ANALYSIS OF AN INEXPENSIVE SOLAR COLLECTOR/STORAGE SYSTEM

M. S. SODHA, F. N. A., G. N. TIWARI *and* S. C. KAUSHIK
Centre of Energy Studies and Physics Department, Indian Institute of Technology, New Delhi 110 029

and

M. A. S. MALIK,
Kuwait Institute for Scientific Research, P.O. Box 12009, Kuwait

(*Received 25 October 1978*)

This paper presents an analysis of the performance of an inexpensive solar collector, which also acts as a storage of heat. The collector consists of a mass of sand/concrete/brick inside which pipes (metallic or plastic) are laid in a plane from which heat can be retrieved by flow of fluids; the upper surface of the concrete is blackened and glazed, while the lower surface is insulated with glass fiber.
The authors have derived an expression for the transient rate at which heat can be retrieved, to keep the temperature of the plane of retrieval constant. Numerical calculations corresponding to a hot day in Kuwait and typical collector parameters, predict a collection efficiency of 22% and 18% corresponding to collection temperatures of 45 °C and 50 °C respectively; the maximum of the rate of retrieval occurs about 6–8 hours after the maximum of solar temperature.

INTRODUCTION

IN solar hot air/water systems, it is necessary to provide a thermal storage in addition to the solar collectors; in this paper, we propose and analyze a compact inexpensive configuration, which combines both the features and can be integrated in architecture with minimal additional cost. The proposed solar collector/storage system consists of a block of concrete (or mass of sand) whose upper surface is blackened and glazed, while the lower surface is insulated. Heat is obtained by flow of a fluid through pipes laid in a plane inside the block. Such a collector cum storage system can form an integral part of architecture of a building.

The authors have obtained an expression for the rate at which heat can be collected to keep the plane at retrieval (at a depth l_1) at constant temperature (T_{00}).

ANALYSIS

The proposed configuration is schematically illustrated by Fig. 1.

The temperature distribution $T(x, t)$ in regions I $(-l_1 \leqslant x \leqslant 0)$ and II $(0 \leqslant x \leqslant l_2)$ is described by the one dimensional equation of heat conduction viz.,

$$K \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t} . \qquad \dots (1)$$

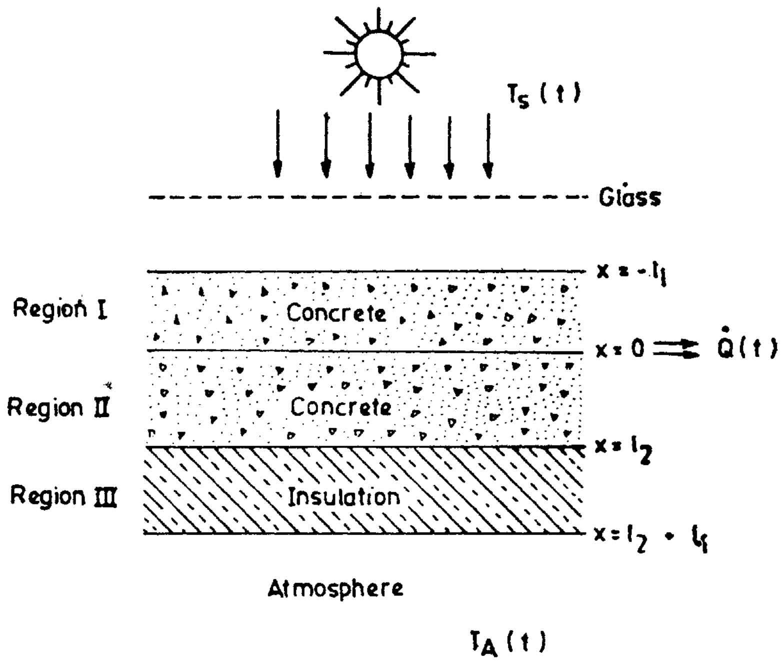


FIG. 1. Configuration of solar collector/storage system.

Whence the periodic solutions for $T(x, t)$ in regions I and II may be expressed as

$$T(x, t) = T_0 - T_1 x + \sum_{m=1}^{\infty} \left[A_m \exp(\alpha_m x) + A^1 m \exp(-\alpha_m x) \right] \exp(im\, wt), \quad \ldots(2)$$

in region I and

$$T(x,t) = \theta_0 - \theta_1 x + \sum_{m=1}^{\infty} \left[B_m \exp(\alpha_m x) + B'_m \exp(-\alpha_m x) \right] \exp(im\, wt), \quad \ldots(3)$$

in region II

where

$$\alpha_m = -\alpha\sqrt{m}\,(1 + i), \quad \alpha = \left(\frac{w\rho c}{2k}\right)^{\frac{1}{2}}$$

Similarly, the temperature distribution in region III ($l_2 \leqslant x \leqslant l_2 + l_i$) may be expressed as

$$T(x, t) = \Phi_0 - \Phi_1 x + \sum_{m=1}^{\infty} \left[C_m \exp(\alpha_{im} x) + C'_m \exp(-\alpha_{im} x) \right] \exp(im\, wt) \quad \ldots(4)$$

The energy balance on surfaces $x = -l_1$, $x = 0$ and $x = l_2 + l_i$ requires

$$-K\left(\frac{\partial T}{\partial x}\right)_{x=-l_1} = \alpha_0 \; S(t) + h(T_A - T_{x=-l_1}) - \epsilon \Delta \; R = h(T_{SA} - T_{x=-l_1}), \quad \ldots(5)$$

$$-K\left(\frac{\partial T}{\partial x}\right)_{x=0;\,\text{region I}} = -K\left(\frac{\partial T}{\partial x}\right)_{x=0;\,\text{region II}} + Q(t) \quad \ldots(6)$$

and

$$-K_i\left(\frac{\partial T}{\partial x}\right)_{x=l_2+l_i\,;\,\text{region III}} = h'\left(T_{x=l_2+l_i} - T_A\right), \quad \ldots(7)$$

where

$$T_{SA} = \frac{\alpha_0 S(t) + hT_A - \epsilon \Delta R}{h}$$ is called solair temperature.

The continuity of temperature on the interfaces $x = 0$ and $x = l_2$ and that of heat flux the interface $x = l_2$ may be expressed as

$$T_{x=0};\ \text{region I} = T_{x=0};\ \text{region II} \quad \ldots(8)$$

$$T_{x=l_2};\ \text{regionII} = T_{x=l_2}\ ;\ \text{region III} \quad \ldots(9)$$

and

$$-K\left(\frac{\partial T}{\partial x}\right)_{x=l_2\,;\,\text{region II}} = -K_i\left(\frac{\partial T}{\partial x}\right)_{x=l_2;\,\text{region III}} \quad \ldots(10)$$

We may also express the solair temperature T_{SA} and the atmospheric temperature T_A as Fourier Series in time, thus

$$T_{SA} = T_{SO} + \sum_{m=1}^{\infty} a_m \exp(im\omega t) \quad \ldots(11)$$

and

$$T_A = T_{AO} + \sum_{m=1}^{\infty} bm \exp(im\omega t), \quad \ldots(12)$$

where

$a_m = T_{Sm} \exp(-i\sigma_m)$, $b_m = T_{Am} \exp(-i\sigma'_m)$; T_{Sm}, T_{Am}, σ_m and σ'_m are given in Tables I and II respectively.

For various applications it is useful to keep the temperature in the plane of heat retrieval to be constant; thus

$$T(o, t) = T_{00}. \quad \ldots(13)$$

TABLE I

Fourier analysis of daily variation of solair temperature in Kuwait (June 21, 1975)

m	0	1	2	3	4	5	6
T_{sm} °C	86.9096	80.9464	31.7143	11.7172	5.4195	2.7488	3.0279
σ_m radians	—	03.6968	01.05104	00.8039	4.8155	0.6866	4.0130

TABLE II

Fourier analysis of daily variation of ambient temperature in Kuwait (June 21, 1975)

m	0	1	2	3	4	5	6
T_{AM} °C	38.025	6.5270	1.4076	0.8846	0.1897	0.3365	0.2424
σ'_m radians	—	4.2962	1.7424	1.8559	5.7313	5.7927	5.1403

Substituting for T from eqns. (2) to (4) in the boundary conditions (5), (7)–(9), (10) and (13), we obtain

$$kT_1 = h\,(T_{S0} - T_0 - T_1\,l_1); \left(1 - \frac{\alpha_m' k}{h}\right) \exp\,(-\alpha_m\,l_i)\,A_m + \left(1 + \frac{k\alpha_m}{h}\right) \exp\,(\alpha_m\,l_1)\,A^1{}_m = a_m$$

$$k_i\,\Phi_1 = h'\left[\Phi_0 - \Phi_1\,(l_2 + l_i) - T_{A0}\right]; \left(1 + \frac{k_i\,\alpha_{im}}{h'}\right) \exp\left[\alpha_{im}\,(l_2 + l_i)\right] C_m + \left(1 - \frac{k_i\,\alpha_{im}}{h'}\right) \exp\left[-\alpha_{im}\,(l_2 + l_i)\right] C'_m = b_m$$

$$T_0 = T_{00} \quad ; A_m + A'_m = 0$$

$$\theta_0 = T_{00} \quad ; B_m + B'_m = 0$$

$$\theta_0 - \theta_1\,l_2 = \Phi_0 - \Phi_1\,l_2 \;;\; B_m \exp\,(\alpha_m\,l_2) + B'_m \exp\,(-\alpha_m\,l_2) = C_m \exp\,(\alpha_{im}\,l_i) + C'_m \exp\,(-\alpha_{im}\,l_i)$$

$$K\theta_1 = K_i\,\Phi_1 \quad ; K\alpha_m\left[B_m \exp\,(\alpha_m\,l_2) - B'_m \exp\,(-\alpha_m l_2)\right] = K_i\alpha_{im}\left[C_m \exp\,(\alpha_{im}\,l_2) - C'_m \exp\,(-\alpha_{im} l_2)\right] \quad \ldots(14)$$

The above equations lead to

$$\theta_0 = T_0 = T_{00} \;;\; T_1 = h\,(k + l_1\,h)^{-1}\,(T_{S0} - T_{00})$$

$$\Phi_0 = \left\{T_{00} + T_{A0}\,h' l_2 \left(\frac{k_1}{k} - 1\right)\left[k_i + h'\,(l_2 + l_2)\right]^{-1}\right\} \Big/ \left\{1 + h'\left(\frac{k_i}{k} - 1\right)\left[k_i + h'\left(l_2 + l_i\right)\right]\right\}$$

$$\Phi_1 = h'\,(\Phi_0 - T_{A0}) \,/\, [k_i + h'\,(l_2 + l_2)\;;\; \theta_1 = k_i\,\Phi_1/k$$

$$-A_m = A'_m = a_m \left\{\left(1 + \frac{k\alpha_m}{h}\right) \exp\,(\alpha_m\,l_1) - \left(1 - \frac{k\alpha_m}{h}\right) \exp\left(-\alpha_m\,l_1\right)\right\}^{-1}$$

$$B_m = -B'_m = 2\,b_m \Big\{\Big(1+\frac{k_i\alpha_{im}}{h'}\Big)\Big[\Big(1+\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp(\alpha_m l_2) - \Big(1-\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp\Big(-\alpha_m l_2\Big)\Big]\exp(\alpha_{im} l_i) + \Big(1-\frac{k_i\alpha_{im}}{h'}\Big)\Big[\Big(1-\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp(\alpha_m l_2) - \Big(1+\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp(-\alpha_m l_i)\Big]\exp(-\alpha_{jm} l_i)\Big\}$$

$$C_m = \frac{B_m}{2}\exp(-\alpha_{im} l_2)\Big[\Big(1+\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp(\alpha_m l_2 - \Big(1-\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp(-\alpha_m l_2)\Big]$$

and

$$C'_m = \frac{B_m}{2}\exp(\alpha_{1m} l_2)\Big[\Big(1+\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp(\alpha_m l_i) - \Big(1+\frac{k\alpha_m}{k_i\alpha_{im}}\Big)\exp(-\alpha_m l_2)\Big]. \quad \ldots(15)$$

From eqn. (6)

$$Q(t) = k\,(T_1 - \theta_1) + 2\sum_{m=1}^{\infty} K\,\alpha_m (A'_m + B_m)\exp(imwt), \quad \ldots(16)$$

where T_1, θ_1, A'_m and B_m are given by eqns. (15)

Numerical Results and Discussion

In order to appreciate the value of this concept, we have carried out numerical calculations to evaluate the transient rate of heat retrieval per unit area corresponding to the following parameters :

(1) Collection Temperature : 40 °C, 45 °C, 50 °C and 55 °C.

(2) Collector Parameters :—

(a) Blackened and glazed top surface corresponding to $\alpha_0 = 0.8$ and $\epsilon = 0.0$

(b) Collector Parameters :

(*i*) Insulation : 5 cm of glass fiber ($k_i = 0.13\ kJ/hr\ m\ °C$,) $\rho_i = 64.08$ kg/m^3, $C_i = 0.67$ KJ/kg °C and $\alpha_i = 3.024 \times 10^{-3}\ m^2/hr$

(*ii*) Collection storage Materials (1) (thickness 30 cms)

Case	Material	(K(KJ/hr m °C)	ρ (Kg/m³)	C(KJ/kg °C)
I	Concrete mortar or plaster	2.597	1858.3	.83
II	Concrete (sand)	6.230	2242.8	.88
III	Brick, common	2.500	1922.4	.84

(*iii*) Depth of plane of retrieval : 20 cm, 25 cm and 30 cm.

(*iv*) Meteorological Parameters : Tables I and II present the coefficients of Fourier series (eqns. 11 and 12) of the solair temperature ($\alpha_0 = 0.8$, $\epsilon = 0$) and the atmospheric temperature; the tables are based on the hourly variation of solar insulation and atmospheric temperature, recorded on June 21, 1975 at Kuwait airport.

(*v*) The overall heat transfer coefficient h between the top surface and the atmosphere (excluding insulation) is equal to 17.38 KJ/m² hr °C.

(*vi*) $h' = 22.08$ KJ/m²/hr °C.

Figs. 2a, 2b and 2c present the variation of $Q(t)$ with time t in the case I, II and III for collection temperatures of 40 °C, 45 °C, 50 °C and 55 °C, the hourly variation of solair temperature has also been illustrated in these figures. Table III tabulates the integrated collection efficiency η, defined as

$$\eta = \int Q(t)\,dt \,/ \int S(t)\,dt$$

for various cases. It is seen that :

(*i*) the maximum of $Q(t)$ occurs 6–8 hrs after the maximum of solair temperature;

(*ii*) higher collection temperatures are obtained at greater depths, of the plane of retrieval and for storage/collector materials with smaller thermal diffusivity;

(*iii*) for a given collection temperaure, η decreases with increasing depth of the plane of retrieval; and

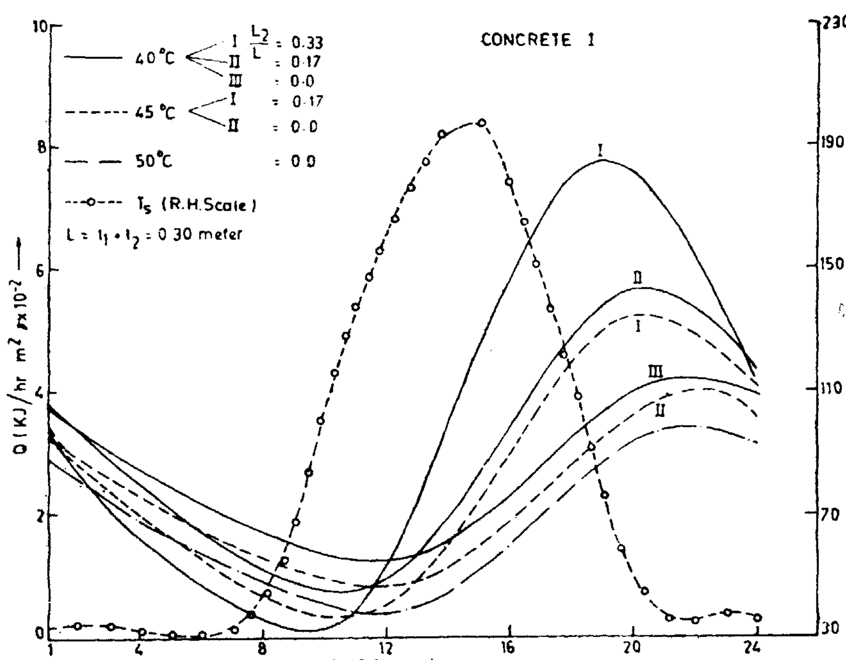
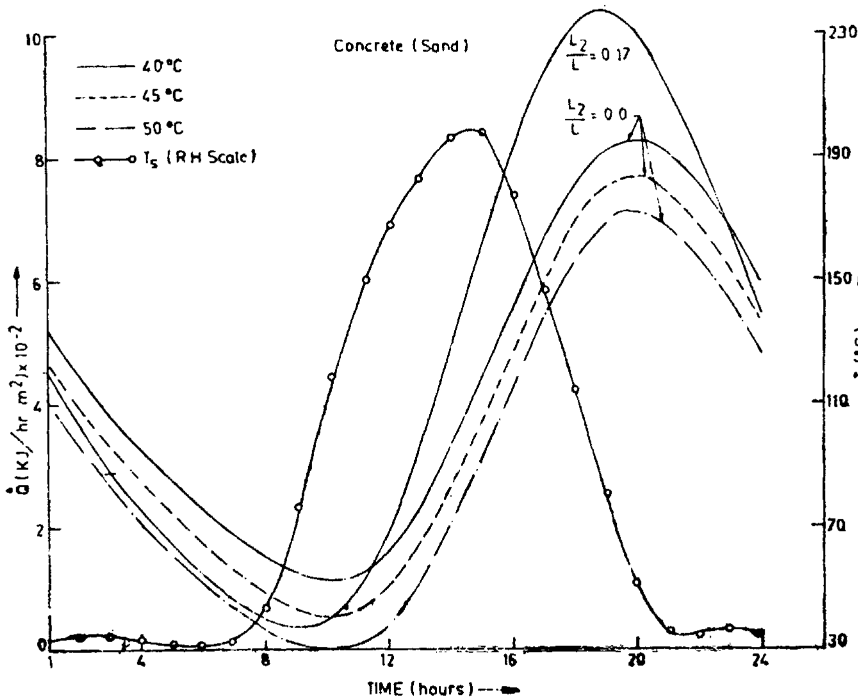
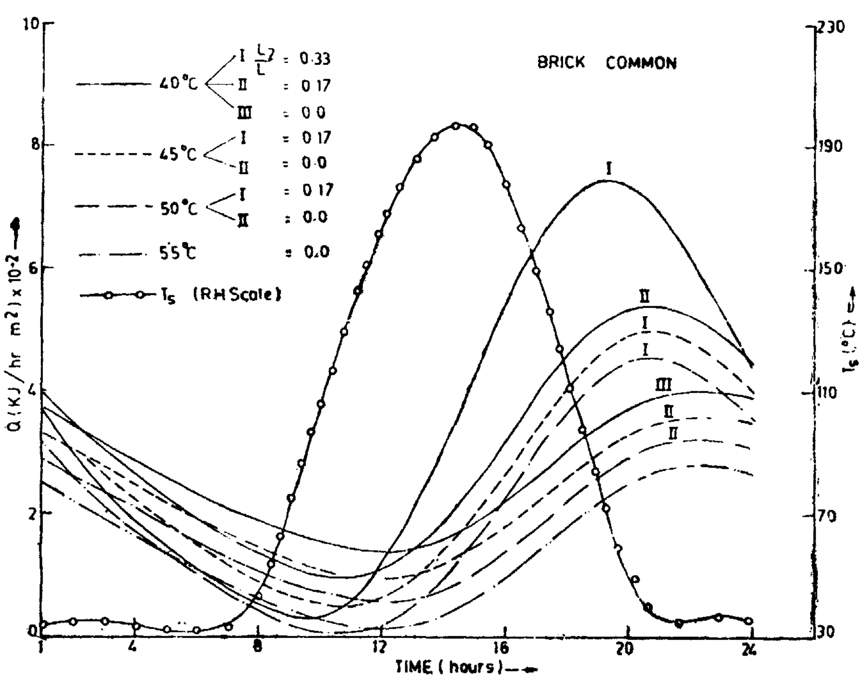


FIG. 2. (a) to (c). Calculated daily hourly variation of heat flux ($Q(t)$) retrieved from the plane $x = 0$ with time for different values of depth i.e. $l_2/L = 0.33$, 0.17 and 0.0 and the specified surface temperatures i.e., 40 °C, 45 °C, 50 °C and 55 °C.

(*a*) Concrete mortar or plaster.



(*b*) Concrete sand.



(*c*) Brick, common.

TABLE III

Collection efficiency, η

	Concrete Mortar I			Concrete Sand II			Brick Common III		
l_2/L \ T_{00}	0.330	1.170	0.000	0.33	0.17	0.00	0.330	0.170	0.000
40 °C	0.312	0.266	0.235		0.42	0.38	0.309	0.265	0.235
45 °C		0.228	0.196			0.33		0.223	0.192
50 °C			0.160			0.28		0.183	0.158
55 °C									0.120

(*iv*) for a given depth, η decreases with increasing collection temperature.

We have for obvious reasons, not discussed cases where $\dot{Q}(t)$ attains negative values for some range of t.

CONCLUSION

The proposed inexpensive collection/storage system, which can be integrated in a roof or other architectural featurs of a building has been analyzed in a mode which keeps the collection temperature constant; reasonable efficiencies are obtained in this mode (Keeping the low cost in mind).

The analysis can be extended to include other modes of operation e.g., insulation of collection temperature corresponding to a specified $\dot{Q}(t)$ variation.

ACKNOWLEDGEMENTS

The authors are grateful to Professor S. P. Sabbarwal for suggesting the configuration investigated. Thanks are also to Dr. R. P. Sharma for various discussions.

Nomenclature

a_m : Defined by eqn. (11)
A'_m, A'_m : Defined by eqn. (2)
b_m : Defined by eqn. (12)
B_m, B'_m : Defined by eqn. (3)
C : Specific heat of concrete, KJ/Kg °C
C_i : Specific heat of insulation, KJ/kg °C
C_m, C'_m : Defined by eqn. (4)
h : Overall heat transfer coefficient between the upper surface of the block and ambient air (excluding insulation), KJ/m^2 hr °C
h' : Convective heat transfer coefficient between lower layer of insulation and air, KJ/m^2 hr °C
K : Thermal conductivity of concrete, KJ/hr m °C
K_i : Thermal conductivity of insulation, KJ/hr m °C
l_1 : Depth of heat transfer pipes, cm

$L=l_1+l_2$	:	Thickness of concrete block, cm
l_i	:	Thickness of insulation, cm
m	:	An integer
$Q(t)$	:	Heat extracted by the pipes per unit area per unit time, KJ/m^2 hr.
ΔR	:	Difference between the long wave radiation, incident on the surface from sky and surroundings, and the radiation emitted by a black body at atmospheric air temperature, KJ/hr m^2.
T_0 T_1	:	Defined by eqn. (2)
T_A	:	Atmospheric temperature, °C
T_{SA}	:	Solar temperature, °C
$T_{A0}T_{Am}$	:	Defined by eqn. (12)
T_{S0}, T_{Sm}	:	Defined by eqn. (11)
T_{00}	:	Temperature of the plane of heat transfer pipes, °C
X	:	Vertical axis downwards
α_0	:	Fraction of solar energy incident on the glass plate, which is absorbed by the upper surface of the concrete (assumed to be constant)
ρ	:	Density of concrete, Kg/m^3
ρ_i	:	Density of insulation, Kg/m^3
θ_0, θ_1	:	Defined by eqn. (3)
Φ_0 Φ_I	:	Defined by eqn. (4)
ω	:	2π (period)$^{-1}$
ϵ	:	Emissivity of the blackened and glazed surface

Reference

Threlkeld, J. K. (1970). *Thermal Environmental Engineering.* Prentice-Hall Inc., New Jersey.