A NOTE ON AMPLIFICATION OF WAVES IN DISSIPATIVE PIEZOSEMICONDUCTORS

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The paper makes use of the equation of motion of plasma, Maxwell's equations, the equations of ferro-elastic body for the amplification of spin waves through ferromagnetic, piezoelectric and semiconducting body characterized by dissipative effect and subjected to a plasma.

1. INTRODUCTION

In recent years there have been a number of studies (Kaliski 1969a—c) in propagation of waves in piezo-semiconductors. More recently, such studies on amplification of waves take into account of plasma stream, *vide*, Kaliski and Sek (1970). This paper also is an attempt in this direction taking into consideration dissipative characteristics of the material which, in turn, is considered through a constitutive equation containing stress rate and strain rate parameters.

The present paper shows that both for non-resonance case and resonance case (under the condition of no collision between plasma and the body) the amplification depends on acoustic parameters and may be larger than that in the case in which the stress rate and strain rate parameters are not taken into account.

2. STATEMENT OF THE PROBLEM AND FUNDAMENTAL EQUATIONS

Let us consider an electron plasma flow through a ferropiezo-semiconducting body characterized by strain rate and stress rate attributes. Our problem is to investigate the waves propagating through such piezoelectric materials.

The equation of motion of plasma, Maxwell's equations and the equation of ferro-elastic body taking into consideration the piezoelectric effects, take the form.

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}_0, \nabla)\, \vec{v} = \frac{q}{m} \{\vec{E} + \frac{1}{c} (\vec{v}_0 \times \vec{B}) + \frac{1}{c} (\vec{v} \times \vec{B}_0)\} - \nu \vec{v} \quad \ldots(2.1)$$

$$\left.\begin{aligned} \operatorname{rot} \vec{E} &= -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \\ \operatorname{rot} \vec{H} &= \frac{1}{c} \frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c} (\rho_e \vec{v}_0 + \rho_{0e} \vec{v}) \\ \operatorname{div} \vec{D} &= 4\pi\rho_e \\ \operatorname{div} \vec{B} &= 0 \end{aligned}\right\} \quad \ldots(2.2)$$

$$\left.\begin{aligned} \dot{\mu}_1 &= -\gamma\,[(H_i - D\nabla^2)\,\mu_2 + b(u_{2,3} + u_{3,2}) - M_0 h_2] \\ \dot{\mu}_2 &= \gamma\,[(H_i - D\nabla^2)\,\mu_1 + b(u_{1,3} + u_{3,1}) - M_0 h_1] \end{aligned}\right\} \quad \ldots(2.3)$$

$$\left.\begin{aligned} \rho\ddot{u}_1 &= \sigma_{11,1} + \sigma_{12,2} + \sigma_{31,3} + \frac{b}{M_0}\,\mu_{1,3} - e(E_{1,3} + E_{3,1}) \\ \rho\ddot{u}_2 &= \sigma_{12,1} + \sigma_{22,2} + \sigma_{23,3} + \frac{b}{M_0}\,\mu_{2,3} - e(E_{2,3} + E_{3,2}) \\ \rho\ddot{u}_3 &= \sigma_{31,1} + \sigma_{23,2} + \sigma_{33,3} + \frac{b}{M_0}\,(\mu_{1,1} + \mu_{2,2}) \\ &\quad - e(E_{1,1} + E_{2,2} + 2E_{3,3}) \end{aligned}\right\} \quad \ldots(2.4)$$

$$\epsilon\vec{\nabla}\,.\,\vec{E} + 4\pi e(2u_{1,13} + 2u_{2,23} + u_{3,22} + 2u_{3,33}) = -\,4\pi\rho_e. \quad \ldots(2.5)$$

with the stress-strain relation of the form

$$\left(1 + \lambda_1\,\frac{\partial}{\partial t}\right)\sigma_{ij} = 2\mu_0\left(1 + \lambda_2\,\frac{\partial}{\partial t}\right)e_{ij}, \quad \ldots(2.6)$$

where ρ_{0e}, ρ_e, $\vec{v}_0$, $\vec{v}$ are the unperturbed and perturbed density and velocity of plasma respectively; q, m the electron charge and mass respectively; ν the coefficient of collisions; μ_0 the elastic constant; λ_1, λ_2 the stress rate parameter and strain rate parameter respectively; and σ_{ij} the stress tensor. The other notations have meanings same as before.

Equation (2.6) can be written as

$$\sigma_{ij} = 2\mu_0 f(D_1)\, e_{ij}, \quad \ldots(2.7)$$

where,

$$f(D_1) = \frac{1 + \lambda_1 D_1}{1 + \lambda_2 D_1},\ D_1 \equiv \frac{\partial}{\partial t}.$$

Applying the equation (2.7) in the equations (2.4) and using

$$e_{ij} = \tfrac{1}{2}\,(u_{i,j} + u_{j,i})$$

we get,

$$\left.\begin{aligned} \rho\ddot{u}_1 &= \mu_0 f(D_1)\,\nabla^2 u_1 + \mu_0 f(D_1)\,(\vec{\nabla}\cdot\vec{u})_{,1} + \frac{b}{M_0}\,\mu_{1,3} \\ &\quad - e(E_{1,3} + E_{3,1}) \\ \rho\ddot{u}_2 &= \mu_0 f(D_1)\,\nabla^2 u_2 + u_0 f(D_1)\,(\vec{\nabla}\cdot\vec{u})_{,2} + \frac{b}{M_0}\,\mu_{2,3} \\ &\quad - e(E_{2,3} + E_{3,2}) \\ \rho\ddot{u}_3 &= \mu_0 f(D_1)\,\nabla^2 u_3 + \mu_0 f(D_1)\,(\vec{\nabla}\cdot\vec{u})_{,3} + \frac{b}{M_0}\,(\mu_{1,1} + \mu_{2,2}) \\ &\quad - e(E_{1,1} + E_{2,2} + 2E_{3,3}). \end{aligned}\right\} \quad \ldots(2.8)$$

We can write eqns. (2.8) as

$$\left.\begin{aligned}
\ddot{u}_1 &= a_2^2 f(D_1) \nabla^2 u_1 + (a_1^2 - a_2^2) f(D_1) (\vec{\nabla} . \vec{u})_{,1} + \frac{b}{\rho M_0} \mu_{1,3} \\
&\quad - \frac{e}{\rho} (E_{1,3} + E_{3,1}) \\
\ddot{u}_2 &= a_2^2 f(D_1) \nabla^2 u_2 + (a_1^2 - a_2^2) f(D_1)((\vec{\nabla} . \vec{u})_{,2} + \frac{b}{\rho M_0} \mu_{2,3} \\
&\quad - \frac{e}{\rho} (E_{2,3} + E_{3,2}) \\
\ddot{u}_3 &= a_2^2 f(D_1) \nabla^2 u_3 + (a_1^2 - a_2^2) f(D_1) (\vec{\nabla} . \vec{u})_{,3} + \\
&\frac{b}{\rho \mu_0} (\mu_{1,1} + \mu_{2,2}) - \frac{e}{\rho} (E_{1,1} + E_{2,2} + 2E_{3,3}).
\end{aligned}\right\} \quad \ldots(2.9)$$

where,

$$a_1^2 = \frac{2\mu_0}{\rho}, \; a_2^2 = \frac{\mu_0}{\rho}.$$

If we seek the solution in the form $e^{i(\omega t - \vec{K} \cdot \vec{r})}$, $\vec{K} = |\vec{K}| (\sin\theta, 0, \cos\theta)$ and if $\vec{B}_0 \parallel \vec{v}_0 \parallel x_3$ the set of eqns. (2.1) - (2.3), (2.5), (2.9) becomes:

$$\begin{aligned}
&iKE_2 \cos\theta = -\frac{i\omega}{c} (h_1 + 4\pi u_1) \\
&iK(E_3 \sin\theta - E_1 \cos\theta) = -i \frac{\omega}{c} (h_2 + 4\pi\mu_2) \\
&iKE_2 \sin\theta = i \frac{\omega}{c} h_3 \\
&iKh_2 \cos\theta = i \frac{\omega}{c} D_1 + \frac{4\pi}{c} (\rho_e v_{01} + \rho_{0e} v_1) \\
&iK(h_3 \sin\theta - h_1 \cos\theta) = i \frac{\omega}{c} D_2 + \frac{4\pi}{c} (\rho_e v_{02} + \rho_{0e} v_2) \\
&iKh_2 \sin\theta = -i \frac{\omega}{c} D_3 - \frac{4\pi}{c} (\rho_e v_{03} + \rho_{0e} v_3) \\
&i\epsilon K(E_1 \sin\theta + E_3 \cos\theta) + 4\pi e K^2 [u_1 \sin 2\theta \\
&\qquad + u_3(1 + \cos^2\theta)] = 4\pi\rho_e \\
&(h_1 + 4\pi\mu_1) \sin\theta + h_3 \cos\theta = 0, \\
&\{i(\omega - Kv_0 \cos\theta) + \nu\} v_1 + \frac{q}{mc} (v_0 B_2 - B_0 v_2) \\
&\quad - \frac{q}{m} E_1 = 0
\end{aligned}$$

(equation continued on next page)

$$\left.\begin{aligned}
&\{i(\omega - Kv_0 \cos\theta) + \nu\}\, v_2 - \frac{q}{mc}(v_0 B_1 - B_0 v_1) \\
&\qquad - \frac{q}{m} E_2 = 0 \\
&\{i(\omega - Kv_0 \cos\theta) + \nu\}\, v_3 - \frac{q}{m} E_3 = 0 \\
&i\omega\mu_1 + \gamma[H_K\mu_2 - ibKu_2 \cos\theta - M_0 h_2] = 0 \\
&i\omega\mu_2 - \gamma[H_K\mu_1 - ibK(u_l \cos 2\theta + u_t \sin 2\theta) - M_0 h_1] = 0 \\
&\left\{\omega^2 - K^2 a_2^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} u_t - \frac{ibK}{\rho M_0}\mu_1 \cos 2\theta \\
&\qquad + \frac{ieK}{\rho}\left(E_1 \cos 2\theta - \frac{E}{2}\sin 2\theta\right) = 0 \\
&\left\{\omega^2 - K^2 a_1^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} u_l - \frac{ibK}{\rho M_0}\mu_1 \sin 2\theta \\
&\qquad + \frac{ieK}{\rho}\{E_1 \sin 2\theta + E_3(1 + \cos^2\theta)\} = 0 \\
&\left\{\omega^2 - K^2 a_2^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} u_2 - \frac{ibK}{\rho M_0}\mu_2 \cos\theta \\
&\qquad + \frac{ieK}{\rho} E_2 \cos\theta = 0,
\end{aligned}\right\} \quad \ldots(2.10)$$

where

$$u_t = u_1 \cos\theta - u_3 \sin\theta,\; u_l = u_1 \sin\theta + u_3 \cos\theta$$

and

$$\begin{aligned}
D_1 &= \epsilon E_1 - i4\pi eK(u_1 \cos\theta + u_3 \sin\theta) \\
D_2 &= \epsilon E_2 - i4\pi eKu_2 \cos\theta \\
D_3 &= \epsilon E_3 - i4\pi eK(u_1 \sin\theta + 2u_3 \cos\theta).
\end{aligned}$$

The case which is of interest to us corresponds to $\theta = 0$, in which the set of eqns. (2.10) become

$$\begin{aligned}
KE_2 &= -\frac{\omega}{c}(h_1 + 4\pi\mu_1) \\
KE_1 &= \frac{\omega}{c}(h_2 + 4\pi\mu_2) \\
iKh_2 &= \frac{4\pi}{c}\rho_0 v_1 + i\frac{\omega}{c}(\epsilon E_1 - i4\pi eKu_1) \\
-iKh_1 &= \frac{4\pi}{c}\rho_0 v_2 + i\frac{\omega}{c}(\epsilon E_2 - i4\pi eKu_2)
\end{aligned}$$

(*equation continued on next page*)

$$\left.\begin{aligned}
&4\pi(\rho_e v_0 + \rho_0 v_3) + i\omega(\epsilon E_3 - i8\pi eKu_3) = 0 \\
&i\epsilon KE_3 + 8\pi eK^2 u_3 = 4\pi\rho_e \\
&\{i(\omega - Kv_0) + \nu\}\, v_1 + \frac{q}{mc}(v_0 B_2 - B_0 v_2) - \frac{q}{m} E_1 = 0 \\
&\{i(\omega - Kv_0) + \nu\}\, v_2 - \frac{q}{mc}(v_0 B_1 - B_0 v_1) - \frac{q}{m} E_2 = 0 \\
&\{i(\omega - Kv_0) + \nu\}\, v_3 - \frac{q}{m} E_3 = 0 \\
&i\omega\mu_1 + \gamma(H_K\mu_2 - ibKu_2 - M_0 h_2) = 0 \\
&i\omega\mu_2 - \gamma(H_K\mu_1 - ibKu_1 - M_0 h_1) = 0 \\
&\left\{\omega^2 - K^2 a_2^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} u_1 - \frac{ibK}{\rho M_0}\mu_1 + \frac{ieK}{\rho} E_1 = 0 \\
&\left\{\omega^2 - K^2 a_2^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} u_2 - \frac{ibK}{\rho M_0}\mu_2 + \frac{ieK}{\rho} E_2 = 0 \\
&\left\{\omega^2 - K^2 a_1^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} u_3 + \frac{i2eK}{\rho} E_3 = 0.
\end{aligned}\right\} \qquad \ldots(2.11)$$

Let us write $u = u_1 + iu_2$, $v = v_1 + iv_2$, $\mu = \mu_1 + i\mu_2$, $h = h_1 + ih_2$ and $E = E_1 + iE_2$.

Therefore the set of eqns. (2.11) become

$$\left.\begin{aligned}
&(\omega - \gamma H_K)\,\mu + ibK\gamma u + \gamma M_0 h = 0 \\
&\left\{\omega^2 - K^2 a_2^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} u - \frac{ibK}{\rho M_0}\mu + \frac{ieK}{\rho} E = 0 \\
&KE = -i\frac{\omega}{c} B \\
&Kh = \frac{4\pi}{c}\rho_0 v + i\epsilon\frac{\omega}{c} E + \frac{4\pi}{c} e\omega Ku \\
&\{i(\omega - Kv_0) + \nu\}\, v - \frac{q}{m} E - i\frac{q}{me}(v_0 B - B_0 v) = 0.
\end{aligned}\right\} \qquad \ldots(2.12)$$

From the set of equations (2.9) we get the dispersion equation :

$$\begin{aligned}
&\left\{\omega^2 - K^2 a_2^2 \frac{(1 + i\omega\lambda_2)}{(1 + i\omega\lambda_1)}\right\} [\Omega_p^2 (\omega - Kv_0)(\omega - \gamma\bar{H}_K) + \\
&\quad \times \omega^2 \{(\omega - Kv_0 + \omega_B) - i\nu\} \{4\pi\gamma M_0 n^2 - (1 - n^2)(\omega - \gamma\bar{H}_K)\}] \\
&\quad + \frac{4\pi e^2}{\epsilon\rho}\omega^2 K^2 (\omega - \gamma\bar{H}_K)\{(\omega - Kv_0 + \omega_B) - i\nu\} \\
&\quad - \frac{b^2}{\rho}\gamma K^2 [\Omega_p^2(\omega - Kv_0) - \omega^2(1 - n^2)\{\omega - Kv_0 + \omega_B) - i\nu\}] = 0
\end{aligned} \qquad \ldots(2.13)$$

where

$$\Omega_p^2 = \frac{4\pi\rho_0 q}{\epsilon m}, \ \omega_B = \frac{qB_0}{mc}, \ n^2 = \frac{c^2K^2}{\epsilon\omega^2}, \ H_K = Hi + DK^2,$$

$$\bar{H}_K = H_K + 4\pi M_0.$$

In this equation linearization has been done by assuming the values of the quantities e^2, b^2 and Ω_p^2 to be small.

3. Solution

Let us consider the solution of the dispersion equation (2.13) by perturbation method, remote from plasma spin resonance.

For spin frequency we set

$$\omega(K) = \omega_s(K) + \eta(K), \ \omega_s(K) = \gamma H_K \qquad \text{...(3.1)}$$

and

$$\eta = \eta_1 + i\eta_2. \qquad \text{...(3.2)}$$

We get from the equation (2.13) after simplification,

$$\eta_2 \frac{\partial D}{\partial \omega}(\omega, K)\Big|_{\omega=\omega_s} = \frac{4\pi\gamma M_0 \Omega_p^2 (\omega_s - Kv_0)\nu}{\{(\omega_s - Kv_0 + \omega_B)^2 + \nu^2\}} + \frac{16\pi^2\gamma M_0 e^2 \omega_s^2 K^2}{\epsilon\rho}$$

$$\times \frac{K^2 a_2^2 (\lambda_2 - \lambda_1)\omega_s}{\epsilon\rho\{(\omega_s^2 - K^2a_2^2)^2 + \omega_s^2(\omega_s^2\lambda_1 - K^2a_2^2\lambda_2)^2\}}$$

$$- \frac{b^2\gamma K^2}{\rho} \times \frac{\left(\omega_s^2 - \frac{c^2K^2}{\epsilon}\right)(\lambda_2 - \lambda_1) K^2 a_2^2 \omega_s}{\{(\omega_s^2 - K^2a_2^2)^2 + \omega_s^2(\omega_s^2\lambda_1 - K^2a_2^2\lambda_2)^2\}}. \qquad \text{...(3.3)}$$

In eqn. (3.3), $D(\omega, K)$ is expressed by

$$D(\omega, K) = 4\pi\gamma M_0 \frac{c^2K^2}{\epsilon} - \left(\omega^2 - \frac{c^2K^2}{\epsilon}\right)(\omega - \gamma\bar{H}_K),$$

and

$$\frac{\partial D}{\partial \omega}(\omega, K) \approx \frac{c^2K^2}{\epsilon}.$$

Therefore we can write the equation (3.3) as

$$\eta_2 = \frac{4\pi\gamma M_0 \nu \Omega_p^2 \epsilon(\omega_s - Kv_0)}{c^2K^2[(\omega_s - Kv_0 + \omega_B)^2 + \nu^2]}$$

(*equation continued on next page*)

$$+ \frac{16\pi^2\gamma M_0 e^2 \omega_s^3 K^2 a_2^2)(\lambda_2 - \lambda_1)}{c^2\rho[(\omega_s^2 - K^2a_2^2)^2 + \omega_s^2 (\omega_s^2 \lambda_1 - K^2 a_2^2 \lambda_2)^2]}$$

$$- \frac{b^2\gamma K^2 a_2^2 \omega_s \epsilon(\lambda_2 - \lambda_1)\left(\omega_s^2 - \frac{c^2K^2}{\epsilon}\right)}{c^2\rho[(\omega_s^2 - K^2a_2^2)^2 + \omega_s^2 (\omega_s^2 \lambda_1 - K^2 a_2^2 \lambda_2)^2]}. \quad ...(3.4)$$

Only the first term on the right hand side of the equation (3.4) represents drift amplification. From the equation (3.4) it is evident that strain rate parameter λ_2 and stress rate parameter λ_1 play a role in non-resonance amplification and amplification of spin waves will be large if the difference of λ_1 and λ_2 is large. One thing to be noted here is that both the piezoelectric effect and the spin-acoustic coupling affect the amplification.

Let us now consider two different cases —K large and K small. If K is large it can be easily shown that

$$\eta_2 \approx \frac{b^2\gamma\omega_s(\lambda_2 - \lambda_1)}{\rho a_2^2 (1 + \lambda_2^2)}. \quad ...(3.5)$$

Thus large values of K brings in amplification if $\lambda_1 > \lambda_2$ and attenuation if $\lambda_1 < \lambda_2$. The spin-acoustic coupling is responsible here for bringing about the amplification or the attenuation. Again if K is small then

$$\eta_2 \approx \frac{4\pi\gamma M_0 \nu \Omega_p^2 \epsilon(\omega_s - Kv_0)}{c^2K^2[(\omega_s - Kv_0 + \omega_B)^2 + \nu^2]} \quad ...(3.6)$$

which is independent of λ_1 and λ_2 and the amplification is only of drift type with critical drift velocity $v_{0cr} = \omega_s/K$.

Let us now consider the resonance of spin frequency ω_s with the plasma frequency $Kv_0 - \omega_B$.

We write,

$$\omega(K) = \omega_s(K) + \eta(K),\ \omega_s(K) = \gamma H_K = Kv_0 - \omega_B. \quad ...(3.7)$$

We get from eqn. (2.13), assuming $\nu = 0$ the equation

$$\eta^2 \left[(Kv_0 - \omega_B)^2 - K^2a_2^2 \frac{\{1 + i(Kv_0 - \omega_B)\lambda_2\}}{\{1 + i(Kv_0 - \omega_B)\lambda_1\}}\right] \frac{\partial D}{\partial \omega}\bigg|_{\omega=\omega_s}$$

$$+ \eta \left[\frac{b^2\gamma K^2}{\rho}\left\{(Kv_0 - \omega_B)^2 - \frac{e^2K^2}{\epsilon}\right\}\right.$$

$$\left. - \frac{16\pi\gamma M_0 e^2 K^2}{\epsilon\rho}(Kv_0 - \omega_B)^2 - \frac{4i\pi\gamma\Omega_p^2 \omega_B(\lambda_2 - \lambda_1) K^2 a_2^2}{\{1 + i(Kv_0 - \omega_B)\lambda_1\}^2}\right]$$

$$+ 4\pi\gamma M_0 \Omega_p^2 \omega_B\left[(Kv_0 - \omega_B)^2 - K^2a_2^2 \frac{\{1 + i(Kv_0 - \omega_B)\lambda_2\}}{\{1 + i(Kv_0 - \omega_B)\lambda_1\}}\right] = 0, \quad ...(3.8)$$

which is of the form

$$A\eta^2 + B\eta + c = 0.$$

So that

$$\eta_2 = Im\left[-\frac{B}{2A} \pm \sqrt{\frac{B^2}{4A^2} - \frac{C}{A}}\right]$$

In (3.8) $\frac{\partial D}{\partial \omega}(\omega, K)$ is given by the formula

$$\frac{\partial D}{\partial \omega}(\omega, K) = \frac{c^2K^2}{\epsilon} - (Kv_0 - \omega_B)^2 - 2(Kv_0 - \omega_B)(Kv_0 - \omega_B - \gamma\bar{H}_K)$$

and for spin frequency we preserve,

$$\frac{\partial D}{\partial \omega}(\omega, K) \approx \frac{c^2K^2}{\epsilon}.$$

Assuming K to be small, we get after simplification

$$\eta_2 = \frac{2\pi\gamma M_0\Omega_p^2\, \omega_B a_2^2\, \epsilon(\lambda_2 - \lambda_1)\{1 - \lambda_1^2(Kv_0 - \omega_B)^2\}}{c^2\{(Kv_0 - \omega_B)^2[1 + \lambda_1^2(Kv_0 - \omega_B)^2 - \lambda_1\lambda_2K^2a_2^2 - K^2a_2^2\}\{1 + \lambda_1^2(Kv_0 - \omega_B)^2\}} \quad ...(3.9)$$

$$\pm\sqrt{\frac{4\pi\gamma M_0\Omega_p^2\,\omega_B\epsilon}{c^2K^2}}.$$

Again,

$$\omega_s = Kv_0 - \omega_B = \gamma(H_i + DK^2)$$

Therefore for small K, we can write

$$K \approx \frac{\gamma H_i + \omega_B}{v_0}.$$

Hence (3.9) becomes

$$\eta_2 = \frac{2\pi\gamma M_0\omega_B a_2^2\epsilon\Omega_p^2\, v_0^2(\lambda_2 - \lambda_1)(1 - \lambda_1^2\gamma^2H_i^2)}{c^2\{\gamma^2H_i^2(1 + \lambda_1^2\gamma^2H_i^2)v_0^2 - (1 + \lambda_1\lambda_2\gamma^2H_i^2)(\gamma H_i + \omega_B)^2a_2^2\}(1 + \lambda_1^2\gamma^2H_i^2)} \quad ...(3.10)$$

$$\pm\frac{2\sqrt{\pi\gamma M_0\omega_B\epsilon}\,\Omega_p v_0}{c(\gamma H_i + \omega_B)}.$$

It is evident from the eqn. (3.10) that stress rate parameter and strain rate parameter play a role in the resonance amplification also. It is also clear that by a suitable change of the impressed magnetic field one can make the amplification larger than that in non-dissipative case.

4. Concluding Remarks

Summing up, we note the following distinguishing characteristics for the propagation of the wave between dissipative and non-dissipative piezoelectric materials :

(*i*) Unlike the non-dissipative situation, the acoustic parameters influence the amplification of spin waves in the dissipative situation both for resonance and non-resonance cases. In dissipative situation, the amplification will be large if the difference of stress rate and strain rate parameter is large.

(*ii*) In non-resonance case, the amplification depends on the collision coefficient between plasma and crystal lattice both for dissipative and non-dissipative situations. In dissipative situation a non-drift type amplification, appreciable for large values of wave number, is possible even when the collision coefficient is zero which is not the case for non-dissipative situation.

(*iii*) In dissipative situation amplification is possible for large values of wave number if stress rate parameter is greater than strain rate parameter in non-resonance case and the spin-acoustic coupling is responsible for bringing about the amplification. But in non-dissipative case, amplification is negligible for large values of wave number.

(*iv*) The above results show that the consideration of stress rate besides the strain rate may bring about the possibility of attenuation and amplification, contrary to the case of amplification, only when there is no stress rate besides the strain rate. Further an attenuation in the former case may turn out to be an amplification in the latter case.

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