\right)\left(\frac{\eta}{2\sqrt{\alpha t}} - 1\right)\right]. \qquad \ldots(12)$$

Similarly we can get I_1 (y, t, α, n) for all integral values of n. Eq. (8) can be written as

$$I_2(y,t,\alpha,n) = \frac{t}{a_0^2 + at}\left[I_1(y,t,\alpha,n) - I_{2'}(y,t,\alpha,n-1)\right]. \qquad \ldots(13)$$

By the use of partial fraction and from McLachlan (1963), we get

$$I_2(y,t,\alpha,0) = \frac{t}{2(a_0^2 + at)}\left[e^{\alpha t}\left\{ e^{\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} + \sqrt{\alpha t}\right) + e^{-\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} - \sqrt{\alpha t}\right)\right\} - e^{-a_0^2}\left\{ e^{\frac{\eta S}{2}}\, erfc\left(\frac{\eta + S}{2}\right) + e^{-\frac{\eta S}{2}}\, erfc\left(\frac{\eta - S}{2}\right)\right\}\right]. \qquad \ldots(14)$$

From Eqs. (12) to (14), we can get the value of I_2 $(y, t, \alpha, 1)$ and so on.

The shearing stress at the wall is

$$P_{xy_{y=0}} = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad \ldots(15)$$

From Eqs. (6) and (15), we have

$$P_{xy_{y=0}} = -\frac{A\mu}{\sqrt{\nu t}}\,\Gamma(n+1)\left[\frac{S}{2}\left\{ I_1(0,t,\alpha,n) - mI_2(0,t,\alpha,n)\right\}\right.$$

$$+ \sqrt{t}\, e^{-\left(a_0^2+\frac{S^2}{4^2}\right)} \frac{d}{dt}\left\{\frac{1}{\pi i}\int_{B_{r_3}} e^{z^2 t} \frac{dz}{(z^2-\alpha)^{n+1}} - \frac{m}{\pi i}\int_{B_{r_3}} e^{z^2 t} \frac{dz}{(z^2-\alpha)^{n+1}\left(z^2-\frac{v_0^2}{4\nu}\right)}\right\}\Bigg] \qquad ...(16)$$

Particular cases

(i) When $at \neq 0$ and $n = 0$—From Eq (6) we have

$$u(y,t) = A \left\{ e^{-\frac{S\eta}{2}} \{I_1(y,t,\alpha,0) - m\, I_2(y,t,\alpha,0)\} + m\, I_2(0,t,\alpha,0) \right\} \qquad ...(17)$$

With the use of Eqs. (11) and (14), Eq. (17) can be written as

$$\frac{u(y,t)}{A} = \frac{1}{2(a_0^2+at} \Bigg[\exp\left(at - \frac{S\eta}{2}\right) at \Bigg\{ e^{\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} + \sqrt{at}\right) + e^{-\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} - \sqrt{\alpha t}\right)\Bigg\} + a_0^2\, e^{-a_0^2} \Bigg\{ e^{\frac{\eta S}{2}}\, erfc\left(\frac{\eta+S}{2}\right) + e^{-\frac{\eta S}{2}}\, erfc\left(\frac{\eta-S}{2}\right)\Bigg\} + 2a_0^2\left(e^{at} - e^{-a_0^2}\right)\Bigg]$$

The shearing stress at the wall from Eq. (16) is

$$P_{xy,y=0} = -\frac{A\mu}{\sqrt{\nu t}}\Bigg[\frac{S}{2}\{I_1(0,t,\alpha,0) - mI_2(0,t,\alpha,0)\} + \sqrt{t}\, e^{-\left(a_0^2+\frac{S^2}{4^2}\right)} \frac{d}{dt}\left\{\frac{1}{\pi i}\int_{B_{r_3}} e^{z^2 t}\frac{dz}{z^2-\alpha} - \frac{m}{\pi i}\int_{B_{r_3}} e^{z^2 t}\frac{dz}{(z^2-\alpha)\left(z^2-\frac{v_0^2}{4\nu}\right)}\right\}\Bigg] \qquad ...(18)$$

By use of Eqs. (11) and (14), we can write (18) in the form

$$\tau_0' = \frac{P_{xy,\,y=0}}{\left(-\frac{A\mu}{\sqrt{\nu}}\right)} = \frac{S}{2}\left[e^{at} - \frac{a_0^2}{a_0^2+at}\left(e^{at} - e^{-a_0^2}\right)\right] + \frac{1}{\sqrt{\pi}}\, e^{-\left(a_0^2+\frac{S^2}{4^2}\right)} + \frac{1}{a_0^2+at}\left\{e^{at}\sqrt{\alpha t} + at\, erf\sqrt{\alpha t} + \frac{a_0^2}{2}\, e^{-a_0^2}\, S erf\left(\frac{S}{2}\right)\right\}$$

(ii) When $at \neq 0$ and $n = 1$—From equations (6) and (13) we have

$$u(y,t) = A\left[\frac{1}{a_0^2+at}\; e^{-\frac{S\eta}{2}} \{at\, I_1(y,t,\alpha,1)+a_0^2\, I_2(y,t,\alpha,0)\}+m\, I_2(0,t,\alpha,1)\right] \quad \ldots(19)$$

With the help of Eqs. (12) to (14), we can get (19) in the form

$$\frac{u(y,t)}{At} = \frac{e^{at-\frac{S\eta}{2}}}{2(a_0^2+at)^2}\left[at(a_0^2+at)\left\{\left(\frac{\eta}{2\sqrt{\alpha t}}+1\right)+a_0^2 e^{\eta\sqrt{\alpha t}}\right\}erfc\left(\frac{\eta}{2}+\sqrt{\alpha t}\right)\right.$$

$$-\left\{at(a_0^2+at)\left(\frac{\eta}{2\sqrt{\alpha t}}-1\right)-a_0^2\; e^{-\eta\sqrt{\alpha t}}\right\}erfc\left(\frac{\eta}{2}+\sqrt{\alpha t}\right)$$

$$\left.-a_0^2\; e^{-(a_0^2+at)}\left\{e^{\frac{\eta S}{2}}\, erfc\left(\frac{\eta+S}{2}\right)+e^{-\frac{\eta S}{2}}\, erfc\left(\frac{\eta-S}{2}\right)\right\}\right]$$

$$+\frac{a_0^2\, e^{at}}{(a_0^2+at)^2}\left[a_0^2+at-1+\exp(-a_0^2-at)\right]$$

The shearing stress at the wall from Eq. (16) is

$$P_{xy_{y=0}} = -\frac{A\mu}{\sqrt{\nu t}}\; 2\left[\frac{S}{2}\{I_1(0,t,\alpha,1)-m\, I_1(0,t,\alpha,1)\}\right.$$

$$+\frac{t^{3/2}}{a_0^2+at}\; e^{-\left(a_0^2+\frac{S^2}{4}\right)}\frac{d}{dt}\left\{\frac{d}{d\alpha} < \frac{a}{\pi i}\int_{B_{r_3}} e^{z^2 t}\frac{dz}{z^2-\alpha}+\right.$$

$$\left.\left.\frac{m}{\pi i}\int_{B_{r_3}} e^{z^2 t}\; \frac{dz}{\left(z^2-\alpha\right)\left(z^2-\frac{v_0^2}{4\nu}\right)} > \right\}\right] \quad \ldots(20)$$

With the help of Eqs. (12) to (14), we can get Eq. (20) in the form

$$T_0' = \frac{P_{xy_{\,y=0}}}{(-\sqrt{\mu_0 t})A} = \frac{S}{2}\left\{\frac{at}{a_0^2+at}+\frac{a_0^2}{a_0^2+at)^2}(e^{at}-e^{-a_0^2})\right\}$$

$$+\frac{\exp(at)\, erf\sqrt{\alpha t}}{a_0^2+at}\left\{\frac{at}{2\sqrt{\alpha t}}+at\sqrt{\alpha t}+\frac{a_0^2\sqrt{\alpha t}}{a_0^2+at}\right\}$$

$$+\frac{at}{\sqrt{\pi}}\frac{\exp\left(-a_0^2-\frac{S^2}{4}\right)}{a_0^2+at}-\frac{a_0^2\exp(-a_0^2)}{(a_0^2+at)^2}\;\frac{S}{2}\, erf\left(\frac{S}{2}\right)$$

(iii) *When at* $\neq 0$ *and* $n = 2$—From the equation (6) we have

$$u(y,t) = 2A\left[e^{-\frac{\eta S}{2}} \frac{1}{a_0^2 + at} \{at\, I_1(y,t,\alpha,2) + a_0^2 I_2(y,t,\alpha,1)\} + m\, I_2(0,t,\alpha,2) \right] \quad \ldots(21)$$

Differentiating (7) with respect to α when $n = 1$, we get

$$I_1(y,t,\alpha,2) = \frac{1}{2}\left[\frac{d}{d\alpha} I_1(y,t,\alpha,1) + t\, I_1(y,t,\alpha,1) \right] \quad \ldots(22)$$

The value of Eq. (21) can be obtained with the help of the relations (12) to (14) and Eq. (22) as

$$\frac{u,(y,t)}{At^2} = \frac{2e^{-\frac{\eta S}{2}}}{a_0^2 + at} \{at\, \phi_1 + a_0^2 \phi_2\} + \frac{a_0^2}{a_0^2 + at}\left[e^{at} - \frac{2}{(a_0^2 + at)^2} \left\{ (a_0^2 + at - 1)\, e^{at} + e^{-a_0^2} \right\} \right]$$

where

$$\phi_1 = \frac{e^{at}}{4} \left[e^{\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} + \sqrt{\alpha t} \right) \left\{ \frac{\eta^2}{4\alpha t} + \frac{\eta}{\sqrt{\alpha t}} - \frac{\eta}{4(\alpha t)^{3/2}} + 1 \right\} \right.$$
$$\left. + e^{-\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} - \sqrt{\alpha t} \right) \left\{ \frac{\eta^2}{4\alpha t} - \frac{\eta}{\sqrt{\alpha t}} + \frac{\eta}{4(\alpha t)^{3/2}} + 1 \right\} \frac{-\eta e}{\sqrt{\pi \alpha t}}^{-\left(\frac{\eta^2}{4} + \alpha t \right)} \right]$$

and

$$\phi_2 = \frac{e^{at}}{2(a_0^2 + at)} \left[e^{\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} + \sqrt{\alpha t} \right) \left\{ \frac{\eta}{2\sqrt{\alpha t}} + 1 - \frac{1}{a_0^2 + at} \right\} \right.$$
$$- e^{-\eta\sqrt{\alpha t}}\, erfc\left(\frac{\eta}{2} - \sqrt{\alpha t} \right) \left\{ \frac{\eta}{2\sqrt{\alpha t}} - 1 + \frac{1}{a_0^2 + at} \right\}$$
$$\left. + e^{-\frac{a_0^2 - at}{a_0^2 + at}} \left\{ e^{\frac{\eta S}{2}}\, erfc\left(\frac{\eta + S}{2} \right) + e^{-\frac{\eta S}{2}}\, erfc\left(\frac{\eta - S}{2} \right) \right\} \right]$$

Discussion of Results

In this problem we have studied the flow of an electrically conducting, incompressible, viscous liquid, due to the time-varying motion of an infinite flat porous plate, in the presence of a uniform magnetic field. The magnetic lines of force are taken to be fixed reltive to the plate. General expression of the velocity and skin-friction have been obtained when the plate moves with the velocity $e^{at}\, t^n$ and discussions have been made for a few particular cases.

Figure 1 shows the effect of magnetic field strength on the velocity field. This shows that for a fixed value of *'at'* and S, an increase in the magnetic field strength increases the velocity at any point of the fluid. Figure 2 shows the velocity distribution for different values of *'at'* when a_0 and S are fixed. This shows that with an increase in the value of *'at'*, the velocity at any point increases. Figure 3 shows the effect of suction velocity on the velocity field. This shows that for a fixed value of a_0 and *'at'*, an increase in the suction velocity decreases the velocity at any point of the fluid. Figure 4 shows the velocity for different values of n in the expression for the plate velocity $At\ n\ e^{at}$ for fixed values of a_0 *'at'* and S. The shows that the velocity at any point decreases as the value of n increases.

Tables I and II show the effect of a_0, *'at'* and suction velocity on the skin-friction. These two tables show that for all values of *'n'*, 'at' and S, the skin-friction decreases as a_0 increases, but for fixed values of a_0 and S, the skin-friction increases with the increase in the value of *'at'*. Also both the tables show that for the fixed values of a_0 and *'at'*, for all values of n, skin-friction increases with increase in the value of suction veloity.

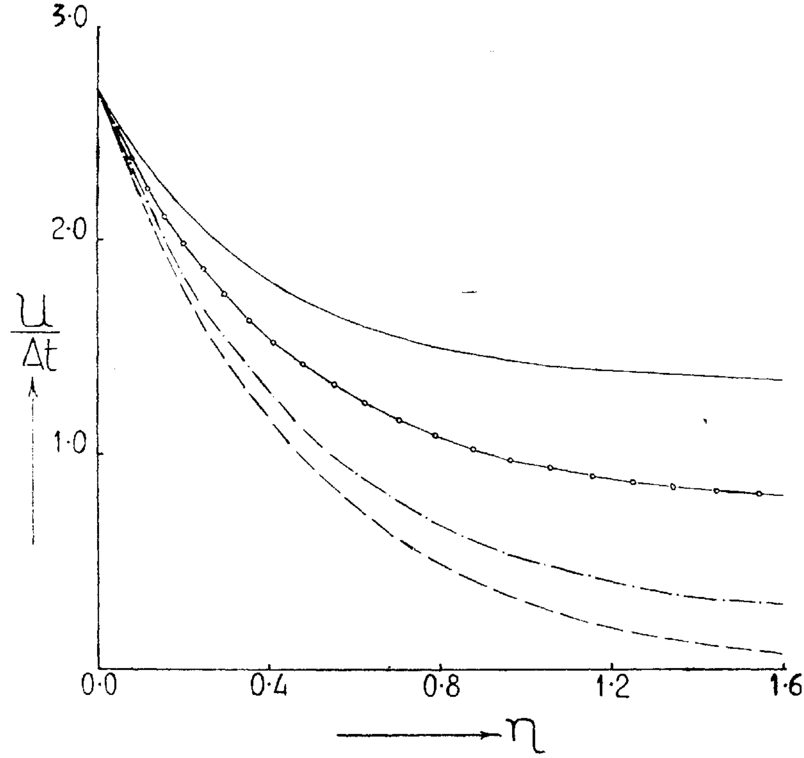

FIG. 1 Velocity distribution for different values of magnetic number a_0, $at = 1.0$, $S = 1.0$, $n = 1$. $a_0 = 0.01$— — — — —, $a_0 =$ —·—·—·—·, $a_0 = 1.0$——0——, $a_0 = 1.5$ ———

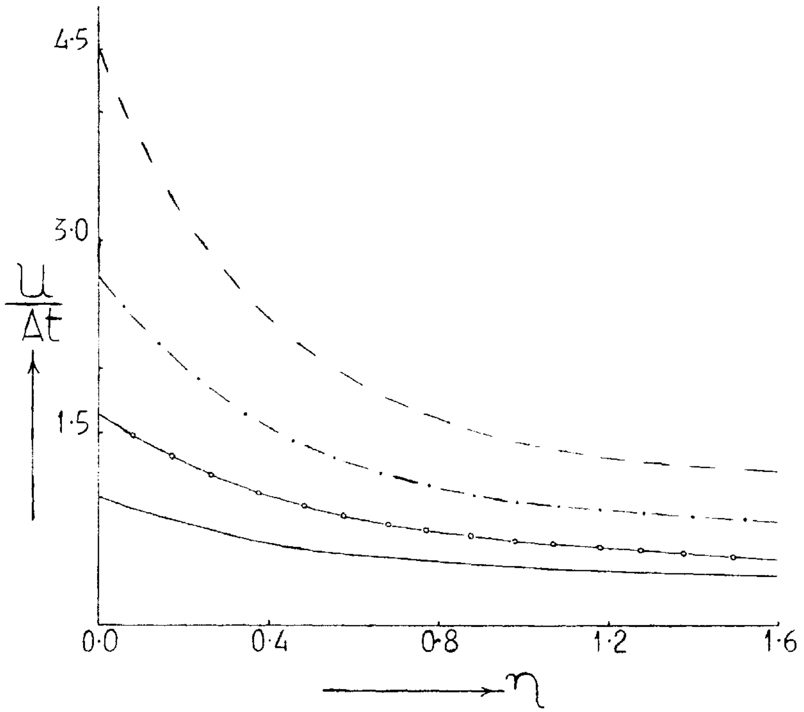

FIG. 2. Velocity distribution for different values of 'at' for a_0 = 1.0, η = 1.0, S = 1.0, at = 0.0 ———— at = 0.5 ——0——, at = 1.0 —. —. —I —, at = 1.5 — — — — — —

TABLE I

n = 0 *values of the skin-friction*

S	at \ a_0	0.0	0.5	1.0	1.5
0.0	0.0	0.56418	1.36148	2.85768	5.60029
	0.5	0.43939	1.18232	2.59609	5.21255
	1.0	0.20755	0.82511	2.04324	4.35305
	1.5	0.05946	0.54726	1.55132	3.50997
2.0	0.0	0.78000	1.82304	3 60828 . .	6.82473
	0.5	0.66139	1.56178	3.23758	6.28916
	1.0	0.31242	1.04517	2.46393	5.11869
	1.5	0.08951	0.65230	1.79549	4.00480
4 0	0.0	0.93939	2.37814	4.49441	8.24893
	0.5	0.93470	2.01666	3.99165	7.53591
	1.0	0.44152	1.30588	2.95166	5.99386
	1.5	0.12649	0.77371	2.07225	4.55967

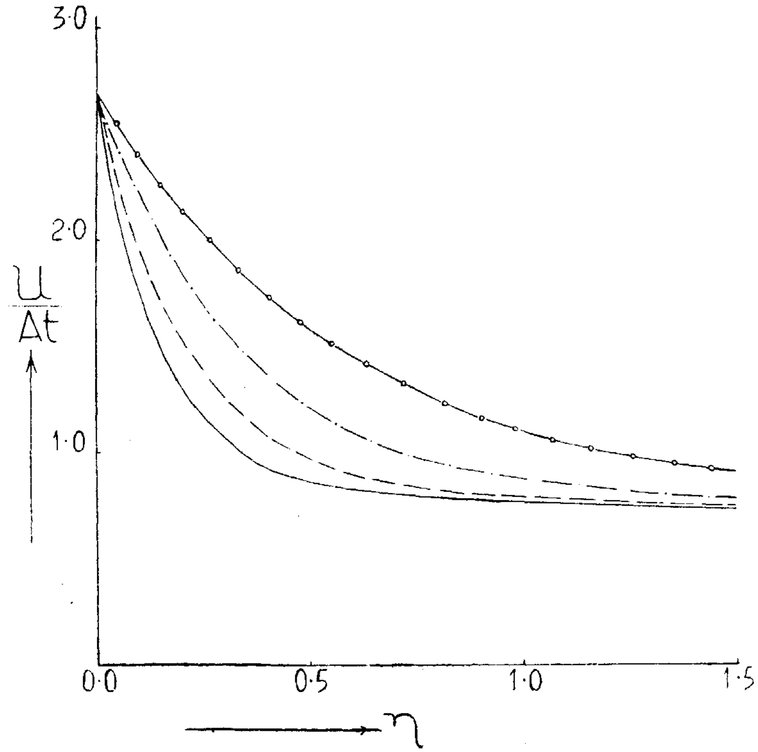

FIG. 3. Velocity distribution for different values of suction parameter S for $n = 1$, $a_0 = 1.0$, $at = 1.0$, $S = 0.0$ ——○——, $S = 2.0$ —. —. —. —, $S = 4.0$ — — — — $S = 6.0$ ———

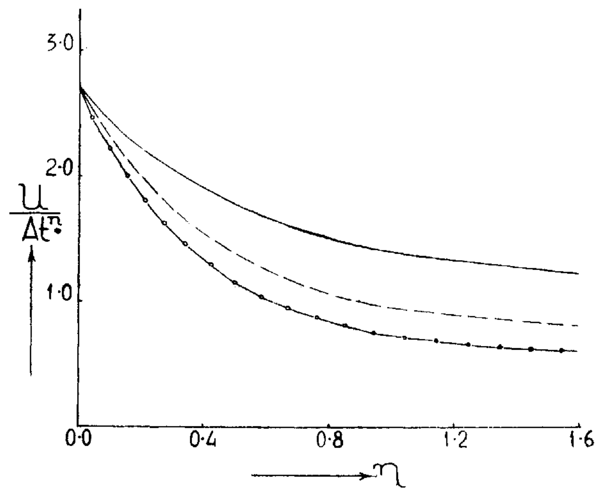

FIG. 4. Velocity distribution for different values of n for $a_0 = 1.0$, $at = 1.0$ $S = 1.0$, $n = 0.0$ ——— $n = 1.0$ — — — —, $n = 2.0$ ——0——

TABLE II

n = 1 *values of the skin-friction*

S	a_0 \ at	0.0	0.5	1.0	1.5
0.0	0.0	1.12836	2.15878	4.00444	7.27899
	0.5	1.04318	2.02643	3.80223	6.96813
	1.0	0.84373	1.71601	3.31622	6.20460
	1.5	0.64436	1.38553	2.77249	5.31456
2.0	0.0	1.32178	2.44384	4.30751	7.61008
	0.5	1.28441	2.32042	4.12695	7.33092
	1.0	1.01499	1.96287	3.62212	6.57353
	1.5	0.75100	1.55832	3.00992	5.62551
4.0	0.0	1.77315	2.79671	4.71362	8.09914
	0.5	1.56430	2.67386	4.54333	7.83600
	1.0	1.21187	2.25100	3.99428	7.04897
	1.5	0.87175	1.75563	3.28914	6.00682

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