GELFAND PATTERN, CHARMED HADRONS AND MAGNETIC MOMENT

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A SU(4) quartet model whose fourth quark carries a positive fractional charge is studied using the Gelfand technique. Gelfand states for charmed hadrons are obtained. The mass splitting spectrum for hadrons and magnetic moment sum rule for charmed baryons are derived.

INTRODUCTION

THE Discovery of resonances ψ and ψ' has led to the speculation that there might be a new kind of quantum number (Aubert *et al.*, 1974; and Augustine *et al.*, 1974). One possibility is charm (Bjorken & Glashow, 1964; and Gallard *et al.*, 1975) in which case the symmetry group is extended from SU (3) to SU(4) (Bjorken & Glashow, 1964). Thus in the case of SU(4) we have one quark, the charmed quark c in addition to the usual SU(3) quarks u, d, s. There are a number of fractionally charged SU(4) quark models (Litchenberg, 1975; and Moffat, 1965, 1975). Here, we consider the quartet model of Glashow *et al.* (1970), as it has the advantage of removing in a simple way the strangeness changing neutral current in a gauge model of weak interactions. The Gell-Mann-Nishijima relation is then modified to

$$Q = t_3 + \frac{Y}{2} + \frac{C}{2}.$$

The purpose of this note is to derive a generalization to SU(4) of the Okubo mass formula by specifying the hadron states by the Gelfand technique (Gelfand & Tsetlin, 1950). Also by means of t and the Gelfand and Tsetlin methods, the mass spectrum of charmed hadrons are derived and the magnetic moment sum rules for charmed and uncharmed baryons are obtained. The spectrum we obtain is compared with the results of others (Goldhaber, 1977; and De Rujula *et al.*, 1976) and hence the efficacy of this technique is hereby established.

GELFAND TECHNIQUE

Gelfand technique is regarded to be the standard form of representing uniquely all the states of all irreducible representation of $U(n)$ and $SU(n)$. The basic simplicity of the states of $U(n)$ and $SU(n)$ becomes clear for the triangular Gelfand pattern given below of $\frac{n(n+1)}{2}$

$$\begin{pmatrix} \cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots \\ \cdots \end{pmatrix}$$

integers uniquely characterizes each state of every irreducible representation of $U(n)$. For $SU(n)$, we put the uppermost right integer $= 0$. It is really astonishing how much information is contained in Gelfand diagrammatic form for the states !

Let us note that in order to read off the quantum numbers $H_i \rightarrow M_i$, from the Gelfand pattern one uses :

$$H_i \rightarrow M_i = \frac{\sum_{j=1}^{i} m_{j,i}}{i} - \frac{\sum_{j=1}^{i+1} m_{j,\,i+1}}{i+1}$$

where $m_{i,j}$ denotes the Gelfand pattern

$$\begin{pmatrix} m_{13} & & m_{23} & & m_{33} \\ & m_{12} & & m_{22} & \\ & & m_{11} & & \end{pmatrix}.$$

This formula is not nearly so complicated as it looks — just the difference of the average values of the adjacent rows.

The general state of the representation is characterised by

$$\begin{pmatrix} P, & Q, & R & \ldots & 0 \\ a & & b & \ldots & 0 \\ & & c & & \end{pmatrix}$$

where a, b, c etc., stand for integers obeying the "between-ness" rule.

One interesting point arises from the Gelfand pattern when written in terms of quantum numbers like t, t_3, Y and C of $SU(4)$. This invites one to write the pattern directly in terms of the charge, as given by the Gell-Mann-Nishijima rule

$$Q = t + Y/2 + C/2.$$

The method of Gelfand discussed is extremely powerful, and it yields answers readily to a wide variety of specific applications.

Gelfand Technique and $SU(4)$

The present authors associate a unique pattern, (Gelfand & Tsetlin, 1950) to every hadronic state by imposing the condition of lexical ordering. By following this technique, the present authors give a pictorial representation for every charmed hadronic state.

Thus all the states of $SU(4)$ can uniquely be represented in terms of t, t_3, Y and C according to the Gelfand patterns. For $SU(4)$, an orthonormal state of an irreducible representation $(P, Q, R, 0)$ is associated with the Gelfand pattern thus :

$(P,Q,R,0);\, t, t_3, Y, C =$

$$\begin{bmatrix} P & & Q & & R & & 0 \\ & t + \frac{Y}{2} + \frac{P+Q+R}{3} + \frac{C}{2} & & -t + \frac{Y}{2} + \frac{P+Q+R}{6} & & + \frac{C}{2} \; 0 & \\ & & t + \frac{Y}{2} + \frac{P+Q+R}{6} + \frac{C}{2} & & -\, t + \frac{Y}{2} + \frac{P+Q+R}{6} + \frac{C}{2} & & \\ & & & t_3 + \frac{Y}{2} + \frac{P+Q+R}{6} + \frac{C}{2} & & & \end{bmatrix} \quad \ldots(1)$$

$$= \begin{pmatrix} P & & Q & & R & & 0 \\ & m_{13} & & m_{23} & & m_{33} & \\ & & m_{12} & & m_{22} & & \\ & & & m_{11} & & & \end{pmatrix} \quad \ldots \; (2)$$

Here (P, Q, R, O) denotes a frame which corresponds to a unique $SU(4)$ irreducible representation ; and to every Gelfand pattern belonging to the frame $(P, Q, R, 0)$ there corresponds a unique state. The next row of the relation (1) denotes how the frame $(P, Q, R, 0)$ is to be filled in order to specify a single vector. To achieve this the symbols occuring in the pattern (2) should obey the between-ness relation :

$$P \geqslant m_{14} \geqslant m_{13} > m_{12} \geqslant m_{11} \geqslant m_{22} \geqslant m_{33} \geqslant m_{44} \text{ etc.}$$

From the relation (1) and (2), we get :

$$t = \tfrac{1}{2}\,(m_{12} - m_{22}\,)\ ;$$

$$t_3 = m_{11} - \tfrac{1}{2}\,(m_{12} + m_{22});$$

$$Y = 2\,[\tfrac{1}{2}\,(m_1{}^2 + m_{22}) - (m_{13} + m_{23} + m_{33}) + \tfrac{1}{3}\,(P + Q + R)];$$

and

$$C = (m_{13}\, m_{23} + m_{33}) - \tfrac{1}{6}\,(P + Q + R). \quad \ldots \; (3)$$

With this Gelfand pattern, the different particles have the different patterns shown in appendix.

The Mass Formula

In $SU(3)$, we know that mass splitting in the lowest order is proportional to the matrix element of A_8 and D_8 (where A and D have usual meaning). A_8 and D_8 both single out "8" direction hypercharge in $SU(3)$. This we generalize to $SU(4)$, where in addition to the above operators, we have the operator A_{15} which singles out charm. We write the mass operator as :

$$M = M_0 + M_1 < m \mid A_8 \mid m > + \, m_2 < m \mid D_8 \mid m > + \\ M_3 < m \mid A_{15} \mid m >, \quad \ldots \; (4)$$

where m is the Gelfand pattern specifying the states with M_0, M_1, M_2 and M_3 as constants.

The first matrix element is proportional to Y and after normalization we have ;

$$< m \mid A_8 \mid m > = Y/2 \quad \ldots (5)$$

Also by definition, $D_i = d_{8jk} A_j A_k$. ... (5a)

Hence D_i operator is quadratic in generators and takes the form :

$$< m \mid D_8 \mid m > \propto \delta t\,(t+1) + \beta\, Y^2 + \gamma z, \quad \ldots (6)$$

where Z is to be found out. To evaluate this, we use the principle that 'the tensor operators acting on multiplets having only singly occupied points in weight space, show multiplicity'. Thus when the Gelfand pattern is of the form

$$\begin{pmatrix} P & 0 & 0 & 0 \\ & m_{13} & 0 & 0 \\ & & \cdot & \cdot \\ & & \cdot & \end{pmatrix}$$

we have

$$\left\langle \begin{pmatrix} P & 0 & 0 & 0 \\ & m_{13} & 0 & 0 \\ & & \cdot & \cdot \\ & & \cdot & \end{pmatrix} \middle| D_8 \middle| \begin{pmatrix} P & 0 & 0 & 0 \\ & m_{13} & 0 & 0 \\ & & \cdot & \cdot \\ & & \cdot & \end{pmatrix} \right\rangle = \propto Y \quad \ldots (7)$$

Then for $\begin{pmatrix} P & 0 & 0 & 0 \\ & m_{13} & 0 & 0 \\ & & \cdot & \cdot \\ & & \cdot & \end{pmatrix}$, we have from equation (3) :

$$t = \frac{m_{12}}{2} = \frac{Y}{2} + m_{13} - \frac{P}{3} ;$$

$$Y = 2\left(\frac{m_{12}}{2} - m_{13} + \frac{P}{3}\right);$$

and $\qquad C = m_{13} - \frac{1}{2} P \qquad \ldots (8)$

Using the eqn. (8) in the left hand side of the eqn. (6), we have

$$\delta\, t\,(t+1) + \beta Y^2 + \gamma z$$
$$= \delta \left[\frac{Y^2}{4} + \frac{Y}{2}\left(2\, m_{13} - \frac{2P}{3} + 1\right) + \left(m_{13} - \frac{P}{3}\right) + \left(m_{13} - \frac{P}{3}\right)^2 \right] + \beta Y^2 + \gamma z. \quad \ldots (9)$$

For the eqn. (7) to hold good, the quadratic term in the eqn. (9) must vanish; hence $\delta = 1$; $\beta = -1/4$ and

$$\gamma z = \left(m_{13} - \frac{P}{3}\right) + \left(m_{13} - \frac{P}{3}\right)^2$$
$$= \left(m_{13} - \frac{P}{2}\right) + \left(m_{13} - \frac{P}{2}\right)^2 + \left(\frac{P}{6} - \frac{5P^2}{36} + \frac{m_8 P}{3}\right)$$
$$= C + C^2 + \mathrm{f} \quad \ldots (10)$$

where f is a constant. Then from the eqn. (6), after normalization, we get ;

$$< m \mid D_8 \mid m > = \frac{1}{6}\left[t\,(t+1) \equiv \frac{Y^2}{4} - C - C^2 - \mathrm{f} \right] \quad \ldots (11)$$

Lastly, A_{15} is used for generating charm quantum number ; hence, after normalization, we have :

$$< m \mid A_{15} \mid m > = \frac{1}{12} C. \quad \ldots (12)$$

Thus the generalized linear mass formula for baryons will be:

$$M = M_0 + \frac{M_1}{2}\, Y + \frac{M_2}{6}\left[t\,(t+1) - \frac{Y^2}{4} - C - C^2 - \mathrm{f} \right] + \frac{M_3}{12} C$$
$$= M_0 + \frac{M_1}{2}\, Y + \frac{M_2}{6}\left[t\,(t+1) - \frac{Y^2}{4} \right] + \frac{C}{12}\,(M_3 - 2M_2)$$
$$- \frac{M_2}{6} C^2 + \mathrm{f}', \quad \ldots (13)$$

where $\mathrm{f}' = - M_2 \mathrm{f}$; a constant introduced in the Okubo formula because of generalisation to $SU(4)$, and taken to be zero for $SU(3)$ hadrons and non-zero for the hadrons having any charmed content.

Accordingly, for mesons the mass squared formula is written as :

$$M'^2 = M_0'^2 + \frac{M_1'^2}{6}\left[t\,(t+1) - \frac{Y^2}{4} \right] - \frac{M'^2_4}{6} C^2 + g. \quad \ldots (14)$$

This gives the familiar results for mesons :

$$K^{*2} + \rho^2 = K^2 - \pi^2 \;;$$

and for charmed mesons ;

$$D^{*2} - F^{*2} = D^2 - F^2.$$

Then the sum rule is given as :

$$3\,(D^{*2} - F^{*2}) = \rho^2 - K^2,$$

where the particle label stands for the mass of the particle. Regarding $(3/2)^+$ 20-plet baryons, the following sum-rule comes out :

$$\Delta^{++} - \Delta = \Delta^{*+} - \Sigma^{*} = \Sigma^{0}{}_{c} - \Xi^{-}.$$

Masses of Supermultiplets

(*a*) *Mesons* — If the eqn. (14) holds good for vector mesons, the values of constants will be :

$$M'^{2}_{0} = 81900\ (\mathrm{MeV})^2\ ;\ M_1'^2 = -\ 999800\ (\mathrm{MeV})^2;\ g = 2650000\ (\mathrm{MeV})^2$$

We get the following masses for D and F mesons ;

$$D^0 = 1873\,(\mathrm{MeV});\ D^+ = 1871\ (\mathrm{MeV});\ D^{*0} = 2011\,(\mathrm{MeV});\ D^{*+} = 2003\ (\mathrm{MeV});$$

$$F = 1930\,(\mathrm{MeV}).$$

These results, when compared with the recent experimental values (Goldhaber, 1977). are found to be correct.

(*b*) *Baryons* — For baryons, we choose the values for constants as ;

$M_0 = 1015$ (MeV); $M_1 = -380$ (MeV); $M_2 = 180$ (MeV); $M_3 = 480$(MeV); $f' = 1500$ (MeV).

The masses of charmed baryons are then (with $C = 1$) ;

$\Delta_c{}^0$=2.35 (GeV); $\lambda_c{}^0$=2.29 (GeV); $\Delta_c{}^+$= 2.35 (GeV); $Z_c{}^0$ 2.5 (GeV); $\varphi_c{}^-$ 2.59 (GeV).

These results, when compared with the results of others (De Rujula *et al.*, 1976) are found satisfactory.

Magnetic Moment

In SU (3), the magnetic moment of hadron depends on the charges and U-spin, where U-spin connects two quarks (d and s) having the same charge. Regarding the $SU(2)$ sub-groups of $SU(4)$, we have six sub-groups ; three known ones called t, U and V-spin ; and three other sub-groups, called by L, P and R-spin. Out of of the latter, L-spin connects u and c quarks. It is assumed that the magnetic moment depends on L-spin as well.

The magnetic moment operator μ_{op} is a pseudovector in angular momentum properties. In $SU(3)$, μ_{op} transforms in the same manner as Q, namely ;

$$\mu_{op} = a\,F_Q + b\,D_Q, \qquad \ldots\ (15)$$

where F and D have their usual meaning. F_Q corresponds to A_8 and DQ_8 to D operator of the eqn. (4). Thus F_Q gives the Q term, D_Q operator being quadratic in generator, the only quadratic operators available are $U\,(U+1)$, Q^2 and the invariant operator t_2 where t_2 will be $\frac{p\,(p+1)}{9}$ for $\begin{pmatrix} p & 0 & 0 \\ & m_{12} & 0 \\ & \cdot & \end{pmatrix}$ in the Gelfand pattern.

Then using this Gelfand pattern, we have :

$$< m \left| \mu_{op} \right| m > = a\frac{Q}{2} + \frac{b}{6}\left[U(U+1) - \frac{Q^2}{4} - t_2\right]. \quad \ldots (16)$$

We extend this approach to SU (4) and write :

$$\mu_{op} = a F_Q + b D_Q + d A_Q, \quad \ldots (17)$$

where A_Q corresponds to A_{15} of the eqn. (4). We assume that A_Q gives L term. Thus, using the eqn. (13), we write for $SU(4)$;

$$< m \mid \mu_{op} \mid m > = a\frac{Q}{2} + \frac{b}{6}\left[U(U+1) - \frac{Q^2}{4} - t_2\right] + \frac{L}{12}(d-2b) - \frac{b}{6}L^2 \quad \ldots (18)$$

$$= a\frac{Q}{2} + \frac{b}{6}\left[U(U+1) - \frac{Q^2}{4} - t_2\right] - \left(\frac{b}{3} - \frac{d_1}{12}\right)L - \frac{b}{6}L(L-1) + \frac{d_2}{12}L$$

$$= a\frac{Q}{2} + \frac{b}{6}\left[U(U+1) - \frac{Q^2}{4} - t_2 - xL\right] - \frac{b}{6}L(L-1) + \frac{d_2}{12}L.$$

Here we write empirically that x takes such a value that

$$t_2 + xL = 4L \textit{ and } -\frac{b}{6}L(L-1) + \frac{d_2}{12} = \frac{e}{12}L(L-1).$$

Then the eqn. (18) takes the form ;

$$< m \mid \mu_{op} \mid m > = a\frac{Q}{2} + \frac{b}{6}\left[U(U+1) - \frac{Q^2}{4} - 4L\right] + \frac{e}{12}L(L-1). \quad \ldots (19)$$

We now find the magnetic moment of all the baryons in terms of neutron, proton and charmed baryon B_c^{++} magnetic moment. We have, then :

$$< m \mid \mu_{op} \mid m > = \frac{Q}{8}(-6\mu_p - 7\mu_N + 7\mu_{Bc}^{++}) - \tfrac{1}{4}(2\mu_p + \mu_N - \mu_{Bc}^{++})\left[U(U+1) - \frac{Q^2}{4} - 4L\right] - 4\mu_N L(L-1). \quad \ldots (20)$$

This equation leads to :

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \tfrac{1}{2}\,\mu_p + \tfrac{3}{4}\,\mu_n - \tfrac{3}{4}\,\mu_{B_c^{++}}. \qquad \ldots \quad (21)$$

Now the $SU(3)$ result for these baryons is :

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = -(\mu_p + \mu_n). \qquad \ldots \quad (22)$$

If $SU(4)$ is broken in such a manner that $SU(3)$ is conserved, we have, from the eqn. (21) and (22) :

$$\mu_{B_c^{++}} = \tfrac{1}{3}\,(6\,\mu_p + 7\,\mu_n). \qquad \ldots \quad (23)$$

The general expression for the magnetic moment of baryon can be written thus :

$$< m \mid \mu_{0p} \mid m > = \frac{Q}{6}\,(6\,\mu_p + 7\,\mu_n) + \frac{\mu_n}{3}\left[U\,(U+1) - \frac{Q^2}{4} - 4\,L\right] - 4\,\mu_n\,L\,(L-1). \qquad \ldots \quad (24)$$

From this, we get the conventional $SU(3)$ results :

$$\mu_{\Sigma^+} = \mu_p \;;\; \mu_{\Xi^0} = \mu_n \;;\; \mu_{r^-} = -\mu_p;$$

$$\mu_{\Sigma^-} = \mu_{\Xi^0} = -(\mu_p + \mu_n).$$

For $(1/2)^+$ baryons, we get :

$$\mu_{B_c^0} = \mu_n \;;\; \mu_{\varphi_c^-} = \mu_n \;;$$

$$\mu_{\Delta_{cc}^0} = \mu_p \;;\; \mu_{\lambda_{cc}^-} = \mu_p \;;$$

$$\mu_{B_c^{++}} = \mu_{\Delta_{cc}^-} = \tfrac{1}{3}\,(6\,\mu_p + 7\,\mu_n).$$

For $(3/2)^+$ baryons, we have :

$$\mu_{\Delta_c^{++}} = 2\mu_p - 3\,\mu_n \;;\; \mu_{\Delta_c^*} = \mu_n$$

$$\mu_{\Delta_c^+} = \mu_p \;;\; \mu_{\Delta_k^{+0}}\,\mu_n \;;\; \mu_{\Sigma_c^0} = \mu_n;$$

$$\mu_{\Xi_{cc}^{++}} = 2\,\mu_p - 3\,\mu_n \;;\; \mu_{\Sigma_{cc}^+} = \mu_p \;;$$

$$\mu_{\Xi_{ccz}^+} = \mu_p \;;$$

$$\mu_{\Xi_{ccc}^{++}} = 2\,\mu_p - 3\,\mu_n.$$

Conclusion

It is seen that although a different technique is used for finding the mass splitting, the results are the same as derived by others from other considerations. But the technique we are adopting is very elegant and easier to handle. From this technique, we get the $SU(3)$ results correctly. This technique gives the correct values of the masses of hadrons and the magnetic moment results of charmed baryons.

It is evident that for determining the exact magnetic moments of all the charmed baryons, we must know at least the values of one of them. So the sum-rules are also derived.

Appendix

1/2⁺ charmed baryon :

Pattern

	3210 310 31 3	3210 310 31 1	3210 310 31 2	3210 220 22 2	3210 310 21 2	3210 310 21 1
	B_c^{++}	B_c^{+}	B_c^{0}	λ_c^{0}	Ξ_c^{-}	Ξ_c^{0}

Quantum Numbers

Q	2	1	0	1	1	0
t	1	1	1	0	1/2	1/2
Y	0	0	0	0	—1	—1
C	1	1	1	1	1	1
U	0	1/2	1	1/2	1/2	1
L	1/2	1	1/2	1	1	1/2

Pattern

	3210 310 21 2	3210 310 21 1	3210 310 11 1	3210 320 32 3	3210 320 32 2	3210 320 22 2
	θ_c^{-}	θ_c^{0}	φ_c^{-}	Δ_{cc}^{-}	Δ_{cc}^{0}	Υ_{cc}^{-}

Quantum numbers

Q	1	0	0	2	1	1
t	1/2	1/2	0	1/2	1/2	0
Y	—1	—1	—2	—1	—1	—2
C	1	1	1	2	2	2
U	1/2	0	1	0	1/2	1/2
L	1/2	1/2	1/2	1/2	1	1

3/2† charmed baryon

pattern

3300 310 31 3	3300 310 31 1	3300 310 31 2	3300 310 21 2	3300 310 21 1	3300 310 11 1	3300 320 32 3	3300 320 32 2	3300 320 22 2	3300 330 33 3
Δ_c^{++}	Δ_c^{0}	Δ_c^{+}	Δ_c^{*+}	Δ_c^{*0}	Σ_c^{0}	Σ_{cc}^{++}	Σ_{cc}^{+}	Ξ_{cc}^{+}	Ξ_{ccc}^{+}

Contd.

Quantum Numbers

Q	2	0	1	1	0	0	2	1	1	2
t	1	1	1	1/2	1/2	0	1/2	1/2	0	0
Y	0	0	0	—1	—1	—2	—1	—1	—2	—2
C	1	1	1	1	1	1	2	2	2	3
U	0	1	1/2	1/2	1	1	0	1/2	1/2	0
L	3/2	1/2	1	1	1/2	1/2	3/2	1	1	3/2

Charmed Mesons

Petterns

	4200 310 21 1	4200 310 21 2	4200 220 22 2	4200 200 10 1	4200 200 10 0	4200 200 00 0	4200 210 21 1
	D^{o}	D^{+}	F^{+}	D^{-0}	D^{-}	F^{-}	ψ^{*}
Q	0	1	1	0	—1	—1	0
t	1/2	1r2	0	1/2	1/2	0	0
Y	—1	—1	0	1	1	0	0
C	1	1	1	—1	—1	—1	0

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