DESIGN AND PERFORMANCE ANALYSIS OF PLANO-CYLINDRICAL FRESNEL LENS

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Design of plano-cylindrical Fresnel plastic lens as solar collector has been given. The concentration ratio for a lens made of plexiglass (PMMA) has been computed for the design wavelength taking into account reflection losses at both the surfaces. The influence of groove width, wavelength dependaence of refractive index of the lens material, defocussing etc., on the concentration ratio has been investigated.

INTRODUCTION

A variety of solar concentrators have been proposed and commissioned for applications in the temperature range of 200°–450 °C by Authier (1977), Clausing (1976), Steward and Kreith (1975), Kreider (1975), and Rabl (1976). These installations are very expensive. Plastic Fresnel lenses for solar energy concentration are attractive because of low cost, mass production potential and compact design Hastings *et al.* (1976). A plano-cylinderical groove down Fresnel lens will essentially focus the incident radiation on a line patch. It is felt that this kind of lenses will find potential applications as solar concentrators in the temperature range of 200–370°C. Because of the light weight and one axis of tracking, the design of a perfectly tracking concentrator is quite easy. With these aspects of plastic lenses as solar concentrators in view, an analytical study of its design and performance has been carried out. The plexiglass (PMMA) has been taken as the material of the lens as the refraction index values at some select wavelengths are available (1972). These values have been used in the interpolation formula to arrive at the refractive index values over the whole solar spectrum. The influence of groove width, defocussing, refractive index etc., on the concentration ratio has been analytically studied.

THEORY

A cylindrical Fresnel lens is a thin sheet of a transparent material on which prismatic grooves have been created on one surface. The prismatic grooves usually run parallel to the longest side of the rectangular sheet. The prismatic angles are chosen such that the rays of a given wavelength passing through the centres of the grooves will converge to a common line focus. The finite size of the grooves results in a line patch of finite width around the line focus. A groove down lens is used here for the analysis. The monochromatic beam is assumed to be incident normally on the surface of the lens; the refraction takes place only at the groove side. Applying Snell's law of refraction for a ray passing through the centre of the groove, we have

$$\sin \theta_m = n \sin (\theta_m + \beta_m),$$

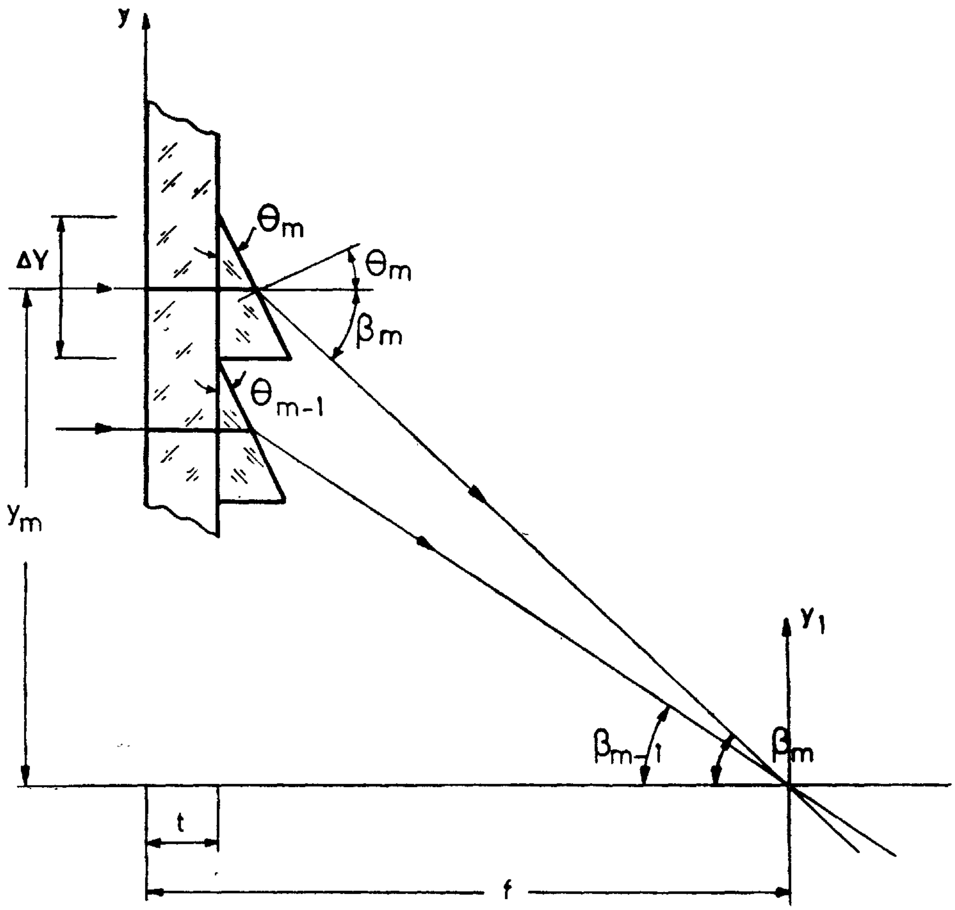

FIG. 1. Fresnel lens design parameters.

where n is the refractive index of the material for the design wavelength and β_m is defined in Fig. 1. Let y_m be the distance of the centre of the mth groove from the lens axis, then

$$\theta_m = \tan^{-1}\left[\frac{y_m}{n\sqrt{y_m^2 + (f-t)^2} - (f-t)}\right], \quad \ldots(1)$$

where f is the focal length of the lens and t its mean thickness. The eqn. (1) can be written in dimensionless variables $y_m' = y_m/f$ and $t' = t/f$, as

$$\theta_m = \tan^{-1}\left[\frac{y'_m}{n\sqrt{y'^2_m + (1-t')^2} - (1-t')}\right]. \quad \ldots(2)$$

The angles θ_m' s of the refracting prisms are fixed such that all the central rays of one particular wavelength have a common foci. Eqn. (2) is the basic equation in the design of Fresnel lenses.

Fresnel Reflection Losses

When a beam encounters a refractive index discontinuity, a part of the beam is reflected and the other part transmitted. The transmitted part may also suffer absorption in the medium. Therefore, the reflected and absorbed components account for the loss. In a Fresnel lens, there is reflection at the front and back surfaces and absorption in the thickness t of the material. The reflection loss at the front surface of the lens is constant, while it varies at the back surface. Since the absorption values of the plexiglass were not available, it was assumed to be completely transparent at all wavelengths. The transmittivity T for the natural light has been calculated using the formula,

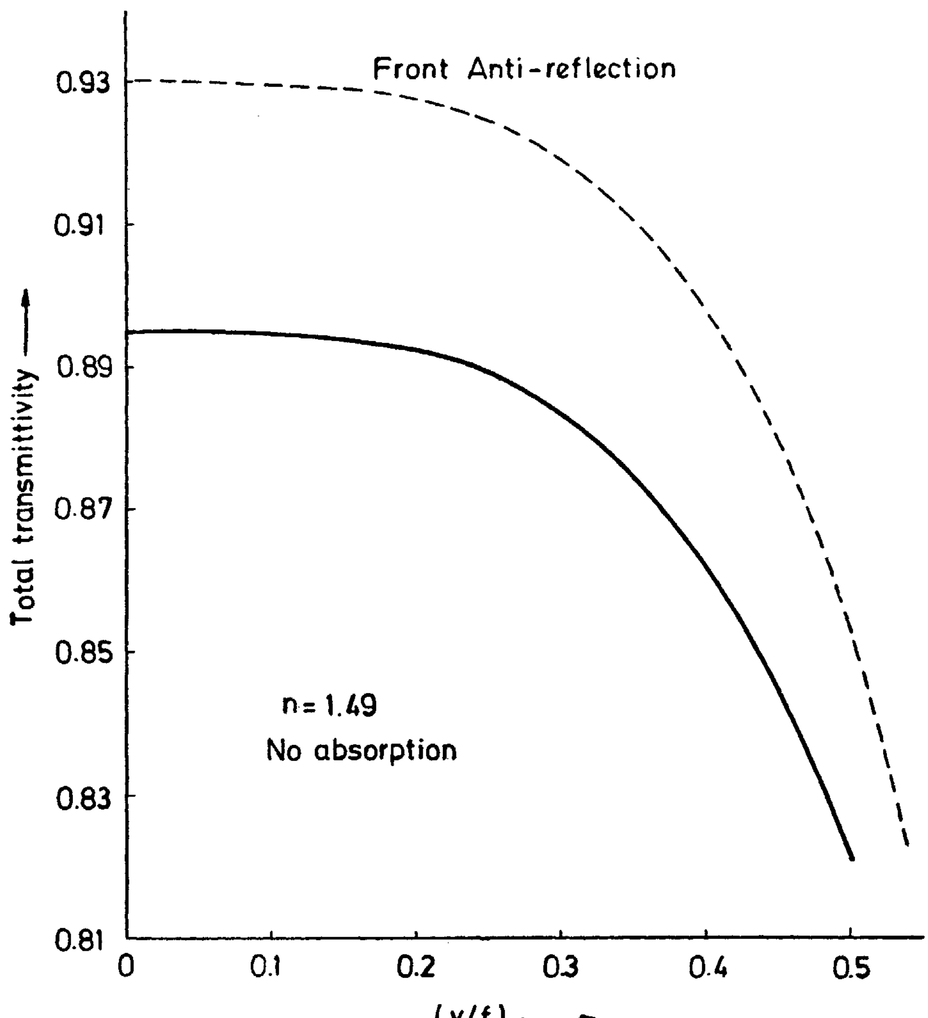

Fig. 2. Variation of transmittivity with the aperture of the lens.

$$T = \frac{8n^2}{(1+n)^2} \; \frac{\cos\theta_m \cos(\theta_m+\beta_m)\,[(1+n^2)(\cos^2\theta_m+\cos^2(\theta_m+\beta_m))+4n\cos\theta_m\cos(\theta_m+\beta_m)]}{[n\{\cos^2\theta_m+\cos^2(\theta_m+\beta_m)\}+(1+n^2)\cos\theta_m\cos(\theta_m+\beta_m)]^2} \quad \ldots(3)$$

The variation of T with y_m' for the design wavelength $\lambda=589.6$ nm ($n=1.491$) has been shown in Fig.2. It will be quite easy to reduce the reflection loss at the front surface considerably by giving an anti-reflection coating. It may be difficult to give an efficient anti-reflection coating at the back surface. Fig. 2 illustrates the improvement in the transmittivity when the front surface is anti-reflection coated. It will be more appropriate to use measured spectral transmittivity for the calculation of the concentration ratio over the whole solar spectrum.

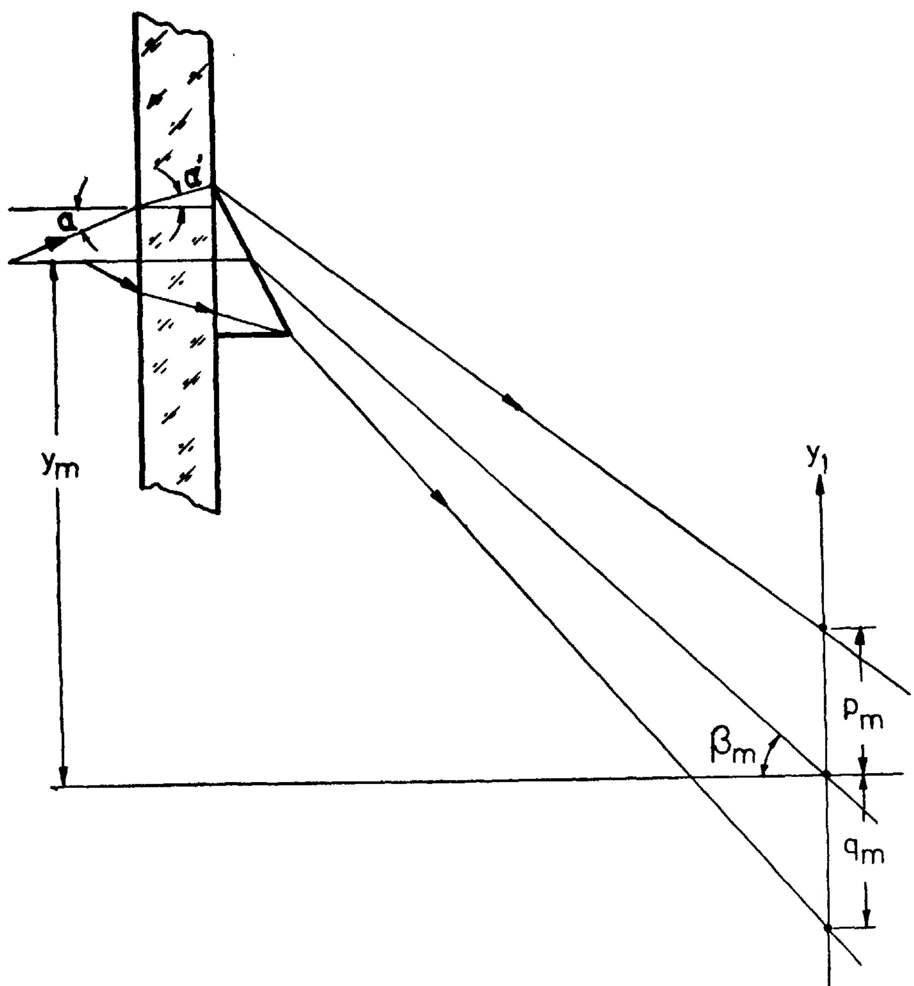

FIG. 3. Calculation of patch size due to the source of finite extent.

Influence of the Width of the Prismatic Grooves

It is assumed that all the prismatic grooves are of the same width $\triangle y' (= \triangle y/f)$. When a collimated beam is incident on the lens, the normalised patch size $\triangle_m$ due to mth groove is given by

$$\triangle_m = \triangle y'(1 - \tan\theta_m \tan\beta_m).$$

For the groove at the axis, the patch size is equal to the groove width $\triangle y'$ itself as the beam passes undeviated. The patch size decreases monotonically for grooves increasingly away from the axis.

Considering a disc source of uniform brightness, the calculation of $\triangle_m$ provides a different insight. This is in fact a physical situation as the sun makes an angle of 32 arc min. The incident beam is no longer collimated. Following Fig. 3 the normalised spread above and below the axis due to decollimation and corresponding to the mth groove at the focal plane are given by

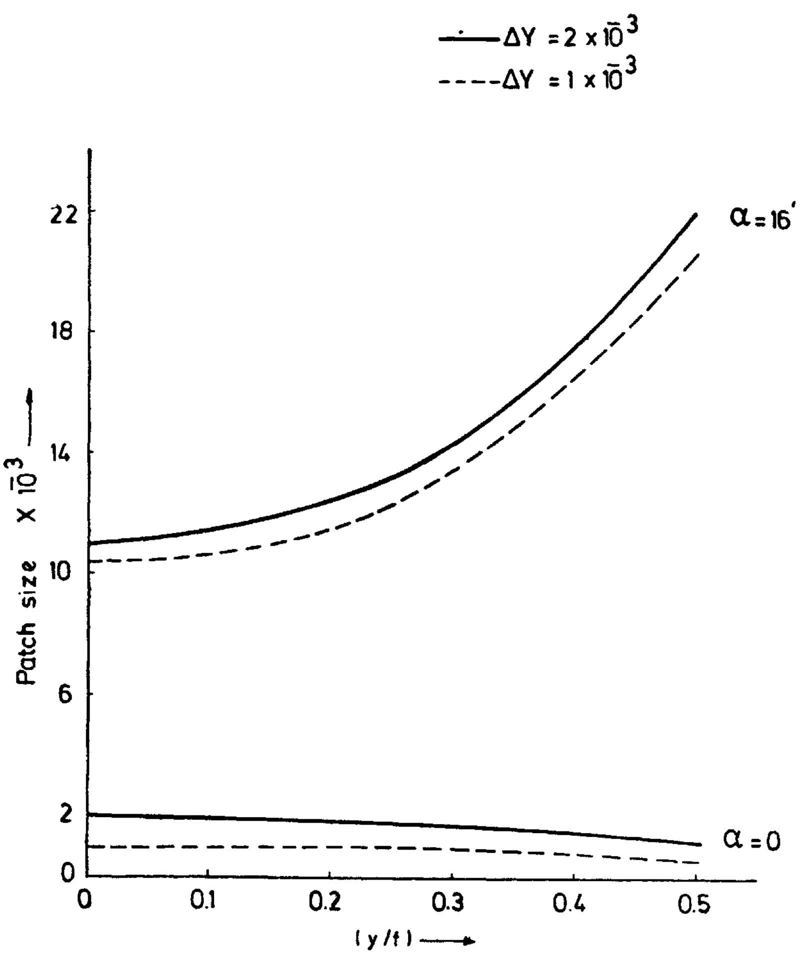

Fig. 4. Variation of patch size with the aperture for $\alpha = 0$ and 16 are minimum and $\triangle y' = 0.001$ and 0.002.

$$p_m = y_m' + \Delta y'/2 - (1-t') \tan \beta_{lm} \quad \ldots(4a)$$

and

$$q_m = (1 - t') \tan \beta_{lm} - y' \tan \theta_m \tan \beta_{lm} - (y'_m - \Delta y'/2), \quad \ldots(4b)$$

where

$$n \sin (\theta_m + \alpha') = \sin (\theta_m + \beta_{lm}), \quad \ldots(5a)$$

$$n \sin (\theta_m - \alpha') = \sin (\theta_m + \beta_{lm}) \quad \ldots(5b)$$

and

$$n \sin \alpha' = \sin \alpha. \quad \ldots(5c)$$

α is the half angular size of the source and is taken as 16 arc min. for the sun. The patch size $\Delta_m(\alpha)$ is now given by

$$\Delta_m (\alpha) = p_m + q_m. \quad \ldots(6)$$

If the patch size is desired at any plane $\Delta f'$ $(= \Delta f/f)$ from the focal plane, eqns. (4*a*) and (4*b*) are rewritten as

$$p_{m\Delta f} = p_m \pm \Delta f' \tan \beta_{lm} \quad \ldots(7a)$$

and

$$q_{m\Delta f} = q_m \pm \Delta f' \tan \beta_{lm}. \quad \ldots(7b)$$

The variation of patch size with y'_m for $\alpha = 0$ and $\alpha = 16$ arc min., and two values of $\Delta y'$ (0.001) and 0.002) has been shown in Fig. 4. The patch size depends linearly on $\Delta y'$ for the collimated beam. For a beam from a bread source the patch size does not depend so strongly on $\Delta y'$. There is an inherent divergence of 0.0093 radians to which the groove width $\Delta y'$ is to be added to arrive at the patch size. In contrast to the case of collimated beam, the patch size rapidly increases with increasing y_m'. It is not possible to decrease the patch size below 0.0093 for any value of groove width. However, there is a minimum value of $\Delta y'$ which is set by diffraction. This is given by

$$\lambda/f \, \Delta y' = 0.0093 + \Delta y'.$$

The solution of this equation gives

$$\Delta y' = (\sqrt{0.0000866 + 4\lambda/f} - 0.0093)/2.0.$$

As an example, let $\lambda = 589.6$ nm and $f = 30$ cm, the diffraction limited $\Delta y'$ is equal to 0.0002, which is about 17 1/mm for this lens. The choice of the minimum $\Delta y'$ for the design of the lens should be judiciously made as the lens has to operate over the whole solar spectrum. However, the Fresnel lenses used in practice are quite coarse, about 1 to 2 1/mm. Fig. 5 illustrates the dependence of concentration ratio on the groove width for the design wavelength. The local concentration ratio has also been computed as a function of relative position both in the focal plane and the defocussed plane for the design wavelength. The irradiance profile has been shown in Fig. 6 for $\Delta f'=0$ and $\Delta f' = \pm 0.02$ for the design wavelength. The maximum concentration ratio is 68.6 per cent.

Effect of Wavelength Dependence of Refractive Index on Concentration Ratio

The Fresnel lens is designed for a particular wavelength. However, when it is used with polychromatic light, say, solar radiation, the rays of wavelength other than the

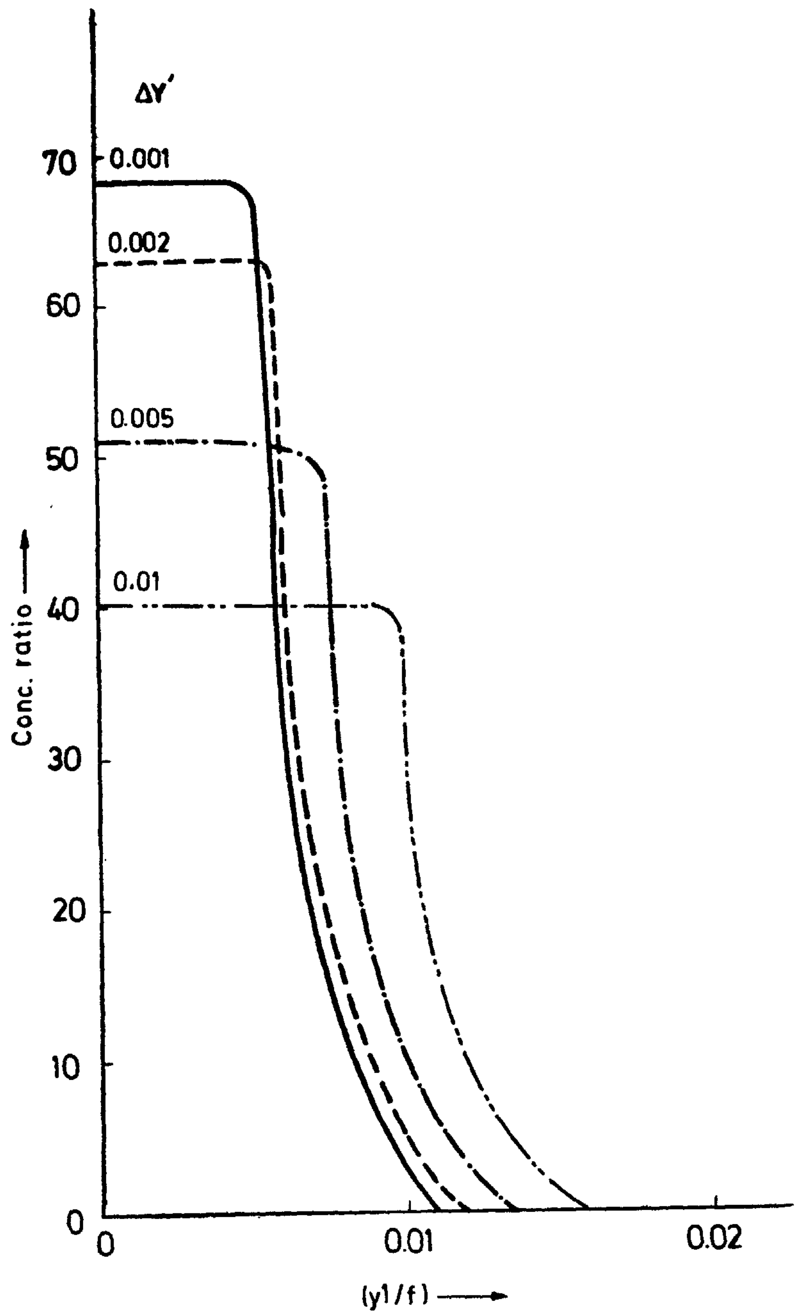

FIG. 5. Dependence of concentration ratio as a function of relative position on the groove width for the design wavelength.

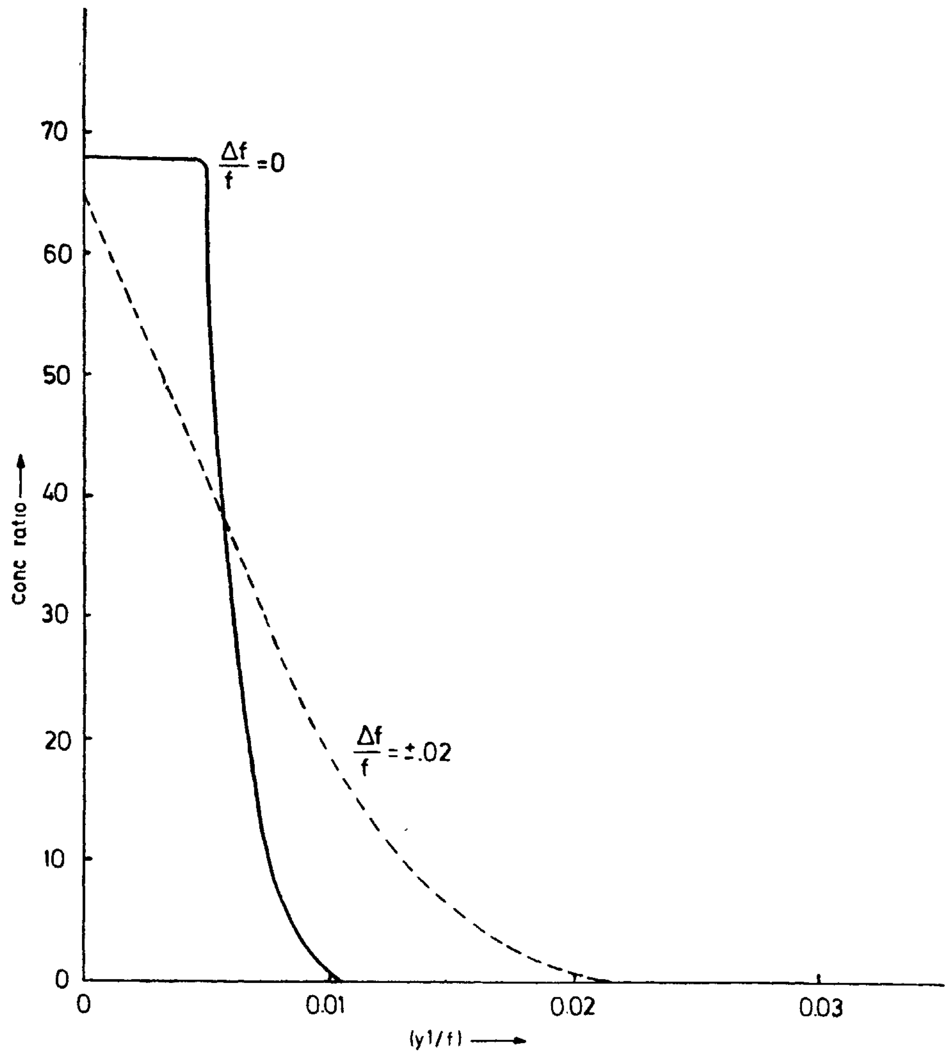

FIG. 6. Dependence of concentration ratio as a function of relative position on defocussing for the design wavelength.

design wavelength will be deviated by different magnitudes. The angle β_m is therefore wavelength dependent. Assuming the variation of n with λ as given by the Cauchy's farmula i.e.,

$$n_\lambda = n_0 + A/\lambda^2,$$

it can be shown that the angle $\beta_m(\lambda)$ for wavelength λ is related with the angle β_m for the design wavelength through the following relation :

$$\beta_m(\lambda) = \beta_m + \frac{A}{\lambda^2} \frac{\sin \theta_m}{\cos [\sin^{-1} (\theta_m + \beta_m)]} \qquad \ldots(8)$$

If $\beta_{mx}(\lambda)$ and $\beta_{mn}(\lambda)$ are the deviations corresponding to the lowest and the

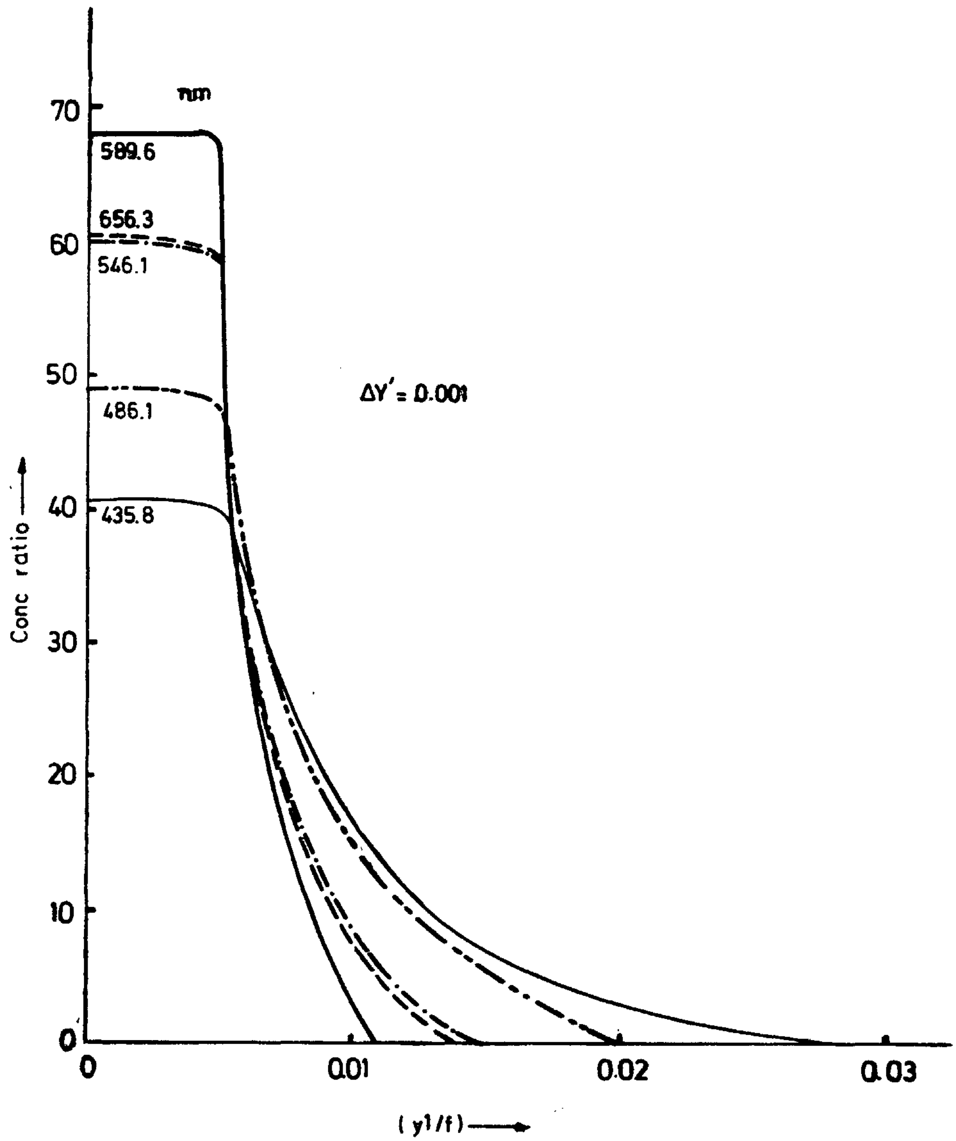

FIG. 7. Dependence of concentration ratio as a function of relative position for various wavelengths.

highest wavelengths present in the polychromatic radiation, then $[\beta_{mx}(\lambda)-\beta_{mn}(\lambda)]$ gives the angular blur due to the wavelength dependence of n. The local concentration ratio for a few wavelengths about the design wavelength as a function of the normalised position at the focal plane has been shown in Fig. 7.

Calculation of the concentration ratio for the solar radiation — Let $w(\lambda)\,d\lambda$ be the solar intensity at wavelength λ in the wavelength interval $d\lambda$. The total energy in the spectrum is obtained by integrating over the whole wavelength range, i.e.,

$$E = \int_0^\infty w(\lambda)\, d\lambda.$$

The spectrum is divided in intervals of $\triangle\lambda$ and the intensity in each interval is denoted by $w_j\,(\triangle\lambda)_j$ for the sake of performing computation on a digital computer. Therefore,

$$E = \sum_j w_j\,(\triangle\lambda)_j.$$

The relative intensity or the weightage factor q_j in the wavelength interval $\triangle\lambda$ can be defined as

$$q_j = \frac{w_j\,(\Delta\lambda)_j}{\sum_j w_j\,(\Delta\lambda)_j} = \frac{w_j\,(\Delta\lambda)_j}{E}\,.$$

The concentration ratio at the focal plane or a defocussed plane has been obtained by summing up the intensity contributions from each groove and for each wavelength interval of the solar spectrum with the proper weightage. For the cylindrical Fresnel

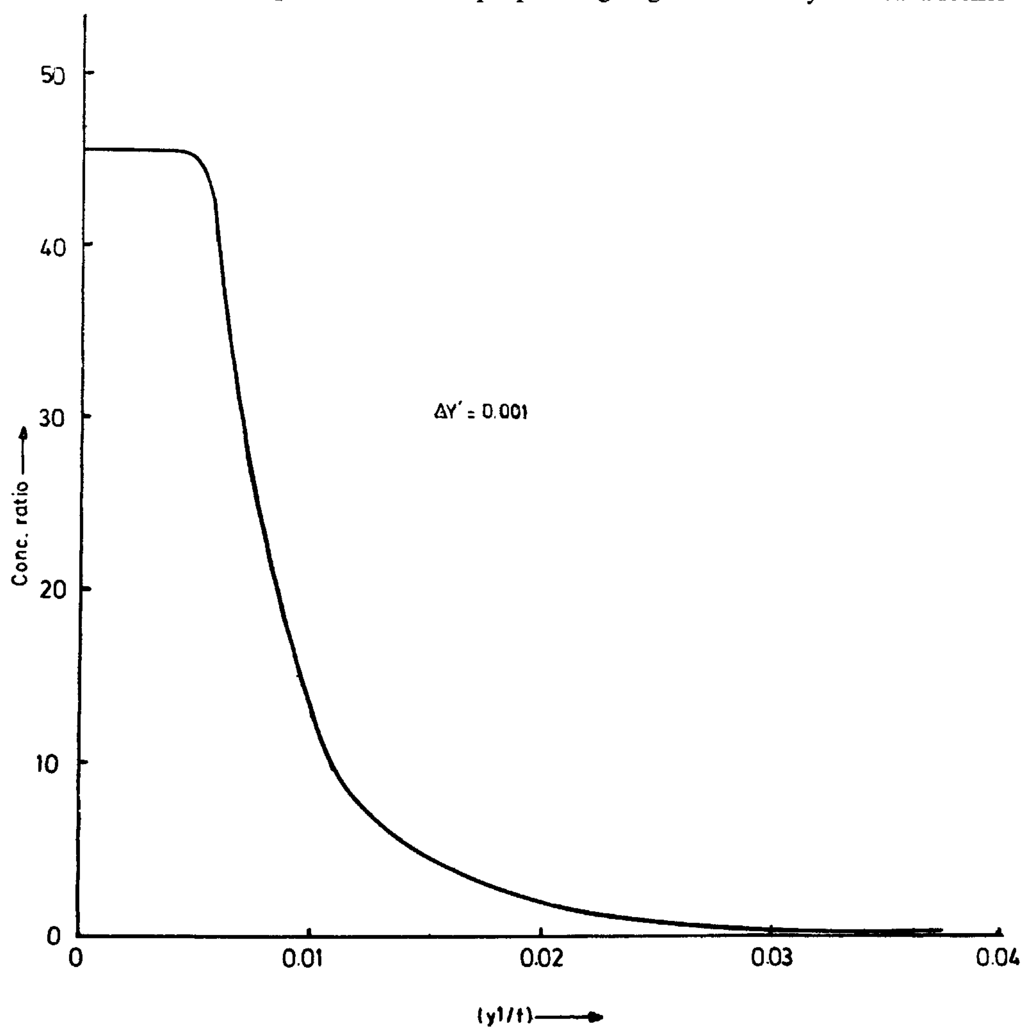

FIG. 8. Dependence of concentration ratio as a function of relative position for the solar spectrum.

lens, the intensity will not be a function of distance along the axis of the concentrator; the problem is thus one dimensional.

The refractive index of plexiglass, assumed material of the lens, is available at five wavelengths in the American Institute of Physics Handbook (1972). The constants of the interpolation formula;

$$n_\lambda^2 = A_0 + A_1\lambda^2 + A_2\lambda^{-2} + A_3\,\lambda^{-4} + A_4\lambda^{-6}.$$

are determined using the known values of the refractive index. The refractive index is then calculated from the above equation for any wavelength in the solar spectrum. The spectral weighting factors q_i for wavelength intervals in the selected dissections of the sea level are calculated from the data given in the book by Robinson (1966). The transmittivity is calculated at each wavelength interval using the computed value of the refractive index and corresponding β value for the design value of θ. The intensity distribution as a function of distance normal to the axis at the focal plane is obtained by summing up contributions from each prismatic groove and for each wavelength interval with proper weight factors. Fig. 8 illustrates the variation of concentration ratio as a function of relative position at the focal plane. *Most of the energy is concentrated in a linear patch of dimension 0.02 times the focal length. There is, however, a considerable spread of the intensity distribution.*

It has been pointed out that the sun is not a disc of uniform brightness Abetti (1938). The sun's intensity distribution is given by an empirical relation of the form

$$\frac{I(r)}{I_0} = \frac{1 + 1.5641\,(1 - r^2/R^2)^{1/2}}{2.5641},$$

where I_0 is the intensity at the center of sun, R its radius and $I\,(r)$ the intensity at any radial coordinate r. The effective angular size of the sun is therefore given by 28.3 arc min. The concentration ratio will correspondingly increase along with the sharpening of the peak.

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References

Abetti, G. (1938). *The Sun.* D. van Nostrand Co., p. 316.

Am. Inst. phys. (1972). *Handbook*, 6–109.

Authier, B. (1977). Etude optique des capeurs solaires spheriques fixes. *J. Opt. Paris*, **8**, 331–337.

Clausing, A. M. (1976). The performance of a stationary reflectro/tracking absorber solar concentrator. *Solar Collectors*, **2**, 304–326.

Hastings, L. J., Allums, S. L., and Cosby, R. M. (1976). An analytical and experimental evaluation of the plane-cylindrical Fresnel lens. *Solar Collectors*, **2**, 275–290.

Kreider, J. F. (1975). Thermal performance analysis of stationary reflector/ tracking absorber (SRTA) solar concentrator. *ASME J. Heat Transfer*, **97**, 451–456.

Rabl, A. (1976). Comparison of solar collectors. *Solar Energy*, **18**, 93–111.

Robinson, N. (1966). *Solar Radiation.* Elsevier & Co.

Steward, W. G., and Kreith, F. (1975). Stationary concentrating cum reflector cum tracking absorber solar energy collector : optical design characteristic. *Appl. Opt.*, **14**, 1509–1512.