WALL CONDUCTANCE EFFECTS ON CONVECTIVE HORIZONTAL CHANNEL FLOW

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In the present paper the combined free and forced convection flow of an electrically conducting fluid between two finitely conducting horizontal parallel walls with a linear axial temperature variation has been studied, when (i) the walls move with uniform different velocities in opposite directions, (ii) the walls move with the same velocities in opposite directions and (iii) one of the plates moves and the other is at rest (Couette flow).
A particular case when both the walls are at rest has also been deduced. An exact solution has been obtained to visualize the behaviour of the flow. The effect of the dimensionless physical parameters characterising flow on the velocity, the skin friction, current density, mass flow rate and temperature distribution have been studied in detail.

INTRODUCTION

Magnetohydrodynamic Couette flow is first studied by Lehnert (1952) assuming no electric field existing in the fluid. The effect of the magnetic field on general Couette, that is, the pressure-induced flow superposed on simple shear in classical hydrodynamics, has been studied by Agarwal (1962). The two dimensional free convection past a semi-infinite hot vertical plate in an electrically conducting fluid in the presence of a transverse magnetic field has been investigated by Gupta (1960), Sparow and Cess (1961), Cramer (1962), Lykoudis (1962), Singh and Cowling (1963) and Riley (1964). These researchers have shown that a magnetic field produces a flattening of the velocity and temperature profiles which also corresponds to a reduction in the local wall shear and heat transfer rate. Cowling (1957) and Pai (1962) have studied the flow problem between non-conducting walls whereas Chang and Yen (1962) have investigated the problem for finitely conducting walls. The heat transfer in MHD channel flows has been discussed by Chang and Lundgren (1959), Yen (1963), Soundalgekar (1969) and Jagadeesan (1964).

It is well known fact that the process of free convection has many applications as a mode of heat transfer. One of the examples is of cooling of nuclear reactors, where liquid sodium (which is electrically conducting) is used as a coolant. However the effect of the buoyancy forces on MHD forced convection flows does not seem to have received much attention. Recently, Gill and Casal (1962) and Gupta (1969) have studied the effect of buoyancy force on a forced convection flow through a horizontal channel with non-conducting walls. The study of Jana (1975) with stationary conducting walls is of interest. It has been shown by Gill and Casal (1962) that viscosity variations and temperature differences inside a forced convection flow of an electrically non-conducting fluid between two parallel plates can be induced by such forces as might significantly increase or decrease the tendency towards instability. Gupta (1969) has shown in his study of forced flow of an electrically conducting fluid between two horizontal parallel plates with a linear axial temperature variation including the buoyancy forces, that the buoyancy and the Lorentz's forces influence the upper and lower boundary layer differently in the entrance region of the channel

In the present paper the wall conductance effects on the convective horizontal channel flow in various situations have been studied. The upper and lower moving plates may have the same or different thickness or electrical conductvities. An exact solution of the governing equations for the combined free and forced convection flow of an electrically conducting fluids between two finitely conducting horizontal parallel walls with a linear axial temperature variation has been obtained. The effect of the dimensionless physical parameters characterizing the flow on the velocity, the skin friction, current density, mass flow rate and the temperature distribution have been studied in detail.

Equations of Motion and Their Solution

We propose to choose a Cartesian coordinate system such that the x-axis is in the direction of the flow and the y-axis perpendicular to the walls $y = \pm l$. At a large distance from the entry, the flow will be fully developed and in the steady state, we can take all the physical variables depending on y only. Then the governing equations for steady fully developed combined free and forced convection flow with uniformly applied transverse magnetic field H_0 can be written as (Gupta 1969),

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \mu_e H_0 \frac{dH_x}{dy}, \qquad \text{...(1)}$$

$$0 = -\frac{\partial p}{\partial y} - g\rho - \mu_e H_x \frac{dH_x}{dy}, \qquad \text{...(2)}$$

$$H_0 \frac{du}{dy} + \nu_m \frac{d^2 H_x}{dy^2} = 0, \qquad \text{...(3)}$$

where u is the horizontal component of velocity and the last terms of Eqs. (1) and (2) are the components of the Lorentz's force $\mu_e \vec{J} \times \vec{H}$ per unit volume.

The equation of state under the Boussinesq approximation is assumed to be

$$\rho = \rho_0 [1 - \beta(T - T_0)], \qquad \text{...(4)}$$

where β is the coefficient of the thermal expansion, T the temperature and ρ_0, T_0 denote the density and temperature in the reference state.

Equation (2) subject to (4) becomes

$$0 = -\frac{\partial p}{\partial y} - g\rho_0 [1 - \beta(T - T_0)] - \mu_e H_x \frac{dH_x}{dy}. \qquad \text{...(5)}$$

Integrating Eq. (5), we get

$$p = -g\rho_0 \int [1 - \beta(T - T_0)]\, dy - \frac{\mu_e H_x^2}{dy} + F(x). \qquad ...(6)$$

Assuming that the wall temperature has a uniform gradient N along the x-direction the temperature of the fluid can be assumed as

$$T - T_0 = Nx + \psi(y). \qquad ...(7)$$

Using (6) and (7), equation (1) can be written as

$$\nu \frac{d^2u}{dy^2} + \frac{\mu_e H_0}{\rho_0} \frac{dH_x}{dy} - g\beta Ny = \frac{1}{\rho_0} \frac{dF(x)}{dx}, \qquad ...(8)$$

where $\nu = \dfrac{\mu}{\rho_0}$.

Since u and H_x are functions of y only, both sides of Eq. (8) must be equal to a constant D, say. Thus, we rewrite Eq. (8) as

$$\nu \frac{d^2u}{dy^2} + \frac{\mu_e H_0}{\rho_0} \frac{dH_x}{dy} - g\beta Ny = D. \qquad ...(9)$$

Introducing the following dimensionless variables

$$\eta = \frac{y}{l}, \quad u_1 = \frac{ul}{\nu}, \quad h_x = \frac{H_x}{\sigma\mu_e\nu H_0}, \qquad ...(10)$$

Eqs. (9) and (3) become

$$\frac{d^2u_1}{d\eta^2} + M^2 \frac{dh_x}{d\eta} - G\eta = -R, \qquad ...(11)$$

$$\frac{d^2h_x}{d\eta^2} + \frac{du_1}{d\eta} = 0, \qquad ...(12)$$

where

$$M^2 = \frac{\sigma\mu_e^2 H_0^2 l^2}{\rho_0\nu}, \quad G = \frac{g\beta N l^4}{\nu^2}, \quad R = \frac{l^3}{\nu}(-D),$$

$$D = \frac{1}{\rho_0} \frac{dF(x)}{dx}, \qquad ...(13)$$

M being the Hartmann number which is the ratio of magnetic and viscous forces. Eq. (7) shows that positive and negative values of N correspond to heating and cooling respectively along the channel walls. Then it follows from (13) that $G \gtrless 0$ according as the channel walls are heated or cooled in the axial direction.

As to the hyrodynamic boundary conditions, they are

$$\left.\begin{aligned} u_1 &= U_1 \quad \text{at } \eta = 1 \\ u_1 &= -U_2 \quad \text{at } \eta = -1 \end{aligned}\right\} \qquad ...(14)$$

Considering σ_1, σ_2 the electrical conductivities and d_1, d_2 the thickness of the upper and lower walls respectively, the boundary conditions on h_x are given by Shercliff (1956), Chang and Lundgren (1959)

$$\left.\begin{aligned} &\frac{dh_x}{d\eta} + \frac{h_x}{\phi_1} = 0 \quad \text{at} \quad \eta = 1 \\ &\text{and} \\ &\frac{dh_x}{d\eta} - \frac{h_x}{\phi_2} = 0 \quad \text{at} \quad \eta = -1 \end{aligned}\right\} \qquad \ldots(15)$$

where $\phi_1 = \frac{\sigma_1 d_1}{\sigma l}$ and $\phi_2 = \frac{\sigma_2 d_2}{\sigma l}$.

The equation of energy including viscous and Ohmic dissipation is

$$u \frac{\partial(T - T_0)}{\partial x} = \alpha \frac{\partial^2(T - T_0)}{\partial y^2} + \frac{\mu}{\rho_0 c}\left(\frac{du}{dy}\right)^2 + \frac{1}{\rho_0 c \sigma}\left(\frac{dH_x}{dy}\right)^2, \qquad \ldots(16)$$

where α is the thermal diffusivity of the fluid and c is the specific heat.

Using (7) and introducing the dimensionless variables Eqs. (10), (16) becomes

$$\frac{d^2\theta}{d\eta^2} = P_r u_1 - k\left[\left(\frac{du_1}{d\eta}\right)^2 + M^2\left(\frac{dh_x}{d\eta}\right)^2\right], \qquad \ldots(17)$$

where

$$P_r = \frac{\nu}{\alpha}, \; k = \frac{\mu \nu^2}{\rho_0 c N \alpha l^3} \text{ and } \theta = \frac{\psi}{Nl} \qquad \ldots(18)$$

As for the temperature boundary conditions, we take the reference temperature T_0 such that the temperature of the lower wall ($\eta = -1$) is $T_0 + Nx$ and this by virtue of (7), implies $\psi(-1) = 0$. Hence using (18), the boundary conditions for θ are given by

$$\theta(-1) = 0 \quad \text{and} \quad \theta(1) = \frac{\psi(1)}{Nl} = \lambda \text{ (say)}, \qquad \ldots(19)$$

where λ is the wall temperature parameter.

Eliminating h_x from (11) and (12), we get

$$\frac{d^3 u_1}{d\eta^3} - M^2 \frac{du_1}{d\eta} = G. \qquad \ldots(20)$$

The solution of (20) subject to the boundary conditions (14) gives the velocity distribution u_1 and then the solution of h_x after substituting the value of velocity distribution u_1 in Eq. (11) subject to the boundary conditions (15) can be obtained. Thus,

$$u_1 = A_1\left(1 - \frac{\cosh M\eta}{\cosh M}\right) + \frac{G}{M^2}\left(\frac{\sinh M\eta}{\sinh M} - \eta\right) + \frac{(U_1 - U_2)}{2} \times \frac{\cosh M\eta}{\cosh M} + \frac{(U_1 + U_2)}{2}\frac{\sinh M\eta}{\sinh M} \qquad \ldots(21)$$

and

$$h_x = \frac{G}{2M^2}\left(\eta^2 - \frac{2\cosh M\eta}{M\sinh M}\right) - \frac{R}{M^2}\eta + \frac{A_1}{M}\frac{\sinh M\eta}{\cosh M}$$
$$- \frac{(U_1 - U_2)}{2M}\frac{\sinh M\eta}{\cosh M} - \frac{(U_1 + U_2)}{2M}\frac{\cosh M\eta}{\sinh M} + B_1, \qquad \ldots(22)$$

where

$$A_1 = \frac{R(\phi + 2) + M\{(U_1 - U_2)\tanh M + M(\phi_1 U_1 - \phi_2 U_2)\}}{M(\phi M + 2\tanh M)}$$

$$B_1 = \frac{G}{2M^2}\left(\frac{2\coth M}{M} - 1\right) + \Big[2R(\phi_1 - \phi_2)(\tanh M - M)$$
$$+ M\Big\{2(U_1 + U_2) + M(\phi_1 - \phi_2)\, U_2 \tanh M + 2M^2\phi_2(\phi_1 U_1 - \phi_2 U_2)$$
$$+ M\phi\Big(U_1 \tanh M + (U_1 + U_2)\coth M + M\phi_2 U_2\Big)\Big\}\Big]$$
$$: M(\phi M + 2\tanh M)$$

$$\phi = \phi_1 + \phi_2. \qquad \ldots(23)$$

Substituting u_1 and h_x from (21) and (22) respectively in (17) and integrating, we get θ after using the boundary conditions (19) as

$$\theta = B_2 + B_3\eta + K_1\eta^2 + K_2\eta^3 + K_3\eta^4 + K_4\cosh M\eta$$
$$+ K_5 \sinh M\eta + K_6(\sinh 2M\eta - 4\eta\cosh M\eta)$$
$$+ K_7\eta\sinh M\eta + K_8\cosh 2M\eta, \qquad \ldots(24)$$

where

$$C_1 = \frac{2A_1 - (U_1 - U_2)}{2\cosh M}, \quad C_2 = \frac{2G + M^2(U_1 + U_2)}{2M^2\sinh M},$$
$$C_3 = \frac{G}{M^2}, \quad C_4 = \frac{R}{M}$$

$$B_2 = \frac{\lambda}{2} - K_1 - K_3 - K_4\cosh M - K_7\sinh M - K_8\cosh 2M$$

$$B_3 = \frac{\lambda}{2} - K_2 - K_5\sinh M - K_6(\sinh 2M - 4\cosh M)$$

$$K_1 = \tfrac{1}{2}\{P_r A_1 - 2k(C_1^2 M^2 + C_3^2 + C_4^2)\}$$
$$K_2 = \tfrac{1}{6}\{C_3(4kMC_4 - P_r)\}$$
$$K_3 = -\tfrac{1}{6}kM^2C_3^2$$
$$K_4 = \frac{1}{M^2}\{4Mk(C_1C_4 - C_2C_3) - P_rC_1\}$$

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$$K_5 = \frac{1}{M^2}\{P_r C_2 - 4Mk(C_3C_1 + C_2C_4 - C_1C_2)\}$$

$$K_6 = kC_1C_2$$

$$K_7 = 4kC_2C_3$$

$$K_8 = -\tfrac{1}{2}k(C_1^2 + C_2^2). \qquad \ldots(25)$$

The dimensionless current density $J_z = -\dfrac{dh_z}{d\eta}$ is given by

$$J_z = -\left[C_3\eta - \frac{C_4}{M} + C_1 \cosh M\eta - C_2 \sinh M\eta\right]. \qquad \ldots(26)$$

The non-dimensional shear stresses at the walls $\eta = 1$ and $\eta = -1$ are given by

$$S_1 = \left(\frac{du_1}{d\eta}\right)_{\eta=1} = M(C_2 \cosh M - C_1 \sinh M) - C_3, \qquad \ldots(27)$$

$$S_2 = \left(\frac{du_1}{d\eta}\right)_{\eta=-1} = M(C_2 \cosh M + C_1 \sinh M) - C_3. \qquad \ldots(28)$$

From the above solutions, the non-dimensional mass flow rate is found to be

$$Q = \int_{-1}^{1} \rho u_1 d\eta$$

$$= \frac{2\rho}{M}(A_1 M - C_1 \sinh M). \qquad \ldots(29)$$

Particular Cases

Case I

When both the conducting walls moving with equal velocities but in opposite directions.

Let

$$U_1 = U_2 = U_0$$

Then from above solutions, we get

$$u_{11} = A_{11}\left(1 - \frac{\cosh M\eta}{\cosh M}\right) + \frac{G}{M^2}\left(\frac{\sinh M\eta}{\sinh M} - \eta\right) + U_0\frac{\sinh M\eta}{\sinh M} \qquad \ldots(30)$$

$$h_{x1} = \frac{G}{2M^2}\left(\eta^2 - \frac{2}{M}\frac{\cosh M\eta}{\sinh M}\right) - \frac{R}{M^2}\eta + \frac{A_{11}}{M}\frac{\sinh M\eta}{\cosh M}$$
$$- \frac{U_0}{M}\frac{\cosh M\eta}{\sinh M} + B_{11} \qquad \ldots(31)$$

where

$$A_{11} = \frac{R(\phi + 2) + M^2 U_0(\phi_1 - \phi_2)}{M(\phi M + 2 \tanh M)},$$

$$B_{11} = \frac{G}{2M^2}\left(\frac{2 \coth M}{M} - 1\right) + [2R(\phi_1 - \phi_2)(\tanh M - M)$$
$$+ MU_0 \{4 + M(\phi_1 - \phi_2) \tanh M + 2M^2\phi_2(\phi_1 - \phi_2)$$
$$+ M\phi(\tanh M + 2 \coth M + M\phi_2)\}] : M(\phi M + 2 \tanh M). \quad \ldots(32)$$

The temperature distribution is given by

$$\theta_1 = B_{21} + B_{31}\eta + K_{11}\eta^2 + K_{21}\eta^3 + K_{31}\eta^4 + K_{41} \cosh M\eta$$
$$+ K_{51} \sinh M\eta + K_{61}(\sinh 2M\eta - 4\eta \cosh M\eta)$$
$$+ K_{71}\eta \sinh M\eta + K_{81} \cosh 2M\eta, \quad \ldots(33)$$

where

$$C_{11} = \frac{A_{11}}{\cosh M}, \; C_{21} = \frac{G + M^2 U_0}{M^2 \sinh M}$$

$$B_{21} = \frac{\lambda}{2} - K_{11} - K_{31} - K_{41} \cosh M - K_{71} \sinh M - K_{81} \cosh 2M$$

$$B_{31} = \frac{\lambda}{2} - K_{21} - K_{51} \sinh M - K_{61}(\sinh 2M - 4 \cosh M)$$

$$K_{11} = \tfrac{1}{2} \{P_r A_{11} - 2k(C_{11}^2 M^2 + C_3^2 + C_4^2)\}$$

$$K_{21} = \tfrac{1}{6} \{C_3(4kMC_4 - P_r)\}$$

$$K_{31} = -\tfrac{1}{6} kM^2 C_3^2,$$

$$K_{41} = \frac{1}{M^2} \{4Mk(C_{11}C_4 - C_{21}C_3) - P_r C_{11}\}$$

$$K_{51} = \frac{1}{M^2} \{P_r C_{21} - 4Mk(C_3 C_{11} + C_{21}C_4 - C_{11}C_{21})\}$$

$$K_{61} = kC_{11}C_{21}$$

$$K_{71} = 4kC_{21}C_3$$

$$K_{81} = -\tfrac{1}{2} k(C_{11}^2 + C_{21}^2). \quad \ldots(34)$$

The dimensionless current density

$$J_{z1} = -\left[C_3\eta - \frac{C_4}{M} + C_{11} \cosh M\eta - C_{21} \sinh M\eta\right]. \quad \ldots(35)$$

The dimensionless shear stresses at the wall $\eta = 1$ and $\eta = -1$ are given by

$$S_{11} = M(C_{21} \cosh M - C_{11} \sinh M) - C_3 \quad \ldots(36)$$

$$S_{21} = M(C_{21} \cosh M + C_{11} \sinh M) - C_3 \quad \ldots(37)$$

The dimensionless mass flow rate is given by

$$Q_1 = \frac{2\rho}{M} (A_{11}M - C_{11} \sinh M) \quad \ldots(38)$$

Case II

When one of the conducting walls is at rest. Let the lower wall be at rest and the upper wall be moving with velocity U_0 i.e., $U_2 = 0,\ U_1 = U_0$

Then

$$u_{12} = A_{12}\left(1 - \frac{\cosh M\eta}{\cosh M}\right) + \frac{G}{M^2}\left(\frac{\sinh M\eta}{\sinh M} - \eta\right) + \frac{U_0}{2}\left(\frac{\cosh M\eta}{\cosh M} + \frac{\sinh M\eta}{\sinh M}\right) \quad \ldots(39)$$

$$h_{x_2} = \frac{G}{2M^2}\left(\eta^2 - \frac{2}{M}\frac{\cosh M\eta}{\sinh M}\right) - \frac{R}{M^2}\eta + \frac{A_{12}}{M}\frac{\sinh M\eta}{\cosh M} - \frac{U_0}{2M}\left(\frac{\sinh M\eta}{\cosh M} + \frac{\cosh M\eta}{\sinh M}\right) + B_{12} \quad \ldots(40)$$

where

$$A_{12} = \frac{R(\phi + 2) + MU_0(\tanh M + M\phi_1)}{M(M\phi + 2\tanh M)}$$

$$B_{12} = \frac{G}{2M^2}\left(\frac{2\coth M}{M} - 1\right) + [2R(\phi_1 - \phi_2)(\tanh M - M) + MU_0\{2 + 2M^2\phi_1\phi_2 + M\phi(\tanh M + \coth M)\}] : M(\phi M + 2\tanh M). \quad \ldots(41)$$

The temperature distribution is given by

$$\theta_2 = B_{22} + B_{32}\eta + K_{12}\eta^2 + K_{22}\eta^3 + K_{32}\eta^4 + K_{42}\cosh M\eta + K_{52}\sinh M\eta + K_{62}(\sinh 2M\eta - 4\eta\cosh M\eta) + K_{72}\eta\sinh M\eta + K_{82}\cosh 2M\eta \quad \ldots(42)$$

where

$$C_{12} = \frac{2A_{12} - U_0}{2\cosh M},\quad C_{22} = \frac{2G + M^2U_0}{2M^2\sinh M}$$

$$B_{22} = \frac{\lambda}{2} - K_{12} - K_{32} - K_{42}\cosh M - K_{72}\sinh M - K_{82}\cosh 2M$$

$$B_{32} = \frac{\lambda}{2} - K_{22} - K_{52}\sinh M - K_{62}(\sinh 2M - 4\cosh M)$$

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$$K_{12} = \tfrac{1}{2}\{P_r A_{12} - 2k(C_{12}^2 M^2 + C_3^2 + C_4^2)\}$$
$$K_{22} = \tfrac{1}{6}\{C_3(4kMC_4 - P_r)\}$$
$$K_{32} = -\tfrac{1}{6} kM^2 C_3^2$$
$$K_{42} = \frac{1}{M^2}\{4Mk(C_{12}C_4 - C_{22}C_3) - P_r C_{12}\}$$
$$K_{52} = \frac{1}{M^2}\{P_r C_{22} - 4Mk(C_3 C_{12} + C_{22}C_4 - C_{12}C_{22})\}$$
$$K_{62} = kC_{12}C_{22}$$
$$K_{72} = 4kC_{22}C_3$$
$$K_{82} = -\tfrac{1}{2}k(C_{12}^2 + C_{22}^2). \qquad \ldots(43)$$

The dimensionless current density is given by

$$J_{z2} = -\left[C_3\eta - \frac{C_4}{M} + C_{12}\cosh M\eta - C_{22}\sinh M\eta\right]. \qquad \ldots(44)$$

The dimensionless shear stresses at the wall $\eta = 1$ and $\eta = -1$ are given by

$$S_{12} = M(C_{22}\cosh M - C_{12}\sinh M) - C_3 \qquad \ldots(45)$$

and

$$S_{22} = M(C_{22}\cosh M + C_{12}\sinh M) - C_3. \qquad \ldots(46)$$

The dimensionless mass flow rate is given by

$$Q_2 = \frac{2\rho}{M}(A_{12}M - C_{12}\sinh M). \qquad \ldots(47)$$

Case III

When both the conducting walls are at rest.

Let

$$U_1 = U_2 = 0$$

Then

$$u_{13} = A_{13}\left(1 - \frac{\cosh M\eta}{\cosh M}\right) + \frac{G}{M^2}\left(\frac{\sinh M\eta}{\sinh M} - \eta\right) \qquad \ldots(48)$$

$$h_{x3} = \frac{G}{2M^2}\left(\eta^2 - \frac{2}{M}\frac{\cosh M\eta}{\sinh M}\right) - \frac{R}{M^2}\eta + \frac{A_{13}}{M}\frac{\sinh M\eta}{\cosh M} + B_{13} \qquad \ldots(49)$$

where

$$A_{13} = \frac{R(\phi + 2)}{M(\phi M + 2\tanh M)},$$
$$B_{13} = \frac{G}{2M^2}\left(\frac{2\coth M}{M} - 1\right) + \frac{2R(\phi_1 - \phi_2)(\tanh M - M)}{M(\phi M + 2\tanh M)}. \qquad \ldots(50)$$

The temperature distribution is given by

$$\theta_3 = B_{23} + B_{33}\eta + K_{13}\eta^2 + K_{23}\eta^3 + K_{33}\eta^4 + K_{43}\cosh M\eta + K_{53}\sinh M\eta + K_{63}(\sinh 2M\eta - 4\eta\cosh M\eta) + K_{73}\eta\sinh M\eta + K_{83}\cosh 2M\eta \quad ...(51)$$

where

$$C_{13} = \frac{A_{13}}{\cosh M}, \quad C_{23} = \frac{G}{M^2 \sinh M}$$

$$B_{23} = \frac{\lambda}{2} - K_{13} - K_{33} - K_{43}\cosh M - K_{73}\sinh M - K_{83}\cosh 2M$$

$$B_{33} = \frac{\lambda}{2} - K_{23} - K_{53}\sinh M - K_{63}(\sinh 2M - 4\cosh M)$$

$$K_{13} = \tfrac{1}{2}\{P_r A_{13} - 2k(C_{13}^2 M^2 + C_3^2 + C_4^2)\}$$

$$K_{23} = \tfrac{1}{6}\{C_3(4kMC_4 - P_r)\}$$

$$K_{33} = -\tfrac{1}{6}kM^2C_3^2,$$

$$K_{43} = \frac{1}{M^2}\{4Mk(C_{13}C_4 - C_{23}C_3) - P_r C_{13}\}$$

$$K_{53} = \frac{1}{M^2}\{P_r C_{23} - 4Mk(C_3C_{13} + C_{23}C_4 - C_{13}C_{23})\}$$

$$K_{63} = kC_{13}C_{23}$$

$$K_{73} = 4kC_{23}C_3$$

$$K_{83} = -\tfrac{1}{2}k(C_{13}^2 + C_{23}^2). \quad ...(52)$$

The dimensionless current density

$$J_{z3} = -\left[C_3\eta - \frac{C_4}{M} + C_{13}\cosh M\eta - C_{23}\sinh M\eta\right] \quad ...(53)$$

The dimensionless shear stresses at the wall $\eta = 1$ and $\eta = -1$ are given by

$$S_{13} = M(C_{23}\cosh M - C_{13}\sinh M) - C_3 \quad ...(54)$$

and

$$S_{23} = M(C_{23}\cosh M + C_{13}\sinh M) - C_3. \quad ...(55)$$

The dimensionless mass flow rate is given by

$$Q_3 = \frac{2\rho}{M}(A_{13}M - C_{13}\sinh M) \quad ...(56)$$

In Case III ($U_1 = U_2 = 0$) Eqs. (48), (49), (50), (51), (53), (54) and (55) reduce respectively to the results (21), (22), (23), (24), (26), (27) and (28) of Jana (1975) with slight change of notation and contants. Also in the absence of the buoyancy force ($G = 0$), Eqs. (48), (49) and (53) reduce to the results (10), (11) and (12) respectively of Chang and Yen (1962) with slight change of notation.

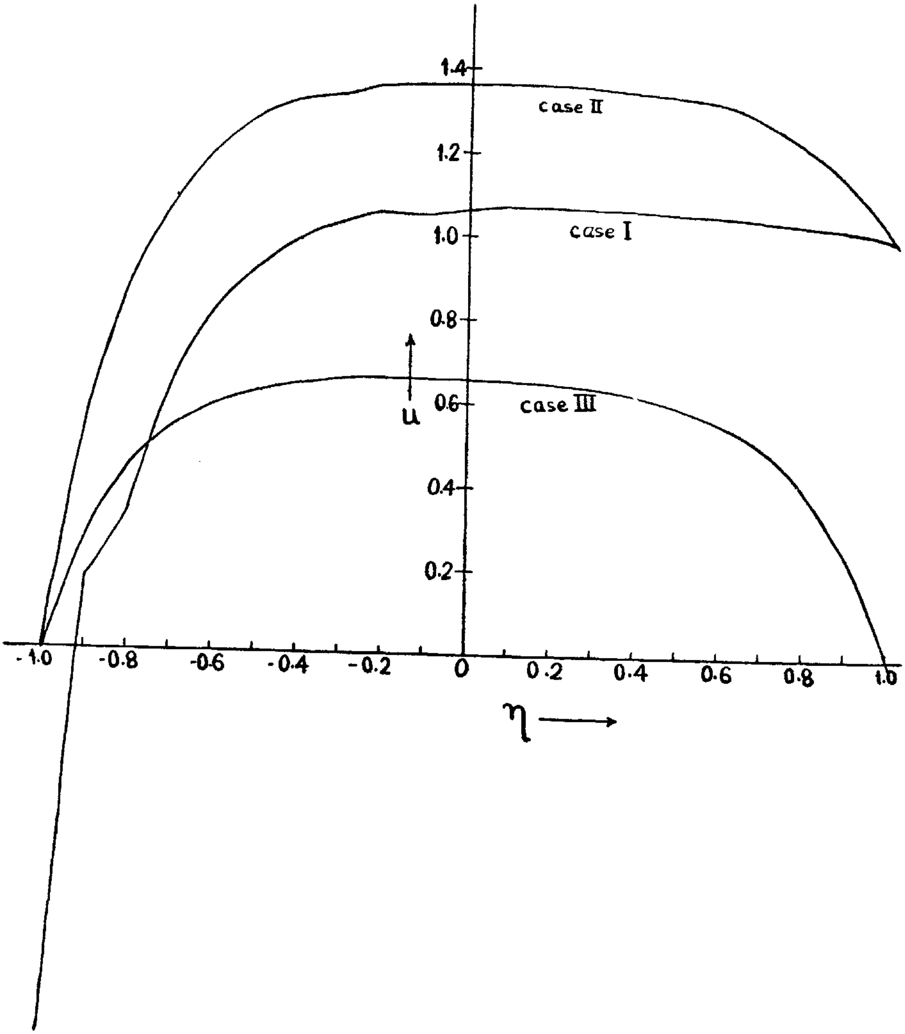

FIG. 1. Velocity profiles.

DISCUSSION

The wall conductance effects on the velocity depends only on the sum $\phi = \phi_1 + \phi_2$ in Case III but in Case II it depends on ϕ as well as ϕ_1, and in Case I the velocity depends on ϕ, ϕ_1 and ϕ_2. The velocity profiles have been plotted against η for $M = 5$, $G = 1$, $R = 10$, $\phi = 2$, $\phi_1 = 1.5$, $\phi_2 = 0.5$ and $U_0 = 1$. The velocity increases in the lower half of the channel for all the three particular cases and

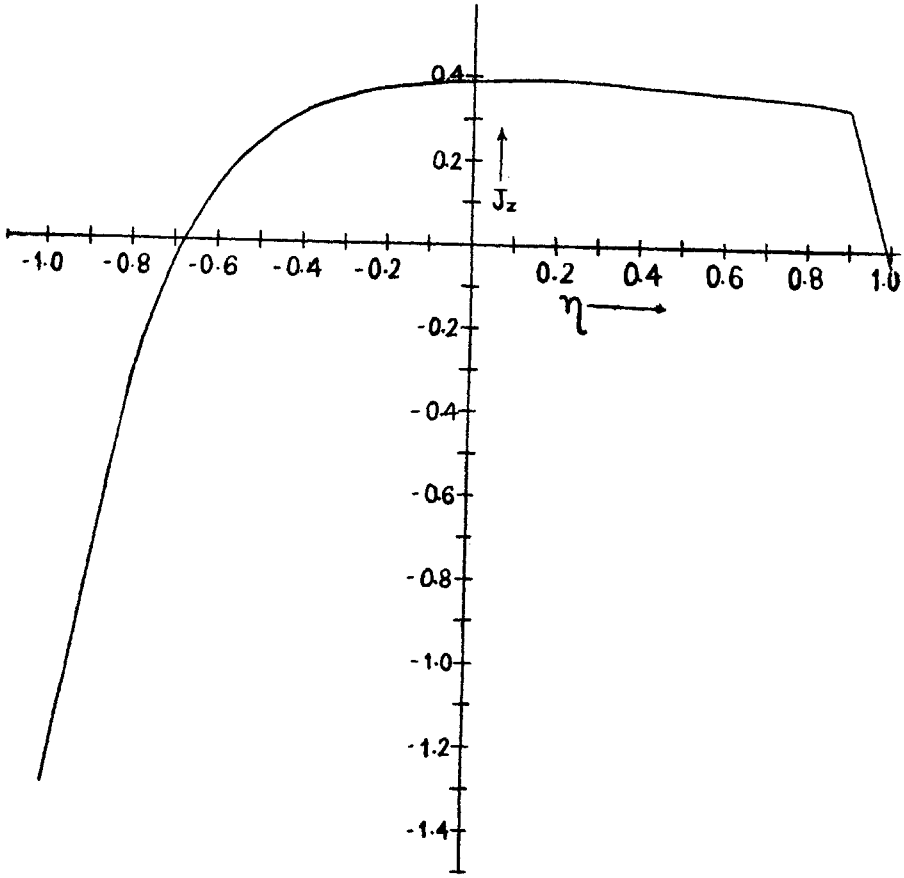

FIG. 2. Current density (J_z) profile.

decreases in the upper half of the channel in Cases II and III. But in Case I it decreases very much slowly as shown in the Fig. 1. The profile is almost symmetrical in Case III. In Case I, behaviour of graph between $-$ 0.9 and $-$ 0.85 is quite interesting, being different from those of Case II and III. The graphs of Case I and Case III intersect at $\eta = -0.75$ (approx.).

In Case I the current density has been investigated and it is obvious to predict it in cases II and III. The current density and temperature depend on ϕ, ϕ_1 and ϕ_2. In Fig. 2 profile of current density J_z against various values of η has been plotted. J_z increases in the lower half of the channel and its symmetry is disturbed in the present problem and the curve decreases almost linearly in between $\eta = 0.9$ and 1.

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