EFFECT OF ELECTROMAGNETIC INTERACTIONS ON CHARM MESON MASS SPECTRA

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(*Received 6 December 1978*)

How the *D* and *F* meson spectrum is influenced by the electromagnetic hyperfine interaction is shown here. A relation between the hyperfine splitting of charmed meson and magnetic moments of baryon is obtained.

INTRODUCTION

THE ratio of the magnetic moments of nucleons agrees perfectly with the prediction based on non-relativistic SU(6) quark wave functions of nucleons (Lichtenberg, 1978). It is assumed that the magnetic moment operator of a baryon is the sum of the magnetic moment operators of the constituent quarks. Then we have the magnetic moment of a nucleon

$$\mu_{(\text{baryon})} = \sum_{q} m_q \vec{\sigma}_q, \qquad \ldots(1)$$

where $\vec{\sigma}_q$ is the quark spin operator and m_q is the magnetic moment operator of a quark q. The assumption here is that the magnetic moment operator m_q of a quark is proportional to its charge. Then m_q is given by

$$m_q = \mu_0 Q_q, \qquad \ldots(2)$$

where μ_0 is the proportionality constant and is otherwise a free parameter.
Hence

$$\vec{\mu}_{(\text{baryon})} = \mu_0 \sum_{q} Q_q \vec{\sigma}_q \qquad \ldots(3)$$

In this paper, $M(F^{+*}) - M(F^{+}) = 100 \pm 8$ (MeV) and SU(4) Charm symmetry scheme is chosen here where the fourth quark carries negative charge $Q_c = -1/3$. In doing so, one is also able to identify the pseudoscalar $\psi^{P.S}$ counterpart of ψ (3.1) particle and other mesons. The weight diagrams for the hadrons are more symmetrical if $Q_c = -1/3$. Though the charge of charmed quark is taken as $-1/3$, it does not change the general condition of various observables. Although here model decay widths and branching ratio results are different, since the model is chosen by Moffat, 1975 and Ahmad and Zadoo, 1977 who have predicted the correct results of various problems, we choose the same model. Moreever in this model, it is possible to explain how in charmed particles weak decays are suppressed (Zadoo, 1977).

HYPERFINE INTERACTION

Isospin symmetry of the strong interactions gives $\mu_u = \mu_d$. Applying the relation (3) to the baryons, including the charmed baryons, we get the quark wave functions, using the conventional SU (6) quark wave functions as

$$\mu_{(p)} = \mu_{(u)};\ \mu_{(n)} = -\tfrac{2}{3}\mu_{(u)}$$

$$\mu_{\lambda} = -\tfrac{1}{3}\mu_{s} \qquad \ldots(4)$$

The experimental values are (Particle Data Group, 1976)

$$\mu_{p} = 2.79\ \frac{Q}{2M_P};\ \mu_n = -1.91\ \frac{Q}{2M_P}$$

$$\mu_{\lambda} = (-\ 0.67 \pm 0.06)\ \frac{Q}{2M_P} \qquad \ldots(5)$$

Within the *Q C D* approach to hadron dynamics, the hyperfine mass splitting of baryons and mesons, is described by the exchange of colour octet vector gluons between quarks (Barnested, 1977; Weisskopf, 1977).

In the case of charmed mesons, the quark gluon coupling constant is expected to be small enough such that, we can describe the hyperfine splitting interactions as obtained in the one-gluon exchange approximation, which is given by the *Q C D*. It is well known that even in the case of uncharmed and charmed baryons and mesons, the hyperfine splitting effects are described qualitatively rather well by the one-gluon exchange approximation.

In the case of charmed mesons, the spin part of the hyperfine splitting interactions is given by the interaction of the colour magnetic moments of the quarks, which are proportional to the electromagnetic moments divided by the quark charges.

For a colour singlet quark-antiquark configuration of the quantum chromodynamic part of the interaction is then proportional to (Auguilar, 1977)

$$-1/3\ g_{st}^2 \sum_{A=1}^{8} \mathrm{tr}\left(\frac{\lambda_A}{2} \cdot \frac{\lambda_A}{2}\right) = -4/3\ g_{st}^2 \qquad \ldots(6)$$

while as according to *QCD*, g_{st} is the coupling constant and $A = 1 ....8$ are the conventional SU(3) matrices, the *QED* part of the interaction is proportional to

$$Q^2 \cdot Q_{q_1} Q_{q_2} \qquad \ldots(7)$$

Adding both the *QCD* and *QED* contributions, we find that for $c\bar{q}$ mesons ($q = u, d, s$)

$$\mathrm{H}^{\mathrm{hyperfine}} \sim (4/3\ k - Q_c\ Q_{\bar{q}}\ \alpha)\ \mu_c\ \mu_{\bar{q}}\ \vec{\sigma}_c\ \vec{\sigma}_{\bar{q}} \qquad \ldots(8)$$

where $k = \left(\frac{g_{st}^2}{4\pi};\ \alpha = \frac{Q^2}{4\pi}\right)$.

Evaluating the expectation values of $\mu_c . \mu_q$ according to the identity $J(J+1) = (\vec{\sigma}_c + \vec{\sigma}_q)^2 = 3/2 + 2\ \vec{\sigma}_c . \vec{\sigma}_q$ (J is the angular momentum of mesons in the case of *s*-wave states), one can calculate the contributions of the magnetic hyperfine interaction eqn. (8) to the masses of the various charmed meson (shown in Table I below); here δm is a universal mass parameter, depending on the unknown details of the wave functions.

According to Table I and eqn. (4), the following relation holds.

$$\frac{F^{+*} - F^{+}}{D^{+*} - D^{+}} = -\ \frac{\mu_s}{\mu_d} = \frac{-\ 3\ \mu_\lambda}{\mu_P} \qquad \ldots(9)$$

TABLE I

Label	Quark Content	Δ M*
D°	$c\,\bar{u}$	$(4/3\,k - 2/9\,\alpha)\ (-3/4)\ \mu_c\ \mu_u\ \delta m$
$D^{\circ *}$	$\bar{c}\,u$	$(4/3\,k - 2/9\,\alpha)\ (1/4)\ \mu_c\ \mu_u\ \delta m$
D^+	$c\,\bar{d}$	$(4/3\,k + 1/9\,\alpha)\ (-3/4)\ \mu_c\ \mu_d\ \delta m$
D^{+*}	$\bar{c}\,d$	$(4/3\,k + 1/9\,\alpha)\ (1/4)\ \mu_c\ \mu_d\ \delta m$
F^+	$c\,\bar{s}$	$(4/3\,k + 1/9\,\alpha)\ (-3/4)\ \mu_c\ \mu_s\ \delta m$
F^{+*}	$\bar{c}\,s$	$(4/3\,k + 1/9\,\alpha)\ (1/4)\ \mu_c\ \mu_s\ \delta m$

$\Delta\, M^*$ is the mass shifts due to the magnetic hyperfine interactions.

From the experimental values of the magnetic moments (Goldhaber, 1977; De Ruajula *et al.* 1976), we obtain

$$F^{+*} - F^+ = (0.72 \pm 0.06)\,(D^{+*} - D^+) \qquad \ldots(10)$$

Thus we observe that $F^{+*} - F^+ = 100 \pm 8$ (MeV), which is in consistence with the recent experimental observations (De Rujula, 1976).

We emphasize that the relation (9) is valid for the chromo-magnetic and electromagnetic contributions to the D^+ and F^+ mass differences. The electromagnetic contributions to the D and D^* masses will cause the mass difference $D^{+*} - D^{\circ *}$ to be less than $D^+ - D^\circ$. In most of the estimates of these mass differences the electromagnetic contributions were neglected and only the tadpole contributions due to the $u - d$ quark mass difference and the electrostatic contributions were taken into account. In this case, we get

$$\frac{(D^+ - D^\circ) - (D^{+*} - D^{\circ *})}{1/3\,[2\,(D^{+*} - D^+) + (D^{\circ *} - D^\circ)]} = -\tfrac{1}{4}\,\frac{\alpha}{k} \qquad \ldots(11)$$

which implies eqn. (10),

$$(D^+ - D^\circ) - (D^{+*} - D^{\circ *}) \approx k^{-1} \otimes 0.75 \text{ (MeV)},$$

where for charmed mesons, $k \approx 0.3 \ldots\ldots 0.5$; so

$$(D^+ - D^\circ) - (D^{+*} - D^{\circ *}) \approx 1.46 \text{ MeV} \ldots\ldots 2.43 \text{ MeV}.$$

Eqn. (11) would allow us to find the quark gluon coupling constant which is relevant for the quark dynamics inside D and F mesons rather accurately. The experimental data of eqn. (10) are consistent with the presence of an electromagnetic contribution to the D mass differences.

CONCLUSION

The relation between the hyperfine splitting of charmed mesons and the magnetic moments of the baryons is derived. Then we have

$$M\,(F^{+*}) - M\,(F^+) = 100 \pm 8 \text{ (MeV)}.$$

An equation analogous to eqn. (11) might be expected to hold good for K and K^* mesons, although here the one gluon exchange approximation may be rather bad, since the relevant value of K is probably of the order

$$\frac{(K^\circ - K^-) - (K^{\circ *} - K^{-*})}{(K^* - K)} = -\tfrac{1}{4}\,\frac{\alpha}{k} \qquad \ldots(12)$$

This shows that the electromagnetic hyperfine interaction causes

$$D^{+} - D^{0} > D^{+*} - D^{0*}; \text{ but } K^{0} - K^{-} < K^{0*} - K^{-*}$$

Eqn. (12) gives ;

$$(K^{0} - K^{-}) - (K^{0*} - K^{-*}) \approx - K^{-1} \otimes 0.73 \text{ (MeV)},$$

which is expected to be $\approx 0.5 \ldots . 1.5$ (MeV), i.e., $K^{0*} - K^{-*} \approx 4.5 \ldots .. 5.5$ (MeV). The present experimental data are in agreement with such a situation.

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