ENERGY DISSIPATION IN A TIDAL CHANNEL DUE TO BOTTOM FRICTION

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A method of estimating the proportion of energy dissipation in the propagation of a tidal wave through a natural channel due to bottom friction and eddy losses has been developed in this paper. This proportion is found to be in the ratio of 7 : 3 respectively for an example case, that has been drawn from the River Hooghly, West Bengal (India).

During such a propagation through shallower tidal reaches, distortion of the wave occurs and "drift current" of significant order is implicitly generated. The direction and magnitude of this current also receives influence from sudden expansions or contractions of the channel sections.

The effects of friction and drift current set up, what is commonly called as "gorging" effect and manifests near the tidal apex, where low water tidal levels do not conform to the behaviour as exhibited at the seaward end of the estuary. A possible explanation of the physical behaviour of this phenomenon has been included.

INTRODUCTION

THE deep water long period tidal waves are almost simple harmonic in nature. The duration for which the system continues to gain potential energy remains equal to that during which it looses. However, as the tide propagates landwards, proportionately shallower depths and constrictions along lateral boundaries bring about a change in the distribution of these durations by way of shortening the former and at the same time extending the latter, although maintaining the total periodicity nearly unaltered.

Many estuaries appear funnel shaped, where the cross-sectional area (or the width) reduces exponentially in the landward direction. But the bars, bends, relatively shallower depths and uneven bottom topography appear in the channel and bring about a huge loss in the energy content of the propagating tidal wave.

In alluvial tidal channel, the shape and depth are implicitly related to energy losses taking place in the channel. However, the contributions of the various energy dissipating agencies could be used as a useful index for various considerations. These primarily are : (a) bed friction losses; and (b) eddy losses (Bandyopadhyay *et al.,* 1979) (i.e., losses contributed by all the agencies other than friction being grouped together).

The present attempt tries to develop a methodology to identify the proportion of contribution from them.

Tidal wave during its propagation in the landward direction encounters long stretches of shallower depths (i.e., where $\frac{\text{depth of water}}{\text{wave length}} < 0.05$) and consequently receives strong influence of friction. This parameter being non-linear in nature distorts the wave shape considerably. Such a condition incipiently brings about a unidirectional flow (drift) component to influence the energy dissipation pattern significantly and cannot be overlooked. It is interesting to note that the drift current component is significant when the tidal compartment is automatically subjected to advancing phases of the Moon (i.e., Neap to Spring tide) or its receding counterpart (i.e., Spring to Neap tide). In such a state, the tidal apex is "filling" in the former and "emptying" in the later case cyclically, and this drift component would obviously exercise differing influences even for the same range of tide. The composite influence of water retaining effects of friction and drift current in turn elevates the low water levels with increasing ranges of tide and produces an effect which is called "gorging" effect.

The methodology developed herein, has been applied for the case study of the River Hooghly of India. In this long waterway, the stretches are shallower and the tidal wave manifests considerable distortion during its propagation through it.

ENERGY DISSIPATION DUE TO CHANNEL BOTTOM FRICTION

The frictional stress at the bottom of the channel may be expressed as :

$$\tau_b = -K\rho/v_b/v_b, \qquad \ldots(1)$$

where

τ_b = stress at the bottom per unit area.

v_b = velocity at the outer surface of friction layer.

ρ = density of water.

K = constant of proportionality.

Assuming that the square law of friction holds good, the time mean rate of dissipation of tidal energy per unit area of channel bottom per unit time, could be presented in the form :

$$< \tau_b \, v_b > = K \, \rho/v_b/v_b^2 \qquad \ldots(2)$$

The total energy dissipated per unit time by friction can be obtained by integrating eqn. (2) over the bottom area i.e.,

$$\int_0^l \int_0^b < \tau_b v_b > \mathrm{d}l \, \mathrm{d}b, \qquad \ldots(3)$$

where l is the length of the channel and b is the width of the channel.

In the case of tidal flow exhibiting distorted wave shape, the velocity component for a first degree of approximation may be expressed as :

$$v_b = v_0 + v_1 \cos \omega t, \qquad \ldots(4)$$

where ω is the angular speed, v_0 and v_1 are the steady and oscillatory parts respectively of the velocity v_b.

Further, the term $/v_b/v_b^2$ could be expressed by a Fourier's series of the form:

$$/v_b/v_b^2 = \sum_{n=0}^{\infty} A_n \cos n\omega t = A_0 + A_1 \cos \omega t \cos \omega t + A_2 \cos 2\omega t + A_n \cos n\omega t, \qquad \ldots(5)$$

$$\text{where } A_0 = \frac{1}{2\pi} \int_0^{2\pi} /v_b/v_b^2 \, d\theta \qquad \ldots(6)$$

$$\text{and } A_n = \frac{1}{\pi} \int_0^{2\pi} /v_b/v_b^2 \cos n\theta \, d\theta, \qquad \ldots(7)$$

for $n = 1, 2, 3$ etc.

$$\theta = \frac{2\pi t}{T} = \omega t, \text{ where T is the time period.}$$

On evaluating the integrands (6) and (7) and substituting in eqn. (4), one gets (neglecting higher harmonics) :

$$A_0 = v_0^3 + \frac{6}{\pi} v_0^2 v_1 + \frac{3}{2} v_0 v_1^2 + \frac{4}{3\pi} v_1^3 \qquad \ldots(8)$$

$$\text{and } A_1 = \frac{4}{\pi} v_0^3 + 3 v_0^2 v_1 + \frac{8}{\pi} v_0 v_1^2 + \frac{3}{4} v_1^3 \qquad \ldots(9)$$

The total energy dissipation per unit time by bottom friction therefore may be written as :

$$K\rho \left(A_0 + A_1 \cos \frac{2\pi t}{T} \right) \times \text{bottom area}$$

$$= K\rho \left[\left(v_0^3 + \frac{6}{\pi} v_0^2 v_1 + \frac{3}{2} v_0 v_1^2 + \frac{4}{3\pi} v_1^3 \right) + \left(\frac{4}{\pi} v_0^3 + 3 v_0^2 v_1 + \frac{8}{\pi} v_0 v_1^2 + \frac{3}{4} v_1^3 \right) \cos \frac{2\pi t}{T} \right] \times \text{bottom area} \qquad \ldots(10)$$

PROCEDURE

The procedure mainly consists of :—

(a) Assessment of the parameters v_b, K and ρ.

(b) Computation of the energy dissipation per unit time with the help of these parameters.

For the estimation of the energy dissipation due to bed friction alone, the reachwise values of v_b (i.e., v_0 and v_1), K and ρ are to be assessed for the stretch of the channel under study. No doubt, the assessment poses problem, since all these parameters are functions of tidal state (i.e., time variant), and judicious considerations are to be exercised for obtaining their values from observational data.

Assessment of the Parameters — For assessing the flow velocity close to bed (v_b), conventional approaches assumed that the velocity distribution in the vertical directions at the observational point remains logarithmic and obtained the requisite data by extrapolating the values available at any other known depth. But Bowden and Fairbairn (1956) have favoured the use of actual values of velocity observed just at the outer limit of the "skin friction" layer, accepting the layer thickness at a level of 2 m (and on a subsequent occasion at a level of 0.75 m) above the bed. Subsequent investigators (Yalin & Russell, 1966) have relied on laboratory experiments, that were conducted for the exploration of the various parameters of this layer that develop in the presence of damped harmonic type of flow velocity profile. From the relationship established thereof, the thickness works out to be 1.4 m for a tidal wave period of 745 minutes.

In this context, it may be mentioned that, with the strength (phases) of the tide, the characteristics of the layer continue to change and has also been supported by the laboratory investigations mentioned above. The present approach however has favoured the concept of the formation of a layer of thickness which varies with the phases of the tide. For the reaches of the River Hooghly which has been exemplified, the average ratio of velocity values at the boundary of the layer occupying on the channel bed a thickness of 20 per cent of the total depth with that of the values at the surface has been found to compare favourably with the average ratio of the velocity values exhibited at a height of 1.4 m from the bottom with that of the surface over a complete tidal cycle. Therefore, the observational data at a depth ratio of 0.8 from the surface has been considered to be adequate. The values of v_0 and v_1 were then obtained by using harmonic analysis of the abovementioned observational data (i.e., velocity-time curve).

Regarding the choice of the value of the parameter K, classical investigators (Grace, 1936; and Proudman, 1953) have adopted a numerical value of 2×10^{-3}. However, it is interesting to note, that by balancing the energy slope to friction slope, one could assess the value for every dynamic state, as the relationship may be written as :—

$$K = g/C_c^2, \qquad \ldots(11)$$

where C_c is the Chezy's constant.

The present approach has considered the use of average values of Chezy's constant which could satisfy the corresponding mathematical model (Nag & Sen, 1967) of tidal wave propagation in the reaches under study (*vide* Table I).

The fluid density also varies continuously with time in a tidal reach, yet for a first approximation model, the mean value as observed in those reaches were used i.e., 1.017 gms/cc.

Computation — Having identified and assessed the requisite parameters, eqn. (10) was used for computing the energy dissipation rate due to bottom friction.

TABLE I

Assessment of the values of K (numerical)

Name of the Reach	Moyapur-Akra	Akra-Garden Reach	Garden Reach-Konnagar	Konnagar-Mulajore	Mulajore-Tribeni
(1)	(2)	(3)	(4)	(5)	(6)
Reach Length (Km)	19.50	9.75	19.36	16.90	18.50
(Chezy's Constant) $C_e(m^{1/2}/\text{Sec.})$	80	72	75	72	65
K(Computed)	00.00153	00.00189	00.00174	00.00189	00.00232

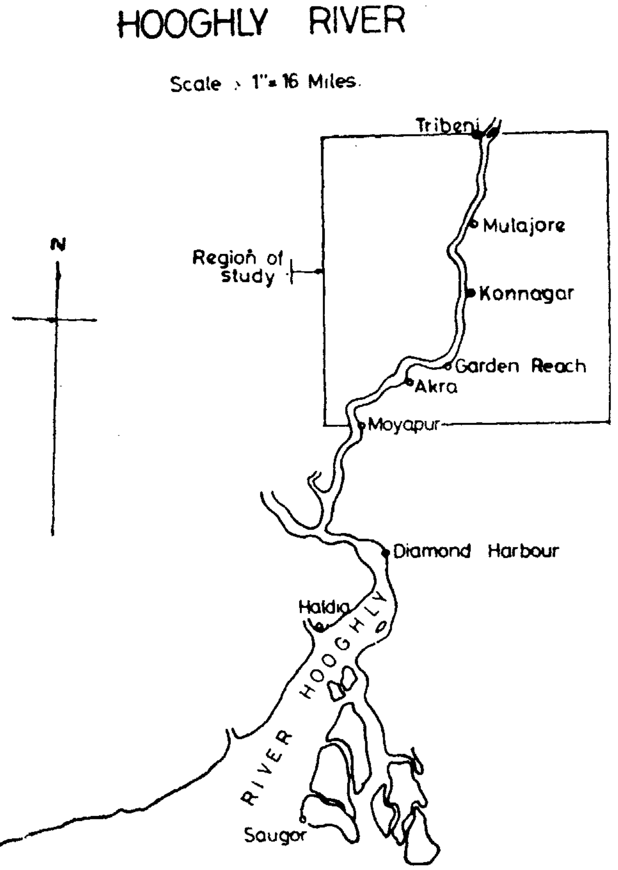

FIG. 1

The Example Case

The example has been drawn from the River Hooghly of India.

The Reach — The tidal reach of the river Hooghly extends about 290 km in the landward direction from the seaface at Saugor. The amplitude of the tidal wave at the entrance to this estuary varies from about 5 m to 2 m between Spring and Neap. During the propagation, the tidal flow is absorbed by four tributaries according to their respective channel capacities. The present study has been purposely left confined within a stretch free from energy sharing, occupying a length of 84 km. from Moyapur in the downstream side to Tribeni in the upstream direction (Fig. 1). During the landward travel of the tidal wave, the effects of the predominating bottom friction of the shallower reaches are manifested in strongly distorting the velocity-time curve. A typical case is exemplified in Fig. 2.

The present study encompasses examples for typical spring (4.4 m), average (3.66 m) and neap (2.9 m) tides, wherever simultaneous observations at those reaches were available. Table II shows the time mean rate of energy dissipation contributed by bottom friction of this stretch of the reach.

To have an idea of the contribution of the bottom friction towards total energy dissipation rate, it would be necessary to estimate the reachwise values of the latter also. The salient features of the methodology (Bandyopadhyay *et al.*, 1979) of the estimate is given in *Appendix-I.* The values have been tabulated in Table II (Column-9).

In this context, it may be mentioned that, if one neglects the contribution of the drift current generated from the distortion of the tidal wave (i.e., velocity-time curve), the energy dissipation pattern for the reach takes a new look, as the corresponding

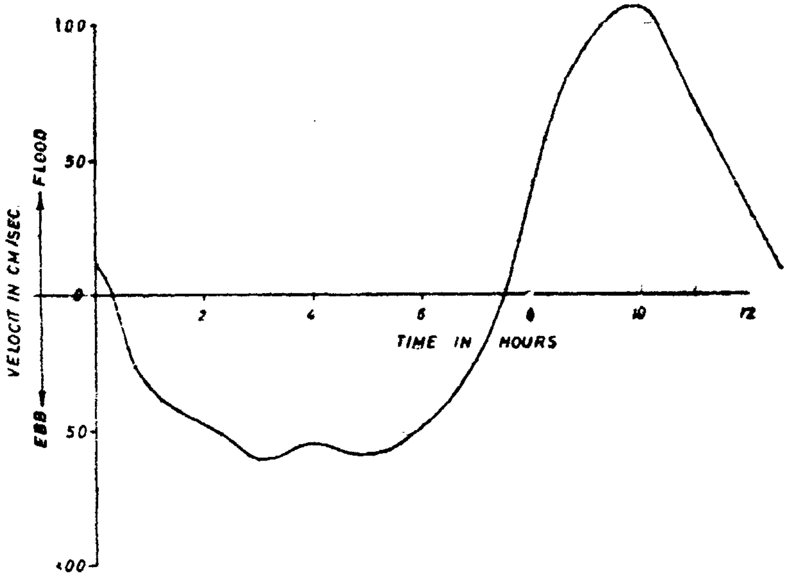

FIG. 2. Typical Velocity *vs.* Time Curve-Reach : Akra-Garden Reach. Tidal range—3.6m

Note : The lowest point on lower quadrant of Fig. 2 may be read as 100.

TABLE II

Assessment of energy dissipation rate

Reach	Tidal Range at seaface m	Date of observation	v_0 m/sec	v_1 m/sec	A_0	A_1	Time Mean Rate of Energy Dissipation due to bottom friction $=<\tau_b v_b>$	Total Energy Dissipation Rate (Eqn. A. 3)	Percentage contribution of bottom friction	A'_0	A'_1	Time Mean Rate of Energy Dissipation due to Bottom Friction (in the absence of drift current)
			←Eqn. 10→							←Eqn. II.1→		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Moyapur-Akra	4.40		-0.152	1.070	0.302	0.544	22.2×10^6	30.0×10^6	74	0.520	0.918	28.9×10^6
-do-	3.66	24-5-64	0.248	1.160	0.187	0.345	17.68×10^6	20.4×10^6	87	3.424	1.170	32.0×10^6
Akra-Garden Reach	4.42	4-5-65	-0.190	1.090	0.279	0.505	8.5×10^6	14.0×10^6	60.7	0.550	0.971	16.3×10^6
-do-	3.30	16-5-64	-0.124	1.006	0.271	0.489	8.28×10^6	11.0×10^6	75.2	0.432	0.754	12.7×10^6
Mulajore-Tribeni	4.40	5-6-73	-0.074	0.900	0.197	0.353	7.47×10^6	9.0×10^6	83	0.309	0.547	11.75×10^6
-do-	2.90	27-1-66	-0.086	0.718	0.056	0.187	2.13×10^6	5.1×10^6	41.8	0.157	0.277	5.89×10^6

TABLE III

State of tidal apex

(Typical examples)

Station	Date of observation	←——— Ranges on				———→	Values of		Remarks
		2nd Preceding day of observation (m)	1st Preceding day of observation (m)	Day of observation (m)	1st Succeeding day of observation (m)	2nd Succeeding day of observation (m)	v_0 (m/s)	v_1 (m/s)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Moyapur	24-5-64	—	3.90	4.05	4.24	4.42	0.248	1.160	Filling
Akra	4-5-65	4.69	4.69	4.57	4.39	3.87	—0.190	1.090	Emptying
	16-5-64	4.94	4.51	3.93	3.29	2.56	—0.124	1.006	-do-
Tribeni	5-6-73	1.80	1.80	1.70	1.50	1.50	—0.074	0.900	-do-
	27-1-66	1.89	1.71	1.58	1.40	1.28	—0.086	0.718	-do-

equation (eqn. 10) gets modified to eqn.-II.1 (*vide Appendix-II*). The estimate of the energy dissipation rate due to bottom friction has also been computed neglecting the drift current and for the sake of comparison has been tabulated (Table II—Column 13).

RESULTS AND DISCUSSIONS

In a tidal wave propagating system as that of the River Hooghly, low water levels gradually become higher as the tidal ranges shift from Spring to Neap. However, this phenomenon reverses in reaches almost close to the tidal apex and is commonly known as "gorging effect." In this area, the low water retention due to higher friction alongwith significant influence of sizeable proportion of generated drift current exhibits such a phenomenon, which due to weaker influence of the latter naturally is not manifested in the lower reaches.

It is interesting to note that, any tidal reach is subjected to the same order of inputs both on advancing and receding Lunar phases (i.e., in tidal states changing from Neap to Spring or Spring to Neap). In the former, the tidal apex continues to fill up, while in the latter it drains out (*vide* Table III) and generates two differently directed influence on drift current even for the same range of tide with consequent repercussion on the dissipation contributed by bottom friction. However, to avoid any additional influence, in this case, the data for the period without upland discharge has been used.

In this attempt, in the absence of any other available data, the average values of C_c was used which is not likely to have introduced any significant error, considering that the evaluation was carried out for the full tidal cycle.

The use of 0.8 ratio for the choice of the depth of observation apparently influences the result, close to high water state, but at that time the condition being slack, is not likely to introduce any serious error.

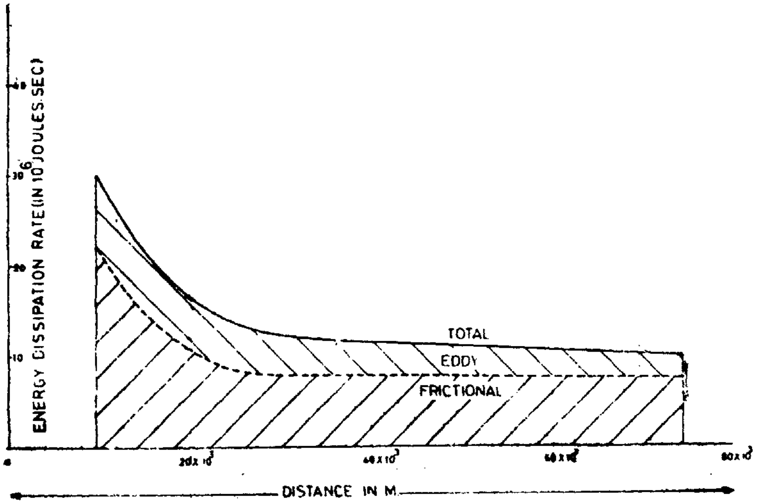

FIG. 3. Rate of energy dissipation along the tidal channel Range:—4.4m

(Energy dissipation rate in 10 Joules/Sec.)

The drift current is further influenced by any storing area available in the adjacent reaches upstream/downstream of the particular reach under study and imparts additional control independent of advancing or receding lunar phases.

CONCLUSION

In a typical spring tide, the contribution of dissipation is shared between bottom friction and eddy losses in the ratio of about 70 per cent and 30 percent (Fig. 3); and the variation from this approximate value in some reaches depends on the area occupied by bends, bars etc., in comparison to the total reach area and to a certain extent on the reach shape.

It would be noticed (Table II) that the order of the first harmonic component of the bed velocity has not changed significantly with changes in tidal range, whereas the percentage contribution of the bottom friction has altered considerably. The higher percentage of contribution is associated with higher depths availble at low water condition and not necessarily with higher tidal ranges.

In long shallow tidal reaches, the drift current becomes sizeable and its neglect vitiates the estimate of dissipation significantly.

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NOMENCLATURE

g : acceleration due to gravity (M/sec^2) (L/T^2)
ρ : density of water (gms/cc) (M/L^3)
H : tidal elevation (M) (L)
u : mean velocity perpendicular to the transverse section. (M/sec) (L/T)
Q : corresponding discharge. (M^3/sec) (L^3/T)
A : area of the transverse section. (M^2) (L^2)
$W = g\rho$; Specific weight of water. (Dynes/cc) (M/L^2T^2)
ϕ_n, Ψ_n ; Phase with respect to low water at the reference station. (Degrees)

Appendix I

Total Energy Dissipation Rate in a Tidal Reach

In a tidal channel, time rate of doing work across any transverse section due to tidal wave propagation is given by—

$$P = g\,\rho \int\int H u \,\mathrm{d}A \qquad \text{... I.1}$$

The time mean value may be expressed (in terms of discharge) as

$$P = \frac{1}{T}\int_0^T W\left[\sum_{n=0}^{\infty} H_n \operatorname{Cos}(n\,\omega t + \phi_n) \times \sum_{n=0}^{\infty} Q_n \operatorname{Cos}(n\,\omega t + \psi_n)\right] \mathrm{d}t$$

for, $n = 0, 1, 2, 3, \ldots\ldots$ etc.

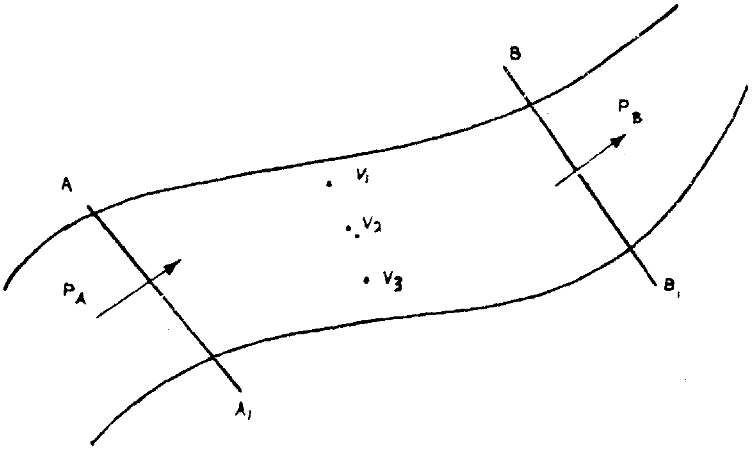

FIG. 4. V_1, V_2, V_3 are the velocity observation points.

$$= W\left[H_0 Q_0 + \sum_{n=1}^{\infty} \tfrac{1}{2} (H_n Q_n \operatorname{Cos} \theta_n) \right] \qquad \ldots \text{I.2}$$

where $\theta_n = \phi_n \sim \psi_n$

and the available energy at that section is given by

$$E = \int_0^T P\, dt \qquad \ldots \text{I.3}$$

If one considers any reach bounded by two transverse sections AA′ and BB′ (Fig. 4), where the difference between the available energies at these sections, indicates the total dissipation suffered by the tide while negotiating through that reach.

Appendix II

The Effect of Drift Current on the Behaviour of Energy Dissipation Rate

In the region where tide propagates through deeper channel, the wave shape does not suffer serious distortion (assuming that the contribution from tidal apex/reach due to lunar incremental or decremental phase is of smaller order) i.e.,
eqn. (10) becomes (Bowden, 1953).

$$K \rho \left(A_0' + A'_1 \cos \frac{2\pi t}{T}\right) \times \text{bottom area}$$

$$= K \rho \left(\frac{4}{3\pi}\, v_1^3 + \frac{3}{4}\, v_1^3 \operatorname{Cos}\frac{2\pi t}{T} \right) \times \text{bottom area} \qquad \ldots \text{II.1}$$

It is evident that the contribution of the drift current has a significant influence due to its *vectorial* ! *value* in comparison to the corresponding first harmonic content.

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