STABILITY OF TIME-DEPENDENT COUETTE FLOW IN PRESENCE OF MAGNETIC FIELD

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The stability of couette flow of an electrically conducting fluid in between two concentric cylinders with a wide gap is discussed by using an initial value method. The walls of the cylinders are electrically non-conducting. The inner cylinder is rotated impulsively while the outer cylinder is kept at rest. The results for hydromagnetic stability are compared with those of hydrodynamic stability. Stabilizing effect of the magnetic field on such flows is confirmed by our results. It is shown that for each Reynolds number there is a suppression of the occurrence of instability and that the appearance of instability disks is delayed by as much as twice the time required in the case of hydrodynamic stability. The MHD couette flow is shown to be stable at $Re = 50$ which proves that for this case the critical Reynolds number lies above 50, whereas for the corresponding hydrodynamic case critical Reynolds number is 22.1

INTRODUCTION

The stability of the time dependent rotational couette flow for hydrodynamic case has been discussed experimentally as well as theoretically by Kirchner and Chen (1970, 1971). They have tackled the problem by two methods. In the initial value method, a perturbation of a definite amplitude and wavelength is fed into the fluid flows, and the linear momentum equations (with specific boundary conditions) and the time evolution of these perturbations are examined to obtain the perturbation kinetic energy of the system. They have brought out the important result that the time of the occurrence of the intrinsic instability is about one-fourth of the time required for a thousand-fold increase in the perturbation energy at which instant the instability disks are clearly visible. It is also shown that the flow is stable below $Re = 22.1$, where Re is the Reynolds number. In the second method, the authors have used the quasi-steady-approach

Their analysis is based on the fact that the methods for the linear stability analysis of rotating couette flows, laminar boundary layer flows and Benard problem cannot be applied in a basically time-dependent flow, as the perturbations are continuously interacting with the basic flow which in itself is evolving with time. Hence, a possible method of solution can be obtained by introducing into the flow field perturbations of small amplitudes but of different wavelengths. For a disturbance of given wavelength and small amplitude the time-rate growth of perturbations is obtained by integrating linear perturbation equations. This gives the time evolution of perturbation kinetic energy. Its growth or decay with respect to time indicates the stability of the MHD flow.

In the present paper, a study of the linear stability of couette flow of an electrically conducting fluid in between the annulus of two concentric cylinders is made. The basic flow originates from the impulsive rotation of inner cylinder while the outer cylinder is kept at rest. Both the cylinders are electrically non-conducting and are separated by a wide gap. For experimental work, mercury can be taken as a suitable fluid along with the physical values of the necessary parameters. Our problem differs from the hydrodynamic problem (cf. Chen & Kirchner 1971) in two respects. The first is the nonavailability of sufficient boundary conditions for solving the parabolic equations of magnetic induction. Secondly, very high resistivity of the fluid renders the time evolution of magnetic perturbations too much time consuming on a computer as the diffusion terms are of very high order compared to the time derivatives of the magnetic perturbation parameters.

Considering the possibility of the existence of irrotational fields outside the non-conducting boundaries, it is possible to change the order of the magnetic induction equation and hence solve it by extending the perturbation velocity symmetrically on both sides. Latter difficulty is overcome by making an approximation for time derivatives as suggested by Hunt (1966).

The effect of the magnetic field in suppressing the instability of such flows is established for the three Reynolds numbers considered in the paper. The critical wave number (for which the rate of growth of the total energy of the flow is the highest) is calculated for each of the three Reynolds numbers. Though the experimental values of the critical wave numbers are not available, the computed values are quite close to the range of the critical wave numbers used by Chandrasekhar (1961), Stuart (1954) and the other authors working on similar flows. The critical times to reach the minimum perturbation energy and the minimum relative perturbation energy for MHD flows are compared with those for the hydrodynamic flows as discussed by Chen and Kirchner (1971).

BASIC EQUATIONS

Mercury as a conducting fluid is considered in between the annulus of two concentric electrically non-conducting cylinders of the inner radius R_1 and the outer radius R_2. The inner cylinder is impulsively rotated with a velocity V_1 and the outer is kept at rest. A constant magnetic field H is applied along the axis of the cylinder (z-axis). Some perturbations in the basic flow are introduced in terms of following parameters :

$$u(r, t, z) = u(r, t) \cos \kappa z; \; v(r, t, z) = v(r, t) \cos \kappa z;$$

$$w(r, t, z) = w(r, t) \sin \kappa z; \; p(r, t, z) = p(r, t) \cos \kappa z;$$

$$h_r(r, t, z) = \phi(r, t) \sin \kappa z; \; h_\theta(r, t, z) = \psi(r, t) \sin \kappa z;$$

$$h_z(r, t, z) = \chi(r, t) \cos \kappa z.$$

Here (u, v, w) denote the perturbation velocity components, p for the pressure,

(h_r, h_θ, h_z) for the perturbation magnetic field components and κ for the wave number of the perturbation.

The linear perturbation equations for the above flow are (cf. Chandrasekhar 1961) :

$$\left(DD_* - \kappa^2 - \frac{1}{\nu}\frac{d}{dt}\right)(DD_* - \kappa^2)u + \frac{\mu H\kappa}{4\pi\rho\nu}(DD_* - \kappa^2)\phi = 2\frac{V}{r\nu}\kappa^2 v \quad ...(1)$$

$$\left(DD_* - \kappa^2 - \frac{1}{\nu}\frac{d}{dt}\right)v + \frac{\mu H\kappa}{4\pi\rho}\psi - (D_* V)u = 0, \quad ..(2)$$

$$\left(DD_* - \kappa^2 - \frac{1}{\eta}\frac{d}{dt}\right)\phi = \frac{H\kappa\, u}{\eta} \quad ..(3)$$

and

$$\left(DD_* - \kappa^2 - \frac{1}{\eta}\frac{d}{dt}\right)\psi = \frac{H\kappa}{\eta}v \quad ..(4)$$

Here V, ν, μ, η, ρ are respectively the basic velocity component, coefficient of viscosity, magnetic permeability, magnetic resistivity and the fluid density. For mercury, η and ν are given by

$$\eta = 7.6 \times 10^3 \text{ cm}^2/\text{sec., and } \nu = 1.1 \times 10^{-3} \text{ cm}^2/\text{sec.} \quad ..(5)$$

Further,

$$D \equiv \frac{d}{dr} \text{ and } D_* \equiv \left(\frac{d}{dr} + \frac{1}{r}\right).$$

The boundary conditions on the nonconducting rigid boundaries are

$$\left.\begin{array}{l} V(r, t) = V_1 \quad \text{at} \quad r = R_1, \\ V(r, t) = 0 \quad \text{at} \quad r = R_2, \\ u = v = D_* u = 0 \text{ at } r = R_1 \text{ and } R_2. \end{array}\right] \quad ..(6)$$

For the magnetic field,

$\psi = 0$ at $r = R_1$ and R_2; further

$(\nabla \times \vec{h})_\theta$ and $(\nabla \times \vec{h})_z$ are determined on the boundaries by the equation

$\nabla \times \vec{h} = \vec{V} \times \vec{h}$; $\vec{V} \equiv (0, V, 0)$.

Using the non-dimensional numbers

$$\tau = \nu t/R_1^2 ;\kappa' = \kappa R_1; \quad Re = R_1 V_1/\nu; \quad r = r' R_1$$

$$u = V_1 u' : \quad \phi = H\phi'; \quad V = V_1 V'; \quad \psi = H\psi' \text{ and}$$

$Rem = R_1 V_1 / \eta$, the non-dimensional form of the equations (1)—(4) (after dropping the dashes) are :

$$\frac{\partial \zeta}{\partial \tau} = (DD_* - \kappa^2)\zeta + \frac{q\kappa}{Rem}(DD_* - \kappa^2)\phi - 2\, Re\, \kappa^2 \frac{vV}{r}, \quad ..(7)$$

$$\frac{\partial v}{\tau\partial} = (DD_* - \kappa^2)\, v + \frac{q\kappa}{Rem}\,\psi - Re\,(D_*\, V)u, \qquad ..(8)$$

$$\frac{\partial \phi}{\tau\partial} = \frac{Re}{Rem}\,(DD_* - \kappa^2)\,\phi - Re\kappa u, \qquad ..(9)$$

$$\frac{\partial \psi}{\partial \tau} = \frac{Re}{Rem}\,(DD_* - \kappa^2)\,\psi + Re\left(\frac{\partial V}{\partial r} - \frac{V}{r}\right)\phi - Re\,\kappa v \qquad ..(10)$$

and the basic flow equation can be written as

$$\frac{\partial V}{\partial \tau} = DD_* V \qquad ..(11)$$

where

$$\zeta = (DD_* - \kappa^2)u. \qquad ..(12)$$

In the equations (9) and (10), Re/Rem term is of the order of 10^6 and hence the time derivatives can be neglected (cf. Hunt 1966) in comparison with the terms on the right hand side. Thus, equations (9) and (10) take the forms

$$(DD_* - \kappa^2)\,\phi = \text{Rem}\;\kappa\; u \qquad ..(13)$$

and

$$(DD_* - \kappa^2)\,\psi = \text{Rem}\left(\frac{\partial V}{\partial r} - \frac{V}{r}\right)\phi\,. \qquad ..(14)$$

The boundary conditions on the velocity field are,

$u = v = D_*\, u = 0$ on $r = 1,\ R(=R_2/R_1)$;

$V(1,\, 0) = 1$; $V(r,\, 0) = 0$ for $1 < r < R$

$V(1,\tau) = 1$; $V(R,\, \tau) = 0$.

Boundary conditions on the magnetic field—Since there are no motions, either of the boundary or of the fluid at $r = R$, there flows no current at $r = R$ and hence $\nabla \times \vec{h} = 0$ at this boundary. Indeed, $\nabla \times \vec{h} = \vec{V} \times \vec{h} = 0$ at $r = R$ since $V \equiv (0, 0, 0)$ here. However, the conditions $(\nabla \times \vec{h})_r = (\nabla \times \vec{h})_\theta = 0$ at $r = R$ yield the boundary conditions;

(*i*) $\psi = 0$, as given by Chandrasekhar (1961) which is true for $r = 1$ also, and

(*ii*) $(DD_* - \kappa^2)\,\phi = 0$ which can also be obtained from the equation (13) and hence is also true for $r = 1$.

$(\nabla \times \vec{h})_z = 0$ at $r = R$ gives the condition $D_*\psi = 0$ at $r = R$ from which one can get

$(DD_* - \kappa^2)\psi = 0$ at $r = R$

Since, in general, $\frac{\partial V}{\partial r}$ is not zero at $r = R$ equation (14) yields

$$\phi = 0 \text{ at } r = R.$$

Thus the boundary conditions for magnetic field are

$$\left.\begin{array}{ll} \psi = (DD_* - \kappa^2)\,\phi = 0 & \text{at } r = 1, R) \\ \text{and } \phi = 0 & \text{at } r = R.) \end{array}\right\} \qquad ..(15)$$

Further operating both sides of equation (13) by D, one gets

$$D(DD_* - \kappa^2)\phi = \text{Rem}\ \kappa\ Du. \qquad ..(16)$$

Equations (7), (8), (14) and (16) are solved by using the boundary conditions (15) together with the boundary conditions for the velocity field. ($D_* u = 0 \Rightarrow Du = 0$ on $r = 1, R$) For solving the parabolic equations (7) and (8), we follow the method suggested by Chen and Kirchner (1971). The second and the third order equations (14) and (16) are solved by a kind of difference method suggested by Fox (1957).

The perturbation energy of the flow is calculated at each time step and is integrated over one wavelength of the fluid along the z-axis.

The kinetic energy E_p of the perturbation flow is given by

$$E_p = \pi \int_1^R r\left(u^2 + v^2 + \frac{1}{\kappa^2}(D_* u)^2\right)dr,$$

where $2\pi\ \rho\ V_1^2\ R_1^3/\kappa$ is a normalising factor.

The basic kinetic energy is

$E_b = 2\pi \int_1^R r\ V^2$ dr by using the same normalising factor.

The magnetic perturbation energy is

$$M_p = \frac{1}{24}\int_1^R r\left(\phi^2 + \psi^2 + \frac{1}{\kappa^2}(D_*\phi)^2\right)dr,$$

which is normalised by the factor $6\ \rho\ \mu\ \pi\ R_1^3\ H^2/\kappa$.

Numerical Method

In equation (12), ζ is given small initial random values at the internal grid points considered along the radius of the cylinder. These random values are generated between 0 and 1 from a library function RANF (—1) available in the computer CDC 3600. They are multiplied by 10^{-2} to make their magnitudes small. From equation (12), the values of u at the internal grid points are calculated using the difference formula (cf. Chen & Kirchner (1971)

$$u^{n}_{j,k+1} = \frac{\omega}{\lambda_j}\left[\alpha_j\, u^{n}_{j+1,k} + \beta_j\, u^{n}_{j-1,k+1} - (\Delta r)^2\, \zeta^{n}_{j}\right]$$

$$- (\omega - 1)\, u^{n}_{j,k} \qquad (j = 1,2,\ldots, J-1), \qquad ..(17)$$

where J is the number of grid points considered and

$$\alpha_j = 1 + (\Delta r)/2r_j,$$

$$\beta_j = 1 - (\Delta r)/2r_j,$$

$$\lambda_j = 2 + (\Delta r)^2\left(\kappa^2 + r_j^{-2}\right)$$

and ω is the optimum relaxation factor for SOR method. ω is determined for each κ considered in this problem. This ω is a function of κ and varies from 1.95 to 1.86 as κ varies from 0.6 to 3. When $\Delta r = 2.5 \times 10^{-2}$, the number of iterations required for $\kappa = 0.6$ is 288 while that for $\kappa = 3.0$, it is 158. For the first calculation, u is assumed to be zero at all points. The time step is denoted by n, while k represents the number of iterations. The iterative process is terminated when the maximum relative error of the variables at two consecutive iterations is within 10^{-6}. The values of ζ at the boundary points are now calculated by making use of $u = Du = 0$ at $r = 1, R$.

Thus,

$$\zeta = \frac{\partial^2 u}{\partial r^2}$$ at $r = 1,\ R$, in the difference form becomes

$$\left.\begin{aligned} \zeta &= \frac{2u^{n}_{1}}{\Delta r^2} && \text{at } r = 1, \\ \text{and} \quad \zeta &= \frac{2u^{n}_{J-1}}{\Delta r^2} && \text{at } r = R. \end{aligned}\right\} \qquad ..(18)$$

Explicit difference scheme is used for integrating the equations (7), (8) and (11) at each time step. The difference equations are taken as

$$Q^{n-1}_{j} = \frac{\Delta\tau}{(\Delta r)^2}\left[\alpha_j\, Q^{n}_{j+1} + \beta_j\, Q^{n}_{j-1} + \left[\frac{(\Delta r)^2}{\Delta\tau} - \psi_j\right] Q^{n}_{j} + F_j\,(Q)\right] \qquad ..(19)$$

where

$$F_j(Q) = \begin{cases} (\Delta r)^2\, q\kappa/Rem\, \psi_j - \dfrac{Re}{2}\,(\Delta r)\left[V_{j+1} - V_{j-1} + \dfrac{V_j}{r_j^2}\,(\Delta r)\right] v_j & \text{for } Q = v \\ \kappa^2\,(\Delta r)^2\, V_j & \text{for } Q = V \\ q\kappa^2\, u_j(\Delta r)^2 - 2Re(\Delta r)^2\, \kappa^2\, v_j V_j/r_j & \text{for } Q = \zeta \end{cases}$$

where q is a non-dimensional number. $q = \mu H^2 R_1^2/(4\pi\rho\nu\eta)$ (Chandrasekhar 1961) To ensure stability of the numerical scheme, we use $(\Delta\tau)/(\Delta r)^2$ less than 0.5.

At each time step, equations (14) and (16) are solved using a difference scheme (cf. Fox 1957). By using the central difference, equation (14) transforms to

$$\psi_j = \psi^{(1)}_j + C_\psi \, \psi^{(2)}$$

where

$$\psi^{(1)}_{j+1} = \{1/\alpha_j)\,(\lambda_j\,\psi^{(1)}_j - \beta_j\,\psi^{(1)}_{j-1} + (\Delta r)^2\,Rem\,[(V_{j+1} - V_{j-1})/2\,(\Delta r) - V_j/r_j]\phi_j - (\Delta r)^2\,Rem\,\kappa\,v_j\} \quad ..(20)$$

and

$$\psi^{(2)}_{j+1} = (1/\alpha_j)\left(\lambda_j\,\psi^{(2)}_j - \beta_j\,\psi^{(2)}_{j-1}\right). \quad ..(21)$$

They are solved under the initial values

$$\psi^{(1)}_0 = 0,\ \psi^{(1)}_1 = 1 \qquad \text{for (20)}$$

and

$$\psi^{(2)}_0 = 0,\ \psi^{(2)}_1 = 0 \qquad \text{for (21)}$$

The constant C_ψ is calculated by using the other boundary condition on ψ. Hence we have

$$C_\psi = -\,\psi^{(1)}_J/\psi^{(2)}_J.$$

In equation (20), ϕ is calculated by

$$\phi_j = \phi^{(1)}_j + C_\phi\,\phi^{(2)}_j,$$

where $\phi^{(1)}_j$ is obtained from the equation

$$\phi^{(1)}_{j+2}\,C_1(r_j) + \phi^{(1)}_{j+1}\,C_2(r_j) + \phi^{(1)}_j\,C_3(r_j) + \phi^{(1)}_{j-1}\,C_4(r_j) = (\Delta r)^3\,Rem\,\kappa\,Du_j \quad ..(22)$$

with the initial values,

$$\phi^{(1)}_{J-1} = \phi^{(1)}_J = 0 \text{ and } \phi^{(1)}_{J+1} = (\Delta r)^2/12)\,Rem\,\kappa\,u_{J+1}\left(1 - \frac{C_{\phi_1}}{C_{\phi_2}}\right). \quad ..(23)$$

Constants C_1, C_2, C_3 and C_4 are the same as given by Fox (1957), pp. 142, Eq. (54).

$\phi_j^{(2)}$ is obtained from,

$$\phi_{j+2}^{(2)} C_1(r_j) + \phi_{j+1}^{(2)} C_2(r_j) + \phi_j^{(2)} C_3(r_j) + \phi_{j-1}^{(2)} C_4(r_j) = 0, \qquad ..(24)$$

with the initial values,

$$\phi_{J-1}^{(2)} = 1, \phi_J^{(2)} = 0 \text{ and } \phi_{J+1}^{(2)} = C_{\phi 1}/C_{\phi 2}$$

where

$$C_{\phi 1} = (\Delta r)^2 + (\Delta r)^2/2(-1/r_J) + (\Delta r)^3/6)\left(3/r_J^2 + \kappa^2\right)$$
$$+((\Delta r)^4/24)\left(-12/r_J^3 - \kappa^2/r_J^3 - \kappa^2/r_J\right. \qquad ..(25)$$

calculated at $r = r_{J+1}$ and $C_{\phi 2}$ is calculated using (25) at $r = r_{J-1}$. The constant C_ϕ is obtained by (J is the suffix at the inner wall)

$$C_\phi = \left(\lambda_J \phi_{J+1}^{(1)} - \alpha_J \phi_{J+2}^{(1)} - \beta_J \phi_J^{(1)}\right)\Big/\left(\alpha_J \phi_{J+2}^{(2)} + \beta_J \phi_J^{(2)} - \lambda_J \phi_{J+1}^{(2)}\right). \qquad ..(26)$$

First, u is calculated from (17) and ζ at the boundaries from (18). Then the equations (22) & (24) are solved to get ϕ_j. The equation for ψ is solved at the next step. Using these values in (19) and taking the initial values of V and v to be zero at the internal grid points, the values for the first time step are calculated. The new values of ζ are used again in (17) along with the values of u at the previous time step for obtaining the values of u at further time steps. The process is continued.

Results and Discussion

Numerical results are obtained for a fixed q (= 0.01) and a radius ratio (R_1/R_2) = 0.2. This value is chosen for making a comparison between the hydrodynamic results of Chen and Kirchner (1971) and our hydromagnetic results. The number of the grid points considered is 160. We could not refine the mesh further due to memory restriction on our computer.

Critical wave numbers for Re equal to 400, 260 and 50 are calculated. For Re = 400, critical κ came out to be 1.41 (Fig. 1). Fig. 2 shows the total energy E_p and the basic flow energy E_b plotted against τ at the critical κ. Fig, 3 shows a decrease in the time of the occurrence of intrinsic instability for an increase in Re computed at Re = 400 and Re = 260. Table I gives a comparison of the critical times required to reach E_{tm} (the minimum total perturbation energy) and E_{rm} (minimum relative energy = E_{tm}/E_b) for the hydromagnetic and the hydrodynamic flows. For Re = 400, E_{tm} is attained at τ = 0.017 whereas for the hydrodynamic couette flow, it is at τ = 0.011. It means that there is a delay in the occurrence of the intrinsic instability for MHD flow compared to the hydrodynamic case. It is apparent

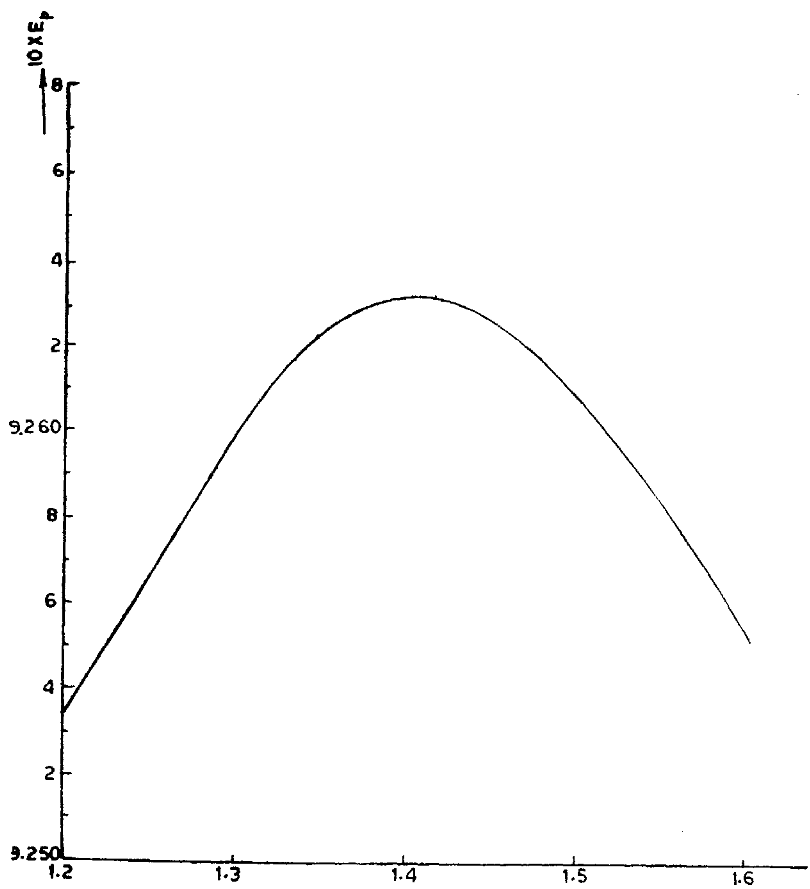

FIG. 1. Critical κ for Re = 400.

that the effect of the magnetic field on the flow field is profound whereas its contribution to the total energy remains uncomparable even after time $\zeta = 0.1$. According to Chen and Kirchner (1971) instability disks occur at the 1000th fold

TABLE I

Comparison of critical times to reach minimum total perturabation energy E_{tm} and minimum relative perturbation energy E_{rm} at calculated critical wave numbers κ_{crit} for two Reynolds numbers (Re)

Re	Critical times					
	Hydrodynamic case			Hydromagneitc case		
	κ_{crit}	E_{tm}	E_{rm}	κ_{crit}	E_{tm}	E_{rm}
260	6 16	0.0203	0.0221	1.40	0.0286	0.0416
400	7.96	0.0115	0.0120	1.41	0.0177	0.0277

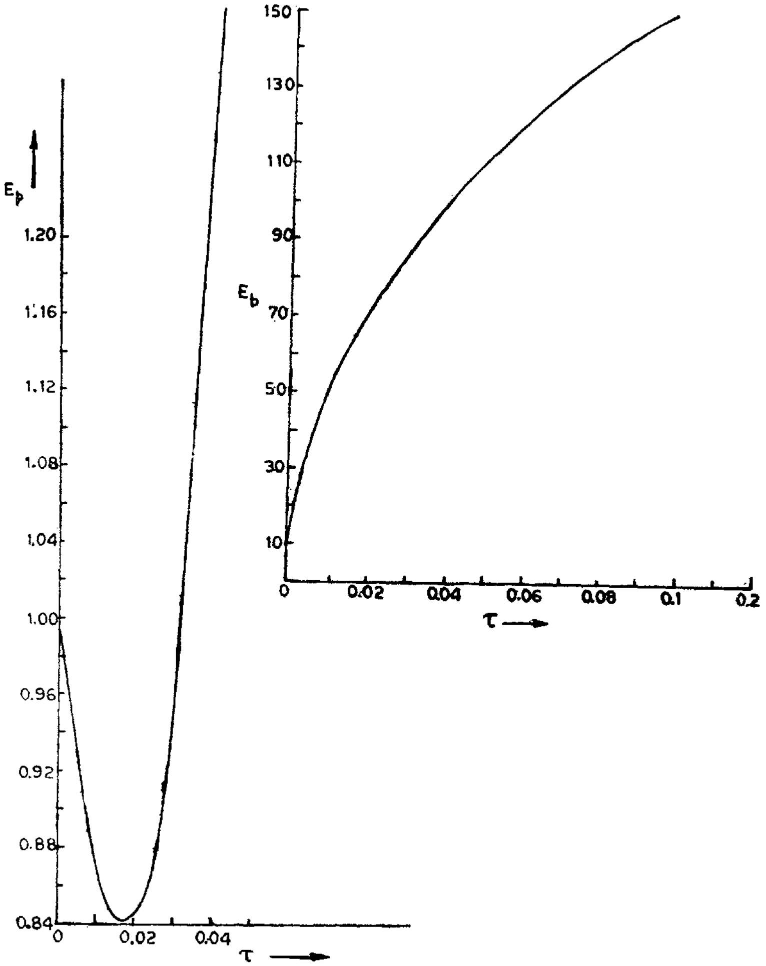

Fig. 2 Comparison of growth of the basic energy and the perturbation energy at $Re = 400$.

increase of the perturbation energy. As in the case of intrinsic instability, there is a long delay of the time to reach a 1000th fold increase in the perturbation energy, thus prolonging the appearance of the instability disks.

From Table I, one finds that the time of the intrinsic instability, in the case of relative energy, is actually doubled compared to the hydrodynamic case. This means that the growth of the basic flow energy is quite fast compared to the growth of the perturbation energy. The reason for this becomes somewhat clear when one observes the values of the parameter after time $\zeta = 0.02$. The growth of the magnetic energy is found to be small. It can be due to two reasons. First, the maintenance of an irrotational field outside the boundary suppresses the oncoming magnetic perturbation and hence makes the growth of magnetic perturbation parameters negligible compared to the basic applied field. Second factor is due to the high resisti-

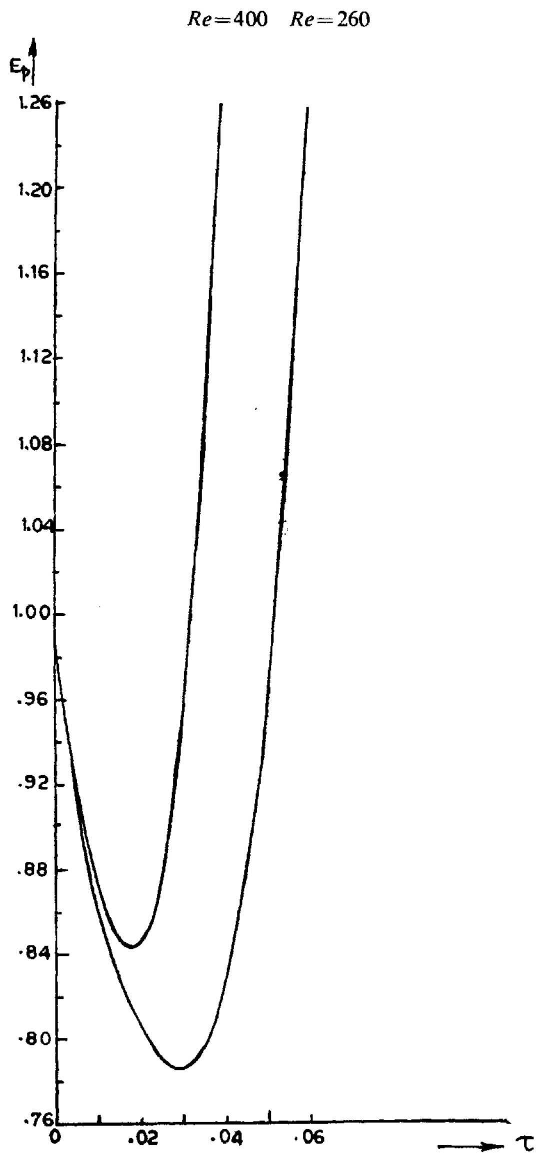

FIG. 3. Perturbation energy curves for $Re = 400$ and $Re = 260$.

vity of the fluid. The perturbation field diffuses fast through the fluid. However, the interaction between the flow field and the magnetic perturbations are significant as the rotating fluid has to cross the magnetic lines which are not carried away by the flow.

The parameters u, ζ, v, V were observed after time $\tau = 0.02$. At this time step, velocity component u is found to have negative amplitude at all the interior grid points; it is smaller near the boundary and has the maximum values in the middle region. The values of u at this time step meet the tolerance limit of accu-

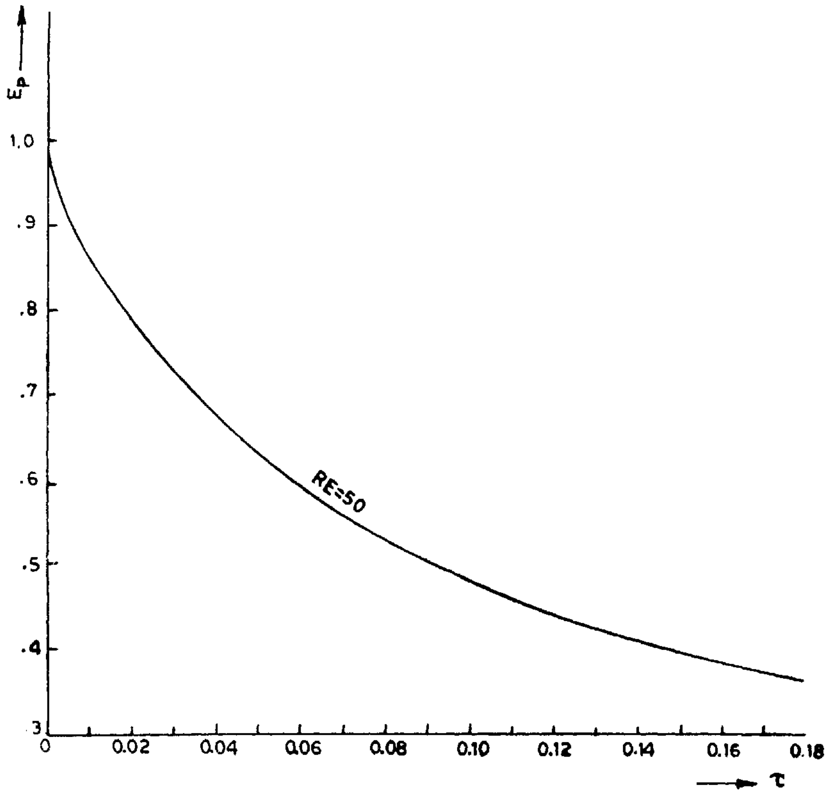

FIG. 4. Perturbation energy curve for $Re = 50$, $\kappa_{crit} = 1.25$.

racy (viz., 10^{-6}). The amplitude of u is small at the inner wall due to the coriolis force generated at the rotating inner cylinder. It is small at the grid points near the wall of the outer cylinder because the fluid encounters reflected oncoming flow due to the stationary outer cylinder. The picture appears to be reversed in the case of the θ-component of the velocity-v. There is a ring of the fluid rotating with the basic velocity developed at that point and hence it influences the flow in both the directions. The viscosity of the fluid drags the fluid after this ring towards the outer cylinder to develop a positive amplitude, whereas the same viscosity diminishes the amplitude towards the inner cylinder. This is due to the influence of the basic velocity from the inner rotating cylinder and due to no contribution from the stationary outer cylinder. This ring appears between 38 and 39th grid points for $J = 160$ at the time $\zeta = 0.02$.

The starting value $E_p = 1$ is reached at $\tau = 0.0511$ for $Re = 260$ and the growth rate remains almost constant after this time step.

For $Re = 50$, the flow is found to be stable. The energy curve decreases exponentially (cf. Fig. 4). The value of the critical Reynolds number for the hydrodynamic case is obtained as 22.1 whereas it is more than 50 for the MHD case. The effect of the magnetic field is observed to suppress the instability of the flow. The critical κ for $Re = 50$ is obtained to be 1.25.

CONCLUSION

We have discussed the stability of time-dependent couette flow in between two concentric cylinders with a wide gap in the presence of a constant magnetic field, the cylinders being electrically non-conducting. An initial value method is used. The criterion for the stability of the flow is obtained by calculating the time evolution of the total perturbation energy. As in the case of the hydrodynamic problem (Chen and Kirchner 1971), the perturbation energy for the MHD case also decreases in the beginning, attains a minimum and then grows exponentially with time for those Reynolds numbers which are above a critical value. If the instability disks were to appear at the 1000th fold increase of perturbation energy from the time of the occurrence of the intrinsic instability, then the effect of the magnetic field is found to prolong this appearance of instability disks to twice (or more) the corresponding time for the hydrodynamic case.

The flow is observed to be stable at $Re = 50$. This implies that the critical Reynolds number may be somewhere above 50. The irrotational field outside the boundaries appear to have a profound effect on the perturbation magnetic field components as well as on the flow field.

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