

DISPERSION OF SOLUTES IN LAMINAR HYDROMAGNETIC FLOWS WITH HOMOGENEOUS AND HETEROGENEOUS CHEMICAL REACTIONS

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A generalized theory for the investigation of the dispersion of soluble matters in an electrically conducting fluid flowing between two parallel plates in the presence of a transverse magnetic field has been presented here. The flow is accompanied by an irreversible first-order chemical reaction. We have discussed here the effect of both homogeneous and heterogeneous reactions under isothermal conditions. When magnetic field remains constant, the numerical results show that for homogeneous reaction in the bulk of the conducting liquid, the effective Taylor diffusion coefficient decreases with increase in the reaction rate constant. Further, if there is a heterogeneous reaction at the catalytic walls, the diffusion coefficient decreases with increase of the wall catalytic parameter when bulk reaction rate and Hartmann number are constant. These results are compatible with the results deduced earlier by Gupta and Gupta (1972) for the case of non-magnetic flows. If we tend Hartmann number to zero, our expressions for the concentration, volumetric rate, effective dispersion coefficient tend to the corresponding expressions obtained by the above authors.

On the other hand if we keep bulk reaction rate constant, the effective diffusion coefficient decreases with the increase of Hartmann number. We obtain the same result even in the presence of heterogeneous chemical reactions of the walls. Making volume reaction rate tend to zero, we have obtained the previous results by Gupta and Chatterjee (1968).

Our work not only generalizes the work of previous authors but also unifies them. As limiting cases we have indicated that, we can deduce the results of Gupta and Chatterjee (1968) and Gupta and Gupta (1972) and whence we can deduce the results obtained by Wooding (1960).

1 INTRODUCTION

Taylor (1953, 1954 *a, b*) initiated the investigation of the dispersion of a soluble matter in a non-conducting viscous fluid flowing through a circular tube under laminar conditions. His results show that the soluble matter can be regarded as dispersing along the tube with an apparent diffusion coefficient $R^2 \bar{V}_x^2 / 48D$, where R is the radius of the circular tube and D is the molecular diffusion coefficient. He has also shown that the condition under which his analysis is valid is $4L/R \gg \bar{V}_x R/D \gg 6.4, L$ being the length in the flow direction. Aris (1956) extended Taylor's results and

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established that the rate of growth of variance of the solute distribution is proportional to the sum of the molecular diffusion coefficient and Taylor diffusion coefficient. His analysis removed the restriction imposed by Taylor. Subsequently Taylor's analysis was extended to the case of different models of non-Newtonian fluids by Fan and Hwang (1965), Fan and Wang (1966), Ghoshal (1971), Ghoshal *et al.* (1971). Gupta and Chatterjee (1968) investigated the dispersion of soluble matter in the hydromagnetic laminar flow between two parallel plates.

All the investigations mentioned above deal with flows where the solute does not chemically react with the liquid through which it is dispersed. But in practice we have to deal with a wide variety of problems where diffusion of a solute takes place with simultaneous chemical reactions, e.g., hydrolysis of ester, gas absorption in an agitated tank with chemical reaction and so on (Bird *et al.* 1960). Cleland and Wilhelm (1956) discussed the problem of a finite first-order homogeneous reaction in a pipe under laminar flow conditions by a finite difference method. He supported his results with experimental data. Katz (1959) investigated the effect of heterogeneous reaction at the wall on the concentration profiles. While combined first order heterogeneous and homogeneous reactions were studied by Walker (1961) and Solomon and Hudson (1967).

The above investigations were confined to chemical reactions taking place under steady state conditions. Gupta and Gupta (1972) discussed the unsteady dispersion of a solute with simultaneous chemical reaction in a laminar flow of a liquid flowing between two parallel plates using Taylor's approach.

One of the important factors is to study how the external magnetic field influences the dispersion. We have presented here a generalised theory which unifies the investigations of some of the above workers.

The paper presents a study of the unsteady dispersion of a solute with simultaneous chemical reaction in a laminar flow of an electrically conducting fluid between two parallel plates in the presence of an external magnetic field. We have solved the problem using Taylor's approach. The results show that the solute is dispersed relative to a plane moving with the mean velocity of the fluid with an effective diffusion coefficient which decreases with increase of the magnetic field (Hartmann number) when reaction rates remain constant. It also decreases with the increases of reaction rates when magnetic field remains constant. We have shown here that if we take the limit reaction rate $\rightarrow 0$ in our results, we obtain the corresponding results obtained by Gupta & Chatterjee (1968) while if Hartmann number tends to zero, we obtain the case discussed by Gupta & Gupta (1972).

The first part of our paper (§ 2) deals with the study of dispersion of a solute through a conducting fluid flowing between two parallel plates in the presence of transverse magnetic field and a homogeneous first order irreversible chemical reaction taking place in the bulk of the fluid. The effective dispersion coefficient is calculated applying Taylor's limiting condition.

In the last part (§ 3) we have extended our investigations for the case of a dispersion through a conducting fluid flowing in the presence of combined first-order heterogeneous and homogeneous chemical reactions.

2. DISPERSION OF A SOLUTE IN AN ELECTRICALLY CONDUCTING FLUID IN THE PRESENCE OF HOMOGENEOUS FIRST-ORDER IRREVERSIBLE CHEMICAL REACTION IN THE BULK OF THE LIQUID

Here we consider a laminar pressure flow of a conducting liquid with conductivity σ and permeability μ between two infinite parallel plates distant $2d$ apart in the presence of a uniform transverse magnetic field H along y -axis. This well-known Hartmann flow (Cowling 1957) where the velocity U_x (x -axis is parallel to the plates) is given by

$$U_x = Z \left[M \coth M - \frac{M \cosh (M \cdot y/d)}{\sinh M} \right], \quad \dots (1)$$

$$\text{where } Z = \frac{P}{\mu^2 \sigma H^2}, \quad M = \mu H d \left(\frac{\sigma}{\rho \nu} \right)^{1/2}. \quad \dots (2)$$

Here P is the pressure gradient along the plates and the Hartmann number M defines the relative strength of the magnetic field compared to viscous effects. So the average velocity.

$$\bar{U} = \frac{1}{2d} \int_{-d}^d U_x dy$$

is given by $\bar{U} = Z (M \coth M - 1)$ (3)

In the present problem a solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in the liquid under isothermal conditions. The equation for the concentration C of the solute satisfies.

$$\frac{\partial c}{\partial t} + U_x \frac{\partial c}{\partial x} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) - k_1 c \quad \dots (4)$$

in which D is the molecular diffusion coefficient (assumed to be constant) and K_1 , is the first order reaction rate constant. Eq. (4) is derived under the assumption that the concentration of the solute present is small, the last term $-K_1 C / \text{mol } C_m^{-3s-1}$ represents the volume rate of disappearance of the solute due to chemical reaction.

We now assume that $\frac{\partial^2 c}{\partial x^2} \ll \frac{\partial^2 c}{\partial y^2}$

It will be advantageous if we consider convection across a plane moving with the mean speed of the flow. Then relative to this plane the fluid velocity is given by

$$v_x = U_x - \bar{U} = Z \left[1 - \frac{M}{\sinh M} \cosh \left(M \cdot \frac{y}{d} \right) \right] \quad \dots (5)$$

We introduce here the following non-dimensional quantities

$$\theta = \frac{t}{\tau}, \quad \hat{t} = \frac{L}{\bar{U}}, \quad \xi = \frac{x - \bar{U} t}{L}$$

$$\eta = \frac{y}{d} \dots \quad \dots (6)$$

Using (5) & (6), equation (4) transforms to

$$\frac{1}{t} \frac{\partial c}{\partial \theta} + \frac{v_x}{L} \frac{\partial c}{\partial \xi} = \frac{D}{d^2} \frac{\partial^2 c}{\partial \eta^2} - k_1 c, \dots \quad (7)$$

where L is a typical length along flow direction.

To solve the equation, we apply Taylor's limiting condition arguments. We assume that the partial equilibrium is established in any cross-section of the channel so that variation of C with η is given

$$- \frac{K_1 d^2}{D} C + \frac{\partial^2 c}{\partial \eta^2} = \frac{d^2}{DL} v_x \frac{\partial c}{\partial \xi} \quad \dots \quad (8)$$

Using (5) and integrating (8), the concentration of the solute is given as

$$C = \frac{AM}{M^2 - \lambda^2} \left[\frac{M \cosh(\lambda \eta)}{\lambda \sinh \lambda} - \frac{\cosh(M \eta)}{\sinh M} - \frac{M^2 - \lambda^2}{M \lambda^2} \right]. \quad \dots \quad (9)$$

$$\text{where } A = \frac{d^2 Z}{DL} \frac{\partial c}{\partial \xi} \quad \text{and } \lambda^2 = \frac{K_1 d^2}{D}.$$

$$\text{The boundary conditions are } \frac{\partial c}{\partial \eta} = 0 \text{ at } \eta = \pm 1$$

The volumetric rate Q of transport of the solute across a section of the channel of unit breadth is

$$Q = \int_{-d}^d c v_x dy = \int_{-1}^1 dc v_x d\eta. \quad \dots \quad (10)$$

So that, using (5) and (9), Q becomes

$$Q = \frac{2d^3 M^2}{(M^2 - \lambda^2) DL} \cdot \frac{\partial c}{\partial \xi} \cdot \frac{\bar{U}^2}{\alpha^2} \left[\frac{M^2}{\lambda (\lambda^2 - M^2)} \coth \lambda + \frac{(3\lambda^2 - 5M^2)}{2M^2(\lambda^2 - M^2)} \alpha + \frac{\alpha^2}{2M^2} - \frac{1}{2} + \frac{1}{\lambda^2} - \frac{1}{(\lambda^2 - M^2)} \right], \quad \dots \quad (11)$$

$$\text{where } \alpha = M \coth M - 1. \quad \dots \quad (12)$$

Let us compare our results (11) with Ficks' law of diffusion. It shows that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* , where

$$D^* = \frac{d^2 \bar{U}^2 M^2}{D \alpha^2 (M^2 - \lambda^2)} \left[\frac{1}{2} - \frac{M^2}{\lambda (\lambda^2 - M^2)} \coth \lambda - \frac{(3\lambda^2 - 5M^2)}{2M^2(\lambda^2 - M^2)} \alpha - \frac{\alpha^2}{2M^2} - \frac{1}{\lambda^2} + \frac{1}{(\lambda^2 - M^2)} \right]. \quad \dots \quad (13)$$

D^* can be expressed conveniently in the following form using (2) and (3).

$$D^* = \frac{d^6 P^2}{D \rho^2 \nu^2} F(M, \lambda),$$

$$\text{where, } F(M, \lambda) = \frac{1}{2M^2(M^2-\lambda^2)} \left[\frac{1}{2} - \frac{M^2}{\lambda(\lambda^2-M^2)} \text{Coth } \lambda \right. \\ \left. - \frac{(3\lambda^2-5M^2)}{2M^2(\lambda^2-M^2)} \alpha - \frac{\alpha^2}{2M^2} - \frac{1}{\lambda^2} + \frac{1}{\lambda^2-M^2} \right].$$

Results and Conclusion

Table I presents numerical values of $F(M, \lambda)$ for different intensities of the magnetic field and reaction rate. It gives values of $F(M, \lambda)$ when both M and λ vary. It is evident from the table that the effective dispersion coefficient decreases with increase in the reaction rate constant λ and Hartmann number M . The results are compatible with the physics of the problem. Increase in λ signifies that increasing number of moles of solute undergoing chemical reaction resulting in a drop in dispersion coefficient. On the other hand, in Hartmann flow (Taylor 1954a), we have seen that the velocity profile becomes flatter with the increase of Hartmann number M , when compared to the profiles of non-magnetic case. This means for a fixed pressure gradient the flow rate decreases with increase in M .

In particular, the results obtained by some of the previous workers (Gupta & Chatterjee 1968; Gupta & Gupta 1972; and Wooding 1960), follow from our results very easily viz., the effective dispersion coefficient decreases if either of M and λ is increased keeping the other constant. However, the effective dispersion coefficient decreases more rapidly when both M and λ increase.

In this limiting case, when $\lambda \rightarrow 0$ i.e., in the absence of first order chemical reaction, we obtain from our results (9) and (11)

$$C = \frac{d^2 Z}{DL} \cdot \frac{\partial c}{\partial \xi} \left[\frac{\eta^2}{2} - \frac{\cosh M\eta}{M \sinh M} \right] + C_0$$

$$Q = \frac{2d^3}{DL} \frac{\bar{U}^2}{\alpha^2} \left[-\frac{5}{6} + \frac{\alpha^2}{2M^2} + \frac{5\alpha}{2M^2} \right] \frac{\partial c}{\partial \xi}$$

TABLE I

Variation of $F(M, \lambda)$ with the Hartmann

λ	$M=10$	$M=20$	$M=30$	$M=40$	$M=50$
0.4	$0.20053241 \times 10^{-4}$	$0.16252829 \times 10^{-5}$	$0.34833709 \times 10^{-6}$	$0.11470206 \times 10^{-6}$	$0.48106741 \times 10^{-7}$
0.8	$.19263901 \times 10^{-4}$	$.15672662 \times 10^{-5}$	$.33647597 \times 10^{-6}$	$.11090118 \times 10^{-6}$	$.46540286 \times 10^{-7}$
1.2	$.18091876 \times 10^{-4}$	$.14809604 \times 10^{-5}$	$.31881779 \times 10^{-6}$	$.10524030 \times 10^{-6}$	$.44206977 \times 10^{-7}$
1.6	$.16695521 \times 10^{-4}$	$.13778089 \times 10^{-5}$	$.29768978 \times 10^{-6}$	$.98463786 \times 10^{-7}$	$.41413152 \times 10^{-7}$
2.0	$.15218858 \times 10^{-4}$	$.12682510 \times 10^{-5}$	$.27521684 \times 10^{-6}$	$.91251252 \times 10^{-7}$	$.38438507 \times 10^{-7}$
2.4	$.13767810 \times 10^{-4}$	$.11600086 \times 10^{-5}$	$.25297117 \times 10^{-6}$	$.84105721 \times 10^{-7}$	$.35490146 \times 10^{-7}$
2.8	$.12407035 \times 10^{-4}$	$.10578335 \times 10^{-5}$	$.23193487 \times 10^{-6}$	$.77338484 \times 10^{-7}$	$.32696412 \times 10^{-7}$
3.2	$.11168109 \times 10^{-4}$	$.96410372 \times 10^{-6}$	$.21256630 \times 10^{-6}$	$.71106399 \times 10^{-7}$	$.30121960 \times 10^{-7}$
3.6	$.10060268 \times 10^{-4}$	$.87958074 \times 10^{-6}$	$.19505665 \times 10^{-6}$	$.65411805 \times 10^{-7}$	$.27788473 \times 10^{-7}$
4.0	$.90798748 \times 10^{-5}$	$.80408836 \times 10^{-6}$	$.17936565 \times 10^{-6}$	$.60395734 \times 10^{-7}$	$.25692457 \times 10^{-7}$

and the effective dispersion coefficient

$$D^* = \frac{P^2 d^6}{D \rho^2 \nu^2} F(M)$$

$$\text{where } F(M) = \frac{1}{M^4} \left[\frac{5}{6} - \frac{\alpha^2}{2M^2} - \frac{5\alpha}{2M^2} \right].$$

These expressions were given by Gupta & Chattrjee while investigating laminar flow between two parallel plates.

If we again make $M \rightarrow 0$ in the above expressions we obtain Wooding's (1960) results.

We may also deduce the corresponding results for non-magnetic case from our results by letting M in Eq. (9) and (11).

$$C = -A \cosh \lambda \eta - \frac{d^2}{2\rho\nu L^2 K_1} \frac{dP}{d\xi} \cdot \frac{\partial c}{d\xi} \left(\frac{1}{3} - \frac{2D}{K_1 d^2} - \eta^2 \right)$$

$$\text{where } A = \frac{dD^{\frac{1}{2}} \left(\frac{dP}{d\xi} \right) \left(\frac{\partial c}{\partial \xi} \right)}{\rho\nu L^2 K_1^{3/2} \sinh \lambda}$$

$$Q = \frac{2d^5 \left(\frac{dP}{d\xi} \right)^2}{\rho^2 \nu^2 L^3 K_1} \left(\frac{1}{3\lambda^2} - \frac{\coth \lambda}{\lambda^3} + \frac{1}{\lambda^4} - \frac{1}{45} \right).$$

The effective dispersion coefficient D^* is given by

$$D^* = \frac{d^6 \left(\frac{dP}{dx} \right)^2}{D \rho^2 \nu^2} F(\lambda),$$

$$\text{where } F(\lambda) = \frac{1}{\lambda^2} \left[\frac{1}{45} - \frac{1}{3\lambda^2} + \frac{\coth \lambda}{\lambda^3} - \frac{1}{\lambda^4} \right].$$

These results were obtained by Gupta and Gupta (1972) while investigating the effect of homogeneous and heterogeneous reactions on the dispersion of a soluble matter in

number and the reaction rate parameter

M=60	M=70	M=80	M=90	M=100
0.23565657 × 10 ⁻⁷	0.12862459 × 10 ⁻⁷	0.76026900 × 10 ⁻⁸	0.47770303 × 10 ⁻⁸	0.31503879 × 10 ⁻⁸
.22807597 × 10 ⁻⁷	.12452462 × 10 ⁻⁷	.73620365 × 10 ⁻⁸	.46266404 × 10 ⁻⁸	.30516482 × 10 ⁻⁸
.21678499 × 10 ⁻⁷	.11841683 × 10 ⁻⁷	.70034866 × 10 ⁻⁸	.44025858 × 10 ⁻⁸	.29045432 × 10 ⁻⁸
.20326268 × 10 ⁻⁷	.11110838 × 10 ⁻⁷	.65750462 × 10 ⁻⁸	.41342138 × 10 ⁻⁸	.27283331 × 10 ⁻⁸
.18886208 × 10 ⁻⁷	.10331000 × 10 ⁻⁷	.61166147 × 10 ⁻⁸	.38483341 × 10 ⁻⁸	.25406219 × 10 ⁻⁸
.17458511 × 10 ⁻⁷	.95584056 × 10 ⁻⁸	.56629829 × 10 ⁻⁸	.35688066 × 10 ⁻⁸	.23544437 × 10 ⁻⁸
.16105250 × 10 ⁻⁷	.8825947 × 10 ⁻⁸	.52329497 × 10 ⁻⁸	.32959395 × 10 ⁻⁸	.21778792 × 10 ⁻⁸
.14857722 × 10 ⁻⁷	.81505575 × 10 ⁻⁸	.48361652 × 10 ⁻⁸	.30479516 × 10 ⁻⁸	.20150121 × 10 ⁻⁸
.13726490 × 10 ⁻⁷	.75379525 × 10 ⁻⁸	.44762904 × 10 ⁻⁸	.28229427 × 10 ⁻⁸	.18672228 × 10 ⁻⁸
.12709865 × 10 ⁻⁷	.69872410 × 10 ⁻⁸	.41527030 × 10 ⁻⁸	.26205926 × 10 ⁻⁸	.17342993 × 10 ⁻⁸

the laminar flow between two parallel plates. Further if we let $\lambda \rightarrow 0$ we obtain Wooding's results (1960).

3. DISPERSION OF A SOLUTE IN A MAGNETOHYDRODYNAMIC FLOW ACCOMPANIED WITH COMBINED HOMOGENEOUS AND HETEROGENEOUS REACTIONS

Here we consider the dispersion of a solute in a conducting fluid flowing between two parallel plates in the presence of a transverse magnetic field. The flow is accompanied by first order irreversible chemical reaction taking place both in the bulk of the fluid as well as at the wall, walls which are assumed to be catalytic. Then the diffusion is governed by the following equation.

$$\frac{\partial c}{\partial t} + V_x \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial y^2} - K_1 c \quad \dots (14)$$

The boundary conditions are

$$\frac{\partial c}{\partial y} + f.c = 0 \text{ at } y = d \quad \dots (15)$$

$$\frac{\partial c}{\partial y} - f.c = 0 \text{ at } y = -d. \quad \dots (16)$$

There are suitable entry and initial condition. To solve the above equation we introduce the dimensionless variables (6) as before and assume the limiting condition of Taylor, to be valid. Then (14) becomes (8), subjected to the boundary conditions.

$$\frac{\partial c}{\partial \eta} + \beta.c = 0 \text{ at } \eta = 1 \quad \dots (17)$$

$$\frac{\partial c}{\partial \eta} - \beta.c = 0 \text{ at } \eta = -1 \quad \dots (18)$$

where $\beta = f.d.$ is the heterogeneous reaction rate parameter corresponding to the catalytic reaction at the walls so, the solution subjected to the boundary conditions (17) and (18) is

$$C(\eta) = A_1 \cosh \lambda \eta - A \left[\frac{M \cosh M\eta}{(M^2 - \lambda^2) \sinh M} + \frac{1}{\lambda^2} \right] \quad \dots (19)$$

where $A_1 = \frac{AM^2}{M^2 - \lambda^2} \left[\beta \left\{ \frac{\coth M}{M} + \frac{1}{\lambda^2} \right\} \right] + 1$
 $(\lambda \sinh \lambda + \beta \cosh \lambda)$,

The volumetric rate of transport of the solute across a section of the channel of unit breadth is

$$Q = \int_{-d}^d CV_x dy \quad \dots (19a)$$

Substituting (19) and (5) in (19a) we obtain

$$Q = \frac{2 d^3 M^2}{DL(M^2 - \lambda^2)} \frac{\partial c}{\partial \xi} \cdot \frac{\bar{U}^2}{\alpha^2} \left[\beta \frac{\sinh \lambda}{\lambda} - \frac{B.M}{M^2 - \lambda^2} \right]$$

$$\left\{ M \cosh \lambda - \lambda \operatorname{Coth} M \sinh \lambda \right\} - \frac{1}{M^2} + \frac{1}{2 \sinh^2 M} + \frac{\operatorname{Coth} M}{2M} \quad \dots \quad (20)$$

where $\alpha = M \operatorname{coth} M - 1$ and

$$B = \frac{\beta \left\{ \frac{\operatorname{coth} M}{M} + \frac{M^2 - \lambda^2}{M^2 \lambda^2} \right\} + 1}{(\lambda \sinh \lambda + \beta \cosh \lambda)} \quad \dots \quad (21)$$

If we compare (21) with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^* = - \frac{\bar{U} d^2}{D} G(\beta, \lambda, M), \text{ where}$$

$$G(\beta, \lambda, M) = \frac{M^2}{(M^2 - \lambda^2)} \cdot \frac{1}{\alpha^2} \left[B \cdot \frac{\sinh \lambda}{\lambda} - \frac{B \cdot M}{M^2 - \lambda^2} \right. \\ \left. \left\{ M \cosh \lambda - \lambda \operatorname{coth} M \sinh \lambda \right\} - \frac{1}{M^2} \right. \\ \left. + \frac{1}{2 \sinh^2 M} + \frac{\operatorname{coth} M}{2M} \right]$$

Results and Conclusion

An extensive numerical computation was carried out for the value of $G(\beta, \lambda, M)$. Table II contains the values of $G(\beta, \lambda, M)$ for different values of β and M when the bulk reaction rate constant λ is kept constant ($\lambda = 1.0$)

From the Table II, we find that the variations of $G(\beta, \lambda, M)$ indicate that even in this magneto-hydrodynamic case, the effective Taylor's dispersion coefficient decreases with the increase of the wall catalytic parameter. On the other hand if this wall catalytic parameter remains unchanged, the effective dispersion coefficient decreases as before, with the increase of Hartmann number. In general, the effective Taylor's dispersion coefficient decreases with the increase of either of β , M or both.

We may deduce as particular cases from our results (19), (20) and (21), the results obtained by a number of research workers before.

As for example by letting $M \rightarrow 0$ in (19) and (20) we get

$$c(\eta) = - \frac{d^2}{\rho \nu L^2 K_1} \cdot \frac{dP}{d\xi} \cdot \frac{\partial c}{\partial \xi} \left[\frac{1 + \beta \left(\frac{1}{3} + \frac{1}{\lambda^2} \right) \operatorname{Cosh} \lambda \eta}{(\lambda \sinh \lambda + \beta \cosh \lambda)} \right. \\ \left. + \frac{1}{2} \left\{ \frac{1}{3} - \frac{2}{\lambda^2} - \eta^2 \right\} \right]$$

$$Q = \frac{2}{5} \frac{\bar{U}^2 d^3}{L \lambda^2 D} \left[A_1 \sinh \lambda + B_1 \cosh \lambda \right]$$

TABLE II

Variation of $G(M, \lambda)$ with heterogeneous reaction rate parameter M and Hartman number λ

β	$M=10$	$M=20$	$M=30$	$M=40$	$M=50$
2	$.19650016 \times 10^{-2}$	$.56849338 \times 10^{-3}$	$.26429741 \times 10^{-3}$	$.15193438 \times 10^{-3}$	$.98515709 \times 10^{-4}$
4	$.19097251 \times 10^{-2}$	$.55117602 \times 10^{-3}$	$.25600724 \times 10^{-3}$	$.14711541 \times 10^{-3}$	$.95349445 \times 10^{-4}$
6	$.18871468 \times 10^{-2}$	$.54410356 \times 10^{-3}$	$.25262159 \times 10^{-3}$	$.14513902 \times 10^{-3}$	$.94056318 \times 10^{-4}$
8	$.18748790 \times 10^{-2}$	$.54026011 \times 10^{-3}$	$.25078113 \times 10^{-3}$	$.14406489 \times 10^{-3}$	$.93353461 \times 10^{-4}$
10	$.18671692 \times 10^{-2}$	$.53784449 \times 10^{-3}$	$.24962518 \times 10^{-3}$	$.14339020 \times 10^{-3}$	$.92911941 \times 10^{-4}$
12	$.18618756 \times 10^{-2}$	$.53618732 \times 10^{-3}$	$.24883123 \times 10^{-3}$	$.14292684 \times 10^{-3}$	$.92603752 \times 10^{-4}$
14	$.18580160 \times 10^{-2}$	$.53497776 \times 10^{-3}$	$.24825247 \times 10^{-3}$	$.14258900 \times 10^{-3}$	$.92387708 \times 10^{-4}$
16	$.18550776 \times 10^{-2}$	$.53405773 \times 10^{-3}$	$.24781195 \times 10^{-3}$	$.14233199 \times 10^{-3}$	$.92219561 \times 10^{-4}$
18	$.18527677 \times 10^{-2}$	$.53333398 \times 10^{-3}$	$.24746533 \times 10^{-3}$	$.14212948 \times 10^{-3}$	$.92087066 \times 10^{-4}$
20	$.18509002 \times 10^{-2}$	$.53274899 \times 10^{-3}$	$.24718525 \times 10^{-3}$	$.14196609 \times 10^{-3}$	$.91980124 \times 10^{-4}$

where

$$A_1 = \frac{3\lambda^6 - (\lambda^2 + 3)(45\lambda^2 + 15\lambda^2\beta + 45\beta)}{3\lambda^5(\lambda \sinh \lambda + \beta \cosh \lambda)}$$

$$B_1 = \frac{45\lambda^2 + 15\lambda^2\beta + 45\beta + \beta\lambda^4}{\lambda^4(\lambda \sinh \lambda + \beta \cosh \lambda)}$$

The results were obtained by Gupta and Gupta (1972), while investigating the effect of homogeneous & heterogeneous reactions on the dispersion of a solute in the laminar flow between two plates.

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M. for a fixed value of reaction rate constant ($\lambda = 1.0$)

M=60	M=70	M=80	M=90	M=100
.69000089 $\times 10^{-4}$.51001930 $\times 10^{-4}$.39225335 $\times 10^{-4}$.31101648 $\times 10^{-4}$.25262867 $\times 10^{-4}$
.66763698 $\times 10^{-4}$.49345224 $\times 10^{-4}$.37946883 $\times 10^{-4}$.30085306 $\times 10^{-4}$.24435587 $\times 10^{-4}$
.65857326 $\times 10^{-4}$.48668524 $\times 10^{-4}$.37424753 $\times 10^{-4}$.29670201 $\times 10^{-4}$.24097738 $\times 10^{-4}$
.65362008 $\times 10^{-4}$.48300894 $\times 10^{-4}$.37140955 $\times 10^{-4}$.29444614 $\times 10^{-4}$.23914097 $\times 10^{-4}$
.65050844 $\times 10^{-4}$.48069749 $\times 10^{-4}$.36962650 $\times 10^{-4}$.29302867 $\times 10^{-4}$.23793715 $\times 10^{-4}$
.64837193 $\times 10^{-4}$.47911140 $\times 10^{-4}$.36840268 $\times 10^{-4}$.29205541 $\times 10^{-4}$.23719512 $\times 10^{-4}$
.64681371 $\times 10^{-4}$.47795525 $\times 10^{-4}$.36751014 $\times 10^{-4}$.29134600 $\times 10^{-4}$.23661759 $\times 10^{-4}$
.64562817 $\times 10^{-4}$.47707464 $\times 10^{-4}$.36683042 $\times 10^{-4}$.29080580 $\times 10^{-4}$.23617805 $\times 10^{-4}$
.64469495 $\times 10^{-4}$.47638226 $\times 10^{-4}$.36620630 $\times 10^{-4}$.29033081 $\times 10^{-4}$.23583212 $\times 10^{-4}$
.64394204 $\times 10^{-4}$.47582289 $\times 10^{-4}$.36586476 $\times 10^{-4}$.29003778 $\times 10^{-4}$.23555239 $\times 10^{-4}$

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