CLEBSCH-GORDAN SERIES FOR SU(5)

MUBARAK AHMAD, M. S. JALLU *and* G. Q. SOFI
Department of Physics, University of Kashmir, Srinagar-190006

(*Received 18 December 1978*)

A method for finding the reduction of the direct products of the two irreducible representations of SU(5), is formulated by the tensor method. This formula is applied for some particular cases to classify the beautiful hadrons under the frame work of SU(5).

INTRODUCTION

IT is now widely accepted that the newly discovered Upsilon resonance $T(9.4)$ [Herb, 1977][4] is a bound state $(b\bar{b})$ of a fifth quark "b" and its antiquark $\bar{b}$, where b represents the beauty quark. The observation of the T particle in the reaction

$$P + \text{Nucleus} \rightarrow T + \text{anything}$$
$$\qquad\qquad\qquad\qquad \lfloor\!\rightarrow \mu^+\mu^-$$

immediately led to its interpretation as a bound state of a quark b and antiquark $\bar{b}$ with the quark mass around 4–5 (GeV).

The purpose of the present work is to develop a method for the calculation of the reduction of direct products of two irreducible representations of SU(5). As it has rank 4, four numbers of lie Algebra, can be diagonalised simultaneously. Thus the irreducible representations are labelled by four non-negative integers (p, q, r, s). The dimensional formula of the representation is written as (Ahmad & Zadoo, 1978; Dalitz, 1975; Feldman & Mathews, 1977; Herb *et al.*, 1977; Moffat, 1975) :

$$D = (1 + p)(1 + q)(1 + r)(1 + s)\left(1 + \frac{p+q}{2}\right)\left(1 + \frac{q+r}{2}\right)\left(1 + \frac{r+s}{2}\right)\left(1 + \frac{p+q+s}{3}\right)\left(1 + \frac{q+r+s}{3}\right)\left(1 + \frac{p+q+r+s}{4}\right) \quad ...(1)$$

We assume that there is another unitary irreducible representation p', q', r' and s'. We have to find the unitary irreducible representation occurring in the Clebsch-Gordan series, which is represented on the right hand side of the expression :—

$$(p, q, r, s) \otimes (p', q', r', s') = \sum_{\oplus} (\alpha, \beta, \gamma, \lambda) \quad ...(2)$$

We shall use the characterization of the irreducible representation of SU(5), as the transformation induced on the irreducible tensorial sets by unitary unimodular transformation, in the complex vector space bounded by hyper planes of SU(5).

IRREDUCIBLE TENSOR

The concept of an irreducible tensor, a tensor which transforms according to an irreducible representation of SU(5) is an irreducible tensor. The direct products of the irreducible representations are decomposed into direct sums of irreducible representations. For this, an irreducible tensor $X^{p,q}_{r,s}$ is used with p, q upper contravaiant indices ; r, s as lower covariant indices. $X^{p,q}_{r,s}$ is denoted by (p, q, r, s) and similary $Y^{p',q'}_{r',s'}$ being the other tensor is denoted by (p', q', r', s').

We use invariant symmetric and antisymmetric tensors δ, $\epsilon_{a,b,c,d}$, $\epsilon^{a,b,c,d}$ for constructing tensors belonging to unitary irreducible representations that appear on the right hand side of (2).

The following processes are used to expand tensors into their irreducible parts :—

1. Contraction of an upper index of X with lower index of Y any number of times using δ's.
2. Contraction of a lower index of X with an upper index of Y any number of times using δ's.
3. Contract an upper index of X and an upper index of Y any possible number of times using $\epsilon^{a,b,c,d}$.
4. Contraction a lower index of X and a lower index of Y using $\epsilon^{a,b,c,d}$, any number of times.

To get unitary irreducible representations on the right hand side of (2), process (1) has been used k times on p and k' times on q where as process (2) is used l times on r and l' times on s. Then the possible values of k, k', l, l' are :—

$$0 \leqslant k \leqslant p \; ; \qquad 0 \leqslant k' \leqslant q$$
$$0 \leqslant l \leqslant r \; ; \qquad 0 \leqslant l' \leqslant s. \qquad \ldots(3)$$

For a definite allowed choice of k, k', l and l', two sets of unitary irreducible representations are to be considered, depending on whether we use process (3) or (4).

Set A

Let the process (3) be used n times, then n lies between :

$$0 \leqslant n \leqslant \min\,[(p + q - k - k' \; ; p' + q' - l - l')]. \qquad \ldots(4)$$

Then

$$\alpha = p + p' - 2k - l - n,$$

$$\beta = q + q' + k - 2k' - n,$$

$$\gamma = r + r' - 2l + l' + n$$

and

$$\lambda = s + s' - 2l' + n. \qquad \ldots(5)$$

Set B

Let the process (4) be used n times ($n = 0$ is not allowed because it is included in the Set A). Then the linear value of n lies between

$$1 \leqslant n \leqslant \min(r - 1, s - 1') ; (r' - k, s' - k') \qquad \ldots(6)$$

corresponding values of α, β, γ and λ are

$$\alpha = p + p' - 2k - l - 2l' + n ;$$

$$\beta = q + q' + k - k' + 2l + l' - n ;$$

$$\gamma = r + r' + 2k' - 2l + l' - n$$

and

$$\lambda = s + s' + k - 2l' - l + n. \qquad \ldots(7)$$

The unitary irreducible representation in the direct sum of (2) falls into two sets. The first set contains unitary irreducible representations $(\alpha, \beta, \gamma, \lambda)$ with values of α, β, γ and λ given by (5) and k, k', l, l' obeying the conditions (3), (6) and (7).

No unitary irreducible representation of one set will coincide with any irreducible representation of the other set.

Applications

The reduction is :

$$5 \otimes 5 = 15 + 10$$

Then $\quad (1, 0, 0, 0) \otimes (1, 0, 0, 0) = \sum_{\oplus} (\alpha, \beta, \gamma, \lambda)$

or $\quad (p', q', r', s') (p, q, r, s) = \sum_{\oplus} (\alpha, \beta, \gamma, \lambda).$

Thus :

$$p' = 1 ; q' = 0 ; r' = 0 ; s' = 0$$

$$p = 1 ; q = 0 ; r = 0 ; s = 0.$$

Here, from eqn. (3), we have :

$$0 \leqslant k \leqslant 1 \; ; 0 \leqslant k' \leqslant 0 \; ; 0 \leqslant l \leqslant 0 \; ; 0 \leqslant l' \leqslant 0$$

Set A

$0 \leqslant n \leqslant 1 - k$

$k = 0 \; ; n = 0 \; ; \alpha = 2 \; ; \beta = 2 \; ; \gamma = 0 \; ; \lambda = 0$ i.e., (2, 0, 0, 0)

$k = 0 \; ; n = 1 \; ; \alpha = 1 \; ; \beta = -1 \; ; \gamma = 1 \; ; \lambda = 1$ i.e., (nil)

$k = 1 \; ; n = 0 \; ; \alpha = 0 \; ; \beta = 1 \; ; \gamma = 0; \lambda = 0$ i.e., (0, 1, 0, 0)

Set B

$0 \leqslant n \leqslant 1$ ($n = 0$ can not be used because of agreement).

$k = 0 \; ; n = 1 \; ; \alpha = 3 \; ; \beta = -1 \; ; \gamma = -1 \; ; \lambda = 1$ i.e., nil

$k = 1 \; ; n = 1 \; ; \alpha = 1 \; ; \beta = 0; \gamma = -1 \; ; \lambda = 2$ i.e., nil.

Hence — *I*

$$(1, 0, 0, 0) \otimes (1, 0, 0, 0) = (2, 0, 0, 0) + (0, 1, 0, 0)$$

$$5 \otimes 5 = 15 + 10$$

II

$$5 \otimes 15 = 40' + 35$$

$$(1, 0, 0, 0) \otimes (2, 0, 0, 0) = \sum_{\oplus} (\alpha, \beta, \gamma, \lambda)$$

or $$(p', q', r', s') \otimes (p, q, r, s) = \sum_{\oplus} (\alpha, \beta, \gamma, \lambda)$$

$p' = 1 \; ; q' = 0 \; ; r' = 0, s' = 0$

$p = 2 \; ; q = 0 \; ; r = 0 \; ; s = 0.$

$$0 \leqslant k \leqslant 2 \; ; 0 \leqslant k' \leqslant 0 \; ; 0 \leqslant l \leqslant 0 \; ; 0 \leqslant l' \leqslant 0$$

Set A

$0 \leqslant n \leqslant \min (2 - k)$.

$k = 0 \; ; n = 0 \; ; \alpha = 3 \; ; \beta = 0 \; ; \gamma = 0 \; ; \lambda = 0$ i.e., 3, 0, 0, 0.

$k = 0 \; ; n = 2 \; ; \alpha = 1 \; ; \beta = -2 \; ; \gamma = 2; \lambda = 2$ i.e., nil.

$k = 1 \; ; n = 1 \; ; \alpha = 0 \; ; \beta = 0 \; ; \gamma = 1 \; ; \lambda = 1$ i.e., 0, 0, 1, 1.

$k = 2 \; ; n = 2 \; ; \alpha = -2 \; ; \beta = 2 \; ; \gamma = 0; \lambda = 0$ i.e., nil.

Set B

$0 \leqslant n \leqslant 1$ ($n = 0$, is not allowed as per agreement).

$k = 0$; $n = 1$; $\alpha = 4$; $\beta = -1$; $\gamma = -1$; $\lambda = 1$ i.e., nil.

$k = 1$; $n = 1$; $\alpha = 0$; $\beta = 0$; $\gamma = -1$; $\lambda = 2$ i.e., nil.

Hence

$$(1, 0, 0, 0) \otimes (2, 0, 0, 0) = (3, 0, 0, 0) + (0, 0, 1, 1)$$

$$5 \quad \otimes \quad 15 \quad = \quad 35 \quad + \quad 40'$$

III

$$5 \quad \otimes \quad \bar{5} \quad = \quad 24 + 1$$

$$(1, 0, 0, 0) \otimes (0, 0, 0, 1) = \sum_{\oplus} (\alpha, \beta, \gamma, \lambda)$$

$p' = 1$; $q' = 0$; $r' = 0$; $s' = 0$

$p = 0$; $q = 0$; $r = 0$; $s = 1$

From eqn. (3) we have :—

$0 \leqslant k \leqslant 0$; $0 \leqslant k' \leqslant 0$; $0 \leqslant l \leqslant 0$; $0 \leqslant l' \leqslant 1$.

Set A

$0 \leqslant n \leqslant (1 - l')$

$k = 0$; $k' = 0$; $l = 0$; $l' = 0$; $n = 0$; $\alpha = 1$; $\beta = 1$; $\gamma = 0$; $\lambda = 1$ i.e., 1, 0, 0, 1.

$k = 0$; $k' = 0$; $l = 0$; $l' = 0$; $n = 1$; $\alpha = 1$; $\beta = 0$; $\gamma = 1$; $\lambda = -1$ i.e., nil.

$k = 0$; $k' = 0$; $l = 0$; $l' = 1$; $n = 0$; $\alpha = 0$; $\beta = -1$; $\gamma = 1$; $\lambda = 2$ i.e., nil.

Set B

$0 \leqslant n \leqslant 1$ ($n = 0$ is not allowed because of agreement).

$k = 0$; $k' = 0$; $l = 0$; $l' = 0$; $n = 1$; $\alpha = 2$; $\beta = -1$; $\gamma = -1$; $\lambda = 2$ i.e., nil.

$k = 0$; $k' = 0$; $l = 0$; $l' = 1$; $n = 1$; $\alpha = 0$; $\beta = 0$; $\gamma = 0$; $\lambda = 0$ i.e., 0, 0, 0, 0.

Thus

$$(1, 0, 0, 0) \otimes (0, 0, 0, 1) = (1, 0, 0, 1) + (0, 0, 0, 0)$$

$$5 \quad \otimes \quad \bar{5} \quad = \quad 24 \quad + \quad 1$$

IV

$$5 \quad \otimes \quad 24 \quad = 70 + 45 + 5$$

$$(1, 0, 0, 0) \otimes (1, 0, 0, 1) = \sum_{\oplus} (\alpha, \beta, \gamma, \lambda)$$

$p' = 1$; $q' = 0$; $r' = 0$; $s' = 0$

$p = 1$; $q = 0$; $r = 0$; $s = 1$

$0 \leqslant k \leqslant 1$; $0 \leqslant k' \leqslant 0$; $0 \leqslant l \leqslant 0$; $0 \leqslant l' \leqslant 1$.

Set A

$0 \leqslant n \leqslant \min(1 - k, 1 - l')$

$k = 0$; $l' = 0$; $n = 0$; $\alpha = 2$; $\beta = 0$; $\gamma = 0$; $\lambda = 1$ i.e., 2, 0, 0, 1

$k = 0$; $l' = 0$; $n = 1$; $\alpha = 1$; $\beta = -1$; $\gamma = 1$; $\lambda = 2$ i.e., nil

$k = 0$; $l' = 1$; $n = 0$; $\alpha = 2$; $\beta = 0$; $\gamma = 1$; $\lambda = -1$ i.e., nil

$k = 1$; $l' = 0$; $n = 0$; $\alpha = 0$; $\beta = 1$; $\gamma = 0$; $\lambda = 1$ i.e., 0, 1, 0, 1

$k = 1$; $l' = 1$; $n = 0$; $\alpha = 0$; $\beta = 1$; $\gamma = 1$; $\lambda = -1$ i.e., nil.

Set B

$0 \leqslant n \leqslant 1$ ($n = 0$ cannot be allowed as per agreement).

$k = 0$; $l' = 0$; $n = 1$; $\alpha = 3$; $\beta = -1$; $\gamma = -1$; $\lambda = 2$ i.e., nil

$k = 1$; $l' = 0$; $n = 1$; $\alpha = 1$; $\beta = 0$; $\gamma = -1$; $\lambda = 2$ i.e., nil

$k = 0$; $l' = 1$; $n = 1$; $\alpha = 1$; $\beta = 0$; $\gamma = 0$; $\lambda = 0$ i.e., 1, 0

$k = 1$; $l' = 1$; $n = 1$; $\alpha = -1$; $\beta = 1$; $\gamma = 0$; $\lambda = 1$ i.e., nil.

Hence

$$(1, 0, 0, 0) \otimes (1, 0, 0, 1) = (2, 0, 0, 1) + (0, 1, 0, 1) + (1, 0, 0, 0)$$

$$5 \quad \otimes \quad 24 \quad = \quad 70 \quad + \quad 45 \quad + \quad 5$$

Physical Applications of C. G. Series

In SU(5) scheme, the $q\bar{q}$ meson states belong to the 24 singlet representation of this SU(5) symmetry. The 24 representation has SU(4) reduction (Dalitz, 1975 ; Feldman & Mathews, 1977),

$$\begin{array}{ccccccc} 24 & = & 4 & + & \underbrace{15 + 1} & & + 4 \\ & & B = 1 & & B = 0 & & B = -1 \end{array}$$

These mesons can easily fit in the above derived representations which give beautiful mesons.

TABLE I

Quantum numbers of mesons

	States	I_3	Q	Y	C	B	Particle (Meson)
T^1_2	$\psi_1\psi_2$	-1	-1	0	0	0	π^-
T^2_1	$\psi_2\psi_1$	$+1$	$+1$	0	0	0	π^+
T^1_3	$\psi_1\psi_3$	$-\frac{1}{2}$	0	$+1$	0	0	K°
T^3_1	$\psi_3\psi_1$	$+1$	0	-1	0	0	$\bar{K}^0$
T^1_4	$\psi_1\psi_4$	$-\frac{1}{2}$	0	0	-1	0	$\bar{F}^0$
T^4_1	$\psi_4\psi_1$	$+\frac{1}{2}$	0	0	$+1$	0	F^0
T^1_5	$\psi_1\psi_5$	$-\frac{1}{2}$	0	0	0	-1	$\bar{B}'$
T^5_1	$\psi_5\psi_1$	$+\frac{1}{2}$	0	0	0	$+1$	B'
T^2_3	$\psi_2\psi_3$	$+\frac{1}{2}$	$+1$	$+1$	0	0	K^+
T^3_2	$\psi_3\psi_2$	$-\frac{1}{2}$	-1	-1	0	0	$\bar{K}^-$
T^2_4	$\psi_2\psi_4$	$+\frac{1}{2}$	$+1$	0	-1	0	F^+
T^4_2	$\psi_4\psi_2$	$-\frac{1}{2}$	-1	0	$+1$	0	$\bar{F}$
T^2_5	$\psi_2\psi_5$	$+\frac{1}{2}$	$+1$	0	0	-1	$\bar{B}''$
T^5_2	$\psi_5\psi_2$	$-\frac{1}{2}$	-1	0	0	$+1$	B''
T^3_4	$\psi_3\psi_4$	0	0	-1	-1	0	D^0
T^4_3	$\psi_4\psi_3$	0	0	$+1$	$+1$	0	$\bar{D}^0$
T^3_5	$\psi_3\psi_5$	0	0	-1	0	-1	$\bar{B}'''$
T^5_3	$\psi_5\psi_3$	0	0	$+1$	0	$+1$	B'''
T^4_5	$\psi_4\psi_5$	0	0	0	$+1$	-1	$\bar{B}''''$
T^5_4	$\psi_5\psi_4$	0	0	0	-1	$+1$	B''''

TABLE IIa

Quantum Numbers of 3/2⁺ *Baryons*

B	States	Practical Baryons	I	I_3	Y	Q	C
	$\psi_1\psi_1\psi_1$	Δ^{++}	3/2	− 3/2	1	− 1	0
	$\psi_1\psi_1\psi_2$	Δ^{+}	3/2	1/2	1	0	0
	$\psi_1\psi_1\psi_3$	Σ^{*+}	1	− 1	0	− 1	0
	$\psi_1\psi_1\psi_4$	Δ_c^{++}	1	− 1	1	− 1	1
	$\psi_1\psi_2\psi_3$	Σ^{*0}	1	0	0	0	0
	$\psi_1\psi_2\psi_4$	Δ_c^{+}	1	0	1	0	1
	$\psi_1\psi_3\psi_4$	Δ_c^{+c}	1/2	− 1/2	0	− 1	1
0	$\psi_1\psi_2\psi_2$	Δ°	3/2	1/2	1	1	0
	$\psi_1\psi_3\psi_3$	$\Xi^{*\circ}$	1/2	− 1/2	− 1	− 1	0
	$\psi_1\psi_4\psi_4$	Σ_{cc}^{++}	1/2	− 1/2	1	− 1	2
	$\psi_2\psi_2\psi_2$	Δ^{++}	3/2	3/2	1	2	0
	$\psi_2\psi_2\psi_3$	Σ^{*-}	1	1	0	1	0
	$\psi_2\psi_2\psi_4$	Δ_c^{0}	1	1	1	1	1
	$\psi_2\psi_3\psi_4$	Δ_c^{*0}	1/2	1/2	0	0	1
	$\psi_2\psi_3\psi_3$	Ξ^{*-}	1/2	1/2	− 1	0	0
	$\psi_2\psi_4\psi_4$	Σ_{cc}^{+}	1/2	1/2	1	0	2
	$\psi_3\psi_3\psi_3$	Ω^{-}	0	0	− 2	− 1	0
	$\psi_3\psi_3\psi_4$	Σ_c^{0}	0	0	− 1	− 1	1
	$\psi_3\psi_4\psi_4$	Ξ_{cc}^{+}	0	0	0	− 1	2
	$\psi_4\psi_4\psi_4$	Ξ_{ccc}^{++}	0	0	1	− 1	3
	$\psi_1\psi_1\psi_5$	Δ_b^{-}	1	− 1	1	− 1	0
	$\psi_1\psi_2\psi_5$	Δ_b^{0}	1	0	1	0	0
	$\psi_1\psi_3\psi_5$	Δ_b^{*-}	1/2	− 1/2	0	− 1	0
	$\psi_1\psi_4\psi_5$	$\Delta_b^{**\bar{c}}$	1/2	− 1/2	1	− 1	1
	$\psi_2\psi_2\psi_5$	Δ_b^{+}	1	1	1	1	0

(continued)

TABLE II(*a*) (*continued*)

B	State	Practical Baryons	I	I_3	Y	Q	C
1	$\psi_2\psi_3\psi_5$	Δ_b^{*0}	1/2	1/2	0	0	0
	$\psi_2\psi_4\psi_5$	Σ_b^{*0}	1/2	1/2	1	0	1
	$\psi_3\psi_3\psi_5$	Σ_b^{*-}	0	0	− 1	− 1	0
	$\psi_3\psi_4\psi_5$	Ξ_b	0	0	0	− 1	1
	$\psi_4\psi_4\psi_5$	Ξ_b^{*0}	0	0	1	− 1	2
2	$\psi_1\psi_5\psi_5$	Σ_{bb}^-	1/2	− 1/2	1	− 1	0
	$\psi_2\psi_5\psi_5$	Σ_{bb}^0	1/2	1/2	1	0	0
	$\psi_3\psi_5\psi_5$	Ξ_{bb}^-	0	0	0	− 1	0
	$\psi_4\psi_5\psi_5$	Ξ^{*-}	0	0	1	− 1	1
3	$\psi_5\psi_5\psi_5$	Ξ_{bbb}^-	0	0	1	− 1	0

TABLE IIb

Quantum numbers of 1/2 *baryons*

B	State	Particle Baryons	I	I_3	Y	Q	C
	$\psi_1\psi_1\psi_2$	N	1/2	− 1/2	1	0	0
	$\psi_1\psi_1\psi_3$	Σ_0^{*0}	1	− 1	0	− 1	0
	$\psi_1\psi_1\psi_4$	Δ_c	1	− 1	1	− 1	1
	$\psi_1\psi_2\psi_3$	Σ^0	1	0	0	0	0
	$(\psi_1\psi_2)_A\psi_3$	Λ^0	0	0	0	0	0
	$(\psi_1\psi_2)_S\psi_4$	C_1^+	1	0	1	0	1
	$(\psi_1\psi_2)_A\psi_4$	C_0^+	0	0	1	0	1
	$(\psi_1\psi_3)_S\psi_4$	S^+	1/2	− 1/2	0	− 1	1
	$(\psi_1\psi_3)_A\psi_4$	A^+	1/2	− 1/2	1	− 1	1
	$\psi_1\psi_2\psi_2$	P	1/2	1/2	1	1	0
	$\psi_1\psi_3\psi_3$	Ξ^-	1/2	− 1/2	− 1	− 1	0
	$\psi_1\psi_4\psi_4$	X_{LL}^{++}	1/2	− 1/2	1	− 1	2
	$\psi_2\psi_2\psi_3$	Σ^-	1	1	0	1	0
	$\psi_2\psi_2\psi_4$	C_1^0	1	1	1	1	1

(*continued*)

TABLE II(*b*) (*continued*)

B	State	Particle Brayons	I	I_3	Y	Q	C
	$(\psi_2\psi_3)_S\psi_4$	S^0	1/2	1/2	0	0	1
	$(\psi_2\phi_3)_A\psi_4$	A^0	1/2	1/2	0	0	1
	$\psi_3\psi_4\psi_4$	X_S^+	0	0	0	− 1	2
	$\psi_2\psi_3\psi_3$	Ξ^0	1/2	1/2	− 1	0	0
	$\psi_2\psi_4\psi_4$	X_a^+	1/2	1/2	1	0	2
	$\psi_3\psi_3\psi_4$	T^0	0	0	− 1	− 1	1
	$\psi_1\psi_1\psi_5$	C_2^{++}	1	− 1	1	− 1	0
	$(\psi_1\psi_2)_S\psi_5$	C_2^+	1	0	1	0	0
	$(\psi_1\psi_2)_A\psi_5$	C_0^{++}	0	0	1	0	0
	$(\psi_1\psi_4)_S\psi_5$	T_2^+	1/2	− 1/2	1	− 1	1
	$(\psi_1\psi_4)_A\psi_5$	T_2^{++}	1/2	− 1/2	1	− 1	1
	$(\psi_2\psi_2)\psi_5$	C_2^0	1	1	1	1	0
	$(\psi_2\psi_3)_S\psi_5$	S_1^0	1/2	1/2	0	0	0
	$(\psi_2\psi_3)_A\psi_5$	S_1^+	1/2	1/2	0	0	0
	$(\psi_2\psi_4)_S\psi_5$	X_b^+	1/2	1/2	1	0	1
1	$(\psi_2\psi_4)_A\psi_5$	X_b^{++}	1/2	1/2	1	0	1
	$(\psi_1\psi_3)_S\psi_5$	X_b^{+*}	1/2	− 1/2	0	− 1	0
	$(\psi_1\psi_3)_A\psi_5$	X_b^{*0}	1/2	− 1/2	0	− 1	0
	$\psi_3\psi_3\psi_5$	T_2^0	0	0	− 1	− 1	0
	$(\psi_3\psi_4)_S\psi_5$	T_2^+	0	0	0	− 1	1
	$(\psi_3\psi_4)_A\psi_5$	T_2^{++}	0	0	0	− 1	1
	$\psi_4\psi_4\psi_5$	$A_1{}^+$	0	0	1	− 1	2
	$\psi_1\psi_5\psi_5$	Z_u^{++}	1/2	− 1/2	1	− 1	0
	$\psi_2\psi_5\psi_5$	Z_d^{++}	1/2	1/2	1	0	0
2	$\psi_3\psi_5\psi_5$	Z_S^{++}	0	0	0	− 1	0
	$\psi_4\psi_5\psi_5$	Z_C^{++}	0	0	1	− 1	1

For baryons, there are 35-plet symmetric with $J = 3/2^+$, 40-plet mixed $J = 1/2^+$ states which also fit in the above representations.
For $3/2^+$ symmetric baryons we have ;

$$35 = \underset{B=0}{20} + \underset{B=1}{10} + \underset{B=2}{4} + \underset{B=3}{1}$$

For $1/2^+$ baryons

$$40 = \underset{B=0}{20} + \underset{B=1}{16} + \underset{B=2}{4}$$

The quantum numbers of these mesons and baryons are shown in the Tables I, II*a*, *b*.

Conclusion

In the present paper and formalism, the desired decomposition of the direct products is obtained in the form of direct sum of all the terms on the right hand side of eqn. (2). Thus we see that all the familiar results of SU(5) are obtained. This method, based on irreducible, tensors is both simpler in its foundations and more efficacious in its applications than other methods based on Young tableaux and weight diagrams etc., which the present authors have established in identifying the hadrons including the heavier ψ and τ particles and their families (Weiskopf, 1977).

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